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THE PHYSICAL REVIEW

*A JOURNAL OF EXPERIMENTAL AND
THEORETICAL PHYSICS*

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THE PHYSICAL REVIEW.

LIGHT PRODUCED BY THE RECOMBINATION OF IONS.

By C. D. CHILD.

CERTAIN phenomena connected with the luminosity of the vapor rising from the mercury arc indicate that this light is produced by the recombination of ions, and that the vibrations of the atoms producing light are not started when the atoms are broken into ions, but when the ions recombine into atoms.¹ Since this conclusion is contrary to that held by some who have investigated the problem of the production of light from a different standpoint, it is desirable to modify in some way the experiment to which reference has been made. In that experiment the current was continuous and the vapor was examined after it had moved away from the region where the current was flowing. The experiment can be modified by stopping the current and examining the vapor in the place where the current has been flowing; that is by examining the light from an alternating current arc at the moment when no current is flowing. If the light is due to recombination it will in either case continue for a time after the current has ceased flowing through the vapor. If the light is due to ionization, it will cease when there is no ionization and this occurs when no current passes through the vapor.

An examination of the light at different phases has already been made by Fleming and Petavel² and they found that at the moment when there is no current there is a very appreciable amount of light given off not only by the carbons, but also by the gas between the carbons. They also found that the minimum of light occurs after the time when the current is zero. It occurs, in fact, at approximately the same time as the minimum of conductivity, that is, the time when there is the least number of ions. These experiments were made before the electron

¹ Phil. Mag. (6), 26, 906; 1913, and (6), 31, 139; 1916.

² Phil. Mag. (5), 41, 339; 1896.

theory of discharge was developed and no attempt was made by Fleming and Petavel to explain their results. It is evident, however, that the phenomena observed by them are what must occur, if the light is due to recombination. It is also possible to suggest other explanations when carbon electrodes are used in air. For example, the light which appears when there is no current may be due to the oxidation of the hot gases between the electrodes. It, therefore, seemed desirable to repeat their experiments using mercury terminals in a vacuum instead of carbon electrodes in air.

Description of Apparatus.—The apparatus used was essentially the same as that employed by Fleming and Petavel and a more complete description can be found in their article. An outline of the apparatus is given in Fig. 1. *A* is an arc enclosed in a box which shuts off as much

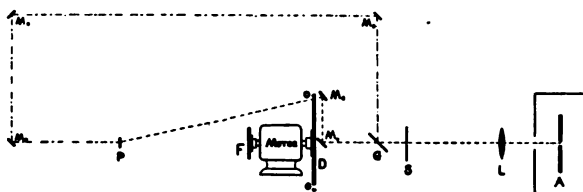


Fig. 1.

of the light as possible. The light from the arc passes through an opening to the lens *L*, which produces the image of the arc on the screen *S*. In this there is an opening which allows the light from the gaseous part of the arc to pass through, but screens off that from the carbons. From this point the light passes to two mirrors, *M*₁ and *M*₂, which are fastened upon an arm that can be rotated about the axis of a synchronous motor. *D* is a disk that is rotated by the motor and in this are two openings, *O* and *O*', through which the light passes from *M*₂ to the Lummer-Brodhun photometer at *P*. Since the motor is in step with the alternations of the arc, this allows the light from some particular phase of the alternation to pass to the photometer and by rotating the arm which carries *M*₁ and *M*₂ the light from different phases can be examined.

For a source of comparison an unsilvered glass is placed at *G*. This reflects a small part of the light to *M*₃ and from there it is reflected to *M*₄, *M*₅, and to *P*. In this way the light from the particular phase which is being studied is compared with the total light given off by the gaseous part of the arc, and errors due to changes in the current or due to changes in the position of the arc are eliminated. Since the light coming from *M*₂ to *P* makes an angle with the photometer bar, the photometer must remain stationary. The mirrors *M*₄ and *M*₅ were, therefore, placed on a

movable carrier and the intensity of the light at the photometer was varied by moving this carrier.

In the experiments on the arc in air the distance from A to L was 22 cm.; from L to S , 128 cm.; from S to M_1 , 34 cm.; from M_1 to M_2 , 17 cm.; from M_2 to P , 125 cm. D was 38 cm. in diameter. The opening O was variable and will be given in connection with the different experiments. The distance from S to the middle of G was 12 cm.; from G to M_3 , 50 cm.; and from M_4 to M_5 , 50 cm.; while the distances from M_3 to M_4 and from M_5 to P were variable.

There was also attached to the shaft of the motor a contact maker F for obtaining the instantaneous values of the current and potential difference of the arc. The rotating part consisted of a fiber wheel 12 cm. in diameter. At the edge of this a piece of copper 1 mm. wide was embedded. This was connected to a ring collector. The stationary brush which made contact with this was 2 mm. wide and was made of woven copper wire. This brush was placed on an arm which could be rotated about the axis of the motor.

The potential was determined by balancing the potential difference existing between the terminals of the arc at the instant that contact was made against a potential difference which was measured by a Weston voltmeter. The balance was determined by a D'Arsonval galvanometer. The current at any instant was determined by passing it through a known non-inductive resistance and balancing the potential difference between the terminals of the wire against a known potential difference.

The motor used was one made by the General Electric Company. It is wound as an ordinary three-phase induction motor with squirrel cage winding and in starting operates as an induction motor. When up to speed it pulls itself into step and runs as a synchronous motor as long as the load is small. This tendency to run as a synchronous motor is aided by a lack of circular uniformity in the arrangement of the iron of the rotor. When up to speed the rotor behaves much as a four-pole permanent magnet would. This has the disadvantage that it does not always drop into step at the same point. At times it drops into step at one point of the phase and at other times at a point 90 degrees from this. In other respects the motor behaved admirably. There was little trouble from hunting. There was a slight fluctuation in speed, but as far as could be determined this was due to an irregularity in the speed of the generator at the power house. The motor made 1,800 revolutions per min. being on a sixty-cycle circuit. The arc was placed between two of the three wires leading to the motor.

The Carbon Arc.—As a test of the apparatus it was first used to repeat

the observations of Fleming and Petavel on the light from the gaseous part of the arc between carbons. Cored carbons 13 mm. in diameter were used. The arc was 7 mm. long and the current was 12 amperes. The opening at *O* was 20 mm. by 16 mm., the former distance being in the direction of the diameter of the wheel. A stationary screen was placed in front of the mirror *M* having an opening of the same size as that on the rotating wheel. The observations which were made are given in the following table. Column 1 gives the phase angle of the alternating current between an arbitrary starting point and the observed point. Column 2 gives the potential difference in volts corresponding with this position of the brush, column 3 gives the current in amperes and column 4 the relative candle power as computed from the setting of the photometer.

TABLE I.

Phase Angle.	Potential Difference.	Current.	Intensity of Light.	Phase Angle.	Potential Difference.	Current.	Intensity of Light.
0	-17	-2.9		192	0		
10	-5	.0	.3	200	-15	-2.0	.25
32	0			210	-24	-3.1	.3
20	14	2.0	.3	220	-31	-6.6	.3
30	24	3.7	.25	230	-35	-9.5	.7
40	31	6.4	.3	240	-37	-11.2	1.4
50	36	9.2	.8	250	-40	-13.4	2.4
60	39	11.9	1.6	260	-45	-15.0	2.9
70	41	13.6	2.4	270	-54	-16.5	3.2
80	48	15.4	3.1	280	-56	-17.4	3.3
90	55	16.9	3.2	290	-58	-16.9	3.3
100	57	17.2	3.3	300	-56	-15.6	3.2
110	58	16.9	3.3	310	-54	-14.2	3.2
120	57	16.0	3.3	320	-52	-12.5	2.7
130	55	13.9	3.2	330	-48	-9.9	1.8
140	52	12.5	3.2	340	-38	-7.9	1.3
150	45	11.0	2.2	350	-30	-5.7	.7
160	36	8.1	1.3	360	-16	-3.7	.4
170	25	5.7	.7	370	0	.0	.3
180	15	3.1	.4	380	12	1.5	.3
190		.0	.3				

These curves are plotted in Fig. 2. They are very similar to those shown by Fleming and Petavel (p. 338). Both sets of curves show that the minimum of light occurs after the time of zero current. The time of zero current can be determined with much accuracy, and while it is not possible to determine the time of minimum intensity of light with the same degree of accuracy, there is no doubt but that the time between these two occurrences is approximately 1/20 of a complete cycle, that is, 1/1200 sec. As has already been stated, we can not consider

that this is a proof that the light is due to recombination since the phenomenon is complicated by the possibility that some ionization is caused by the combustion of the hot gases.

The ratio between the candle power when the light is at a minimum and that when it is at a maximum is approximately 1/10. This is the same as that found by Fleming and Petavel. It has been suggested

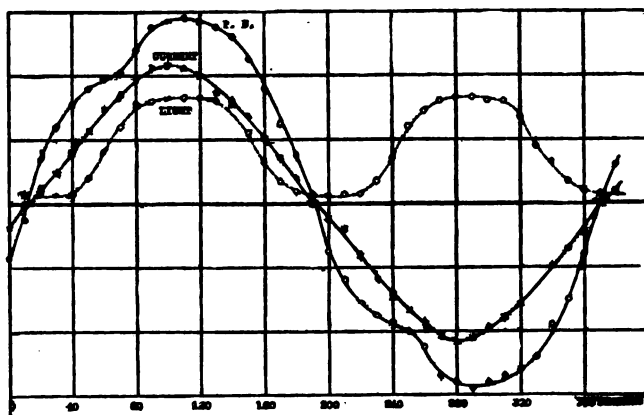


Fig. 2.

that the light which appears to exist at the time of the minimum is due either to stray light or to the fact that the opening at *O* has a very appreciable width. Both of these explanations will be discussed in connection with the flaming arc, where the amount of light at the time of the minimum is much larger than with the carbon arc.

The Flaming Arc.—A study of the flaming arc can give us no proof of the way in which light is produced, since the light at the time of no current may here also be due to oxidation of the hot gases, but, as far as the writer is aware, no data regarding the relation between the current phase of the flaming arc and the intensity of the light has been published. It, therefore, seemed desirable to make such an investigation. The carbons used in this investigation were No. 301, made by Siemens and Halske. These were 8 mm. in diameter. The arc was 9 mm. long and the current was 12 amperes. The opening at *O* was 16.5 mm. in width.

It is difficult to make correct settings of the photometer because of the changes in the color of the light. When the light is at a minimum it is much yellower than the average light with which it is compared. During the remainder of the cycle it is bluer than the average. The observations, however, are sufficiently accurate to give a correct idea of the general relation existing between the current and the intensity of the light. These observations are given in Table II.

TABLE II.

Phase Angle.	Potential Difference.	Current.	Intensity of Light.	Phase Angle.	Potential Difference.	Current.	Intensity of Light.
0	-20	-3.3	100	190			73
10			77	192	0	0	
14	0	0		200	-15	- 1.1	45
20	17		50	210	-32	- 1.3	
30	37	1.1		214			29
34			29	220	-48	- 2.6	42
40	48	2.6	38	230	-49	- 6.6	69
50	49	6.1	58	240	-44	- 9.0	83
60	45	9.7	81	250	-42	-12.5	
70	41	12.3		260	-42	-14.7	90
80	41	14.9	89	270	-43	-15.6	
90	42	16.2		280	-42	-17.6	94
100	40	17.6	92	290	-40	-17.6	
110	40	17.6		300	-40	-16.7	98
120	40	16.9	95	310	-38	-15.8	
130	36	15.0		320	-37	-13.4	105
140	35	14.5	100	330	-34	-11.9	
150	32	12.1		340	-31	- 9.7	110
160	30	9.9	104	350	-26	- 6.2	110
170	28	6.6	104	360	-20	- 3.3	100
180	20	2.6	94	370			77

These data are plotted in Fig. 3. As with the arc between cored carbons we find that the minimum of light does not occur at the time of zero current but about 1/1200 sec. after the current is zero.



Fig. 3.

Accuracy of Measurement for Minimum Intensity.—In this case the minimum amount of light is approximately $\frac{1}{2}$ of the maximum. No appreciable amount of this is due to stray light, for when the opening either at *O* or at the screen in front of this was closed no light that could be detected passed to the photometer.

There is, however, the possibility that the apparent continuance of light is due to the fact that the opening at *O* has a very appreciable width. It does not give instantaneous values of the light, but values for a finite length of time. For example the width of *O* when the preceding measurements were made was 16.5 mm. The width of the opening in front of this was the same. The center of *O* was 17 cm. from the center of the disk *D*. The time elapsing from the instant when light begins to pass through the opening until it entirely ceases is $\frac{2 \times 16.5}{2\pi \times 170}$ or .031 of the time of one complete revolution of the disk. Since there are two cycles for each revolution of the disk, this would be .062 of a cycle. This may be sufficient to give an apparent value different from the real value.

In order to test this the candle power at the time of minimum was examined with different openings at *O*. If the light passing through the opening were to remain the same during the time that one opening passed the other, the amount of light would vary as the product of the widths of the two openings. If the light is a minimum at the instant the two overlapped, the intensity of the light will decrease when the opening is made smaller more rapidly than the width of the opening. To test this observations of the candle power at the point of minimum intensity of light were made with different sizes of openings in the revolving disk at *O*. These are given in Table III. Column 1 gives the size of opening in the revolving disk. Column 2 gives numbers proportional to the intensity of the light passing this opening as nearly as could be determined by taking the average of several settings. Column 3 gives the ratios between the numbers in the two preceding columns.

TABLE III.

Size of Opening in Disk.	Intensity of Light.	Ratios.
2.24	5.60	2.50
1.58	3.48	2.54
.68	1.75	2.58

The intensity of the light decreases but slightly more rapidly than the width of the opening. This indicates that the real minimum is not more than a few per cent. below that given in Table II.

Spectrum at Different Phases.—As has been stated the color of the light at the time of minimum intensity is yellower than that shown at the time of maximum intensity. On examining the spectrum of the light at different phases it was found that the only spectrum shown with the apparatus used was the calcium spectrum and that the one shown at the

time of minimum intensity was that which is known as the flame spectrum, while at the time of maximum intensity the arc spectrum was shown. This phenomenon has already been investigated by Crew and Spence¹ and by Puccianti² and no further knowledge concerning it has been gained by this investigation. It is evident that this fact is quite in harmony with the assumption that the luminosity at the time of no current is due to the oxidation of the hot gases and that it proves nothing concerning the mechanism by which light is produced.

The Mercury Arc.—As is well known it is very difficult to maintain an alternating current arc between mercury terminals. An attempt was made by the writer to do so by using two transformers in series, one giving 4,000 volts and one 2,000 volts. With a slight amount of residual air in the tube this gave a current of several amperes, but with the residual air the sides were soon covered with a film of mercury and became opaque. Without the air it was impossible to maintain an arc. This was true whatever the size of the tubes or the length of the arc, and the attempt to produce the arc in this way was finally abandoned.

An attempt was then made to use an alternating-current arc between a direct-current arc and a third terminal, as shown in Fig. 4. The direct

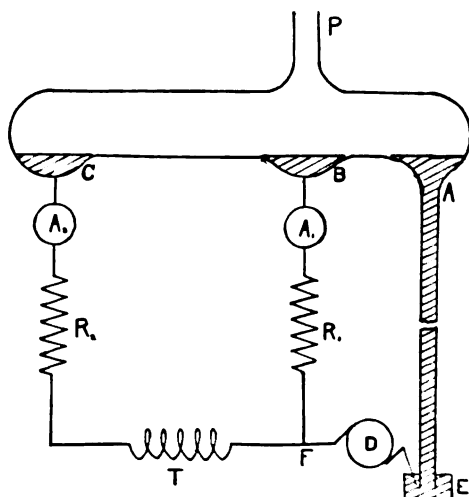


Fig. 4.

current arc was maintained between *A* and *B* by means of the dynamo *D*. A transformer *T* giving 110 volts was inserted between *F* and the terminal *C*. A vertical tube extended from *A* to the cup *E*. The arc was started by raising the cup *E* until the mercury in *A* made contact with that in *B*.

¹ *Astrophys. Jour.*, 22, 190; 1905

² *Physik. Z. S.*, 8, 470; 1907.

By this means a unidirectional pulsating current was easily maintained between *C* and *A*. A cold terminal in the neighborhood of a mercury arc can easily become the positive terminal of a second arc, but can not become the negative terminal. However, there is no objection to this kind of an arc for the work in hand, for all that is needed is a current which passes through zero value at regular intervals.

Different combinations of this arrangement were tried by reversing the polarity of *A* and *B* and by changing the position of the resistances, but the arrangement shown in the diagram gave the most uniform and consistent results. The current to *B* was regulated by means of the resistance R_1 and that to *C* by means of R_2 . The potential difference between *A* and *C* was very nearly independent of the current flowing but it is dependent on the amount of vapor in the tube. As the tube becomes heated the density of vapor becomes greater and the potential difference between the terminals becomes greater.

The tube *P* was connected all the time with the pump, but in spite of this the pressure in the tube varied. Moreover the intensity of the light depends both on the amount of current and on the potential difference. The only way to regulate this is to stop the alternating current arc for a time and to allow the vapor to condense. In making observations the arc was run until the potential difference reached a definite value, the observations were then made and the current stopped for a few minutes allowing the tube to cool. This process was repeated for each observation. By this means data could be obtained at a definite current and potential difference.

The pressure of the vapor was determined by measuring the change in height of the mercury in *A*. As the pressure increased this column was pushed down and the amount of change was measured by a cathetometer reading to 1/20 mm.

With the mercury arc the light from the terminals can be easily avoided by taking that which comes from the vapor midway between the terminals. The screen *S* in Fig. 1 was, therefore, omitted and the arc focused on *P* instead of on *S*. With this arrangement the distance from *A* to *L* was 30 cm. and LM_1 was 22 cm., the other distances remaining approximately the same as before. The inside diameter of the tube was 2.4 cm. The distance from *A* to *B* was 9 cm. and from *B* to *C* was 18 cm.

The data obtained concerning the mercury arc is given in Table IV., the arrangement being as in the preceding tables. During these readings the current as measured by the ammeter A_2 was 10 amperes and the voltage between *C* and *A* was 45 volts. The pressure of the vapor was 1.4 mm.

TABLE IV.

Phase Angle.	Potential Difference.	Current.	Intensity of Light.	Phase Angle.	Potential Difference.	Current.	Intensity of Light.
10	18.4			140			198
20	45.5			150	25.2	17.1	
30	80.5			160			198
40	93.0	1.65		170	25.2	15.3	
50	63.0	4.82	50	180			190
60	33.0	8.3	90	190	24.2	12.6	
70	27.2	11.3	142	200			172
80			160	210	22.3	7.6	
90	25.2	14.6		220	21.3	5.7	114
100			181	230	19.3	3.6	80
110	25.2	17.1		240	17.5	1.8	53
120			194	250	5.8	.43	25
130	25.2	18.0					

Fig. 5 gives the plot for these data. It will be observed that the potential difference and the current continue for more than half of a complete period. This is due to the manner in which the two circuits are connected. When there is no E.M.F. due to the transformer there

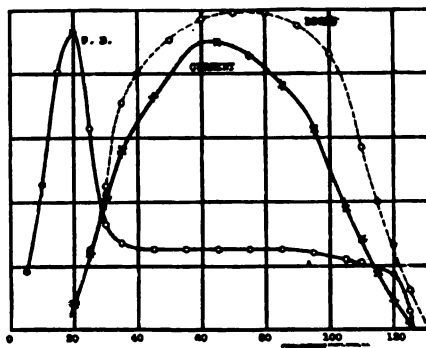


Fig. 5.

is still a potential difference between C and B due to the drop in potential around R_1 . From F there is a divided circuit, part of the current flowing through T , R_2 , and C to B and part flowing through R_1 . The current will continue to flow from F through C until C is approximately at the same potential as B , that is until the E.M.F. of the transformer is equal to the drop in potential through R and in the opposite direction.

The curve for the potential difference shows the usual peculiarities for the alternating current arc in an exaggerated form. The voltage rises to a relatively high value before the current begins to flow. The high voltage is probably required in order to establish the conditions

at the anode essential for the arc form of discharge. A comparatively small number of electrons can pass from a mercury arc to a third terminal without any glow or luminous point on the electrode, but when the discharge becomes larger one or more luminous points are produced by the impact of the negative ions. From these points positive ions move toward the arc, neutralizing the electro-static effect of the negative ions, and the drop in potential near the terminal becomes less. In the case here studied the luminous points must be produced at the beginning of each period. Until they are established the voltage continues to increase and energy is expended in heating the surface of the mercury. When the hot points are once produced less voltage is required and we have a decrease in voltage, as shown in the voltage curve.

Determination of Time of Minimum Intensity.—It is evident from the curves shown in Fig. 5 that the light does not cease at the instant the current ceases, but the exact time when the light vanished could not be determined by means of the photometer. A better arrangement for this purpose is obtained by placing the tube as near to the rotating plate as possible and looking directly at the tube with the rotating plate between the arc and the eye as shown in Fig. 6. As the opening passes the tube one obtains a view of the arc showing the progressive changes during a fraction of a cycle. The width of the opening was decreased to 4 mm., so as to secure as nearly as instantaneous a view as possible.

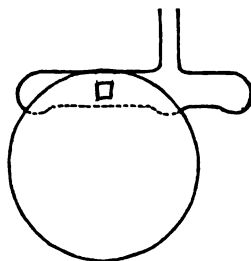


Fig. 6.

With this arrangement it was found that there was a time of minimum intensity occurring shortly after the current ceased, followed by a period of slightly greater luminosity. This gradually died out, being visible, however, during a time equivalent to 34 electrical degrees. The minimum was more pronounced when the pressure of the vapor was small. When the pressure became 1.7 mm. it was no longer possible to detect a point of minimum intensity. A photograph was taken showing this point of minimum luminosity, but it was not found possible to secure one of sufficient value to reproduce. There was a slight irregularity in the speed of the motor of which mention has already been made. This blurred the photograph when a time exposure was taken and with a very short exposure the photograph was too faint to be of value.

This phenomenon seemed to indicate that with the reversal of the E.M.F. a slight current flowed in the direction opposite to that of the main current, that is, flowing from *A* to *C*. A more careful examination with the apparatus previously used for finding the amount of current

showed that such a current existed. When the pressure was low the maximum value of this current was in the neighborhood of .05 ampere. When the pressure increased it became too small to be detected with the apparatus used.

This reverse current is, no doubt, to be explained as follows: Before the ions produced by the flow of current from *B* to *C* have all recombined an electric force is started in the opposite direction. This force is sufficient to cause the electrons to ionize by their impact and a resulting current to flow. The force is not, however, sufficiently high to produce ionization by the impact of positive ions on the cathode *C*. As a consequence no ions are formed to take the place of those which move toward *B*, the field is soon cleared of electrons, and the current ceases.

This action takes place in a small fraction of one cycle, so that we have a momentary current of small magnitude. As the pressure of the vapor increases the mean free path of the electrons becomes less so that there is less chance to ionize the gas. Moreover, the velocity of both the positive and negative ions becomes less. For both of these reasons the reverse current becomes less, as the pressure of the gas becomes greater.

The intensity of the light when at the minimum was much too small to be determined with any apparatus which was at hand. It was, however, quite possible to obtain the time at which the minimum intensity occurred. This was approximately 12 electrical degrees or 1/1800 sec. after the current became zero.

A similar set of readings was taken when the vapor pressure in the tube was 3.23 mm. instead of 1.4 mm. and the current was 8 amperes instead of 10. In this case the potential difference between *A* and *C* as recorded by the A. C. voltmeter was 55 volts instead of 45 volts. The curves found at this pressure were similar to those shown in Fig. 4. The potential difference, however, rose to 104 volts before the current began to flow and was 32 volts when the current was a maximum. The current continued through only 184 degrees instead of 230.

Spectrum at Different Phases.—The light given out at the time of zero current was slightly more yellow than the light of the normal mercury arc. None of the lines of the spectrum disappeared, but the relative intensity changed somewhat. The green line (5,460 microns) and others belonging to the same sub-series being less prominent at the time of zero current. This corresponds to the change observed in the spectrum of the vapor rising from the mercury arc.¹ In both cases the green line is less prominent in the vapor through which no current is passing than it is in the arc itself.

¹ PHYS. REV. (2), 4, 388; 1914.

An attempt was made to study the conduction of the vapor after the main current of the arc had ceased by means of exploring electrodes, but this did not prove successful. In the first place the conductivity of a highly conducting vapor can not be determined with any degree of accuracy by exploring electrodes. In order to remove all of the ions from a region where there are many ions, it is necessary to have a strong field and such a field produces new ions by impact. In the second place, as has been shown, the current does not remain zero for any appreciable length of time, and a knowledge of the conductivity at the time that a current was being started in the opposite direction would not give us any knowledge of the rate at which the ions recombine.

Conclusion.—These results are entirely in harmony with the view that part at least of the light coming from the mercury arc is due to recombination of the ions. The time of maximum intensity occurs after the time of maximum current and the light continues after the current ceases, as is required by such a view. Moreover, if the light is due to recombination, the intensity of light must be greater, the greater the number of ions present. The number of ions present is very approximately proportional to the conductivity of the gas and this in turn is proportional to the current divided by the potential difference, and we find that this relation does in fact exist. Through a large part of the period the voltage is nearly constant and the luminosity changes as the current. At the end of the period both the current and the voltage decrease, but the light is roughly proportional to the ratio between them.

On the other hand these facts are not in harmony with the view that the light is caused by a vibration produced when the atoms are ionized, unless we assume that the atoms continue to vibrate for a comparatively long time after the impact is made which starts the vibration. In the work here described the light continues for at least $1/30$ of a cycle after the current ceases, that is for $1/1800$ sec. The difficulty of explaining this continuance of the light has already been shown by the author.¹ Strutt² also mentions this difficulty, but expresses the belief that it can eventually be explained.

Summary.—The work of Fleming and Petavel on the intensity of the light from an alternating current carbon arc at different phases was repeated and a similar study was made of the flaming arc and mercury arc in a vacuum. In every case the intensity of the light passes through a minimum without entirely vanishing and this minimum occurs after the current passes through the zero value. With the mercury arc the

¹ Phil. Mag. (6), 26, p. 909; 1913.

² Proc. Roy. Soc., A, 91, p. 92; 1914.

minimum intensity occurs approximately $1/1800$ sec. after the current becomes zero. In this case the continuance of the light indicates that the light is due to the recombination of the positive and negative ions, since there is here no chance for oxidation or other known chemical action.

COLGATE UNIVERSITY,

August, 1916.

COUNTER ELECTROMOTIVE FORCE IN THE ALUMINUM
RECTIFIER.

BY ALBERT LEWIS FITCH.

I. INTRODUCTION.

THE anomalous action of aluminum in the electrolytic cell was first discovered by Wheatstone in 1855. Soon after this, Buff found that an electrolytic cell one electrode of which was aluminum would rectify the alternating current. Among the other men who have been interested in this cell may be mentioned Ducretet,¹ Hutin and Leblanc,² Montpellier,³ Nodon,⁴ Guthe,⁵ Greene,⁶ and Schulze.⁷ The latter has perhaps done the largest amount of work of any. His articles have appeared from time to time in a number of magazines.

The earlier experimenters with this cell confined themselves to the study of aluminum but later investigation⁷ has shown that many other metals possess this same property to a greater or less degree. Among these may be mentioned iron, nickel, cobalt, magnesium, cadmium, tin, bismuth, zirconium, tantalum, etc.

A great many electrolytes may be used in the rectifier. The most commonly used are the alums, phosphates, and carbonates; however Greatz and Pollak⁸ have shown that any electrolyte which will liberate oxygen on electrolysis may be used more or less satisfactorily.

It has been found that the ability of the cell to rectify alternating current depends upon the current density at the aluminum anode,⁹ the inductance and resistance of the circuit,¹⁰ and its temperature.¹¹ The cell works best when the current density is high and the inductance, resistance, and temperature are low.

¹ Comptes Rendus, Vol. 80, p. 280.

² French Patents, No. 215945.

³ Electrician, Vol. 22, p. 17.

⁴ Comptes Rendus, Vol. 136, p. 445.

⁵ PHYS. REV., Vol. 15, p. 327.

⁶ PHYS. REV., Vol. 3, series 2, p. 264.

⁷ Zeitschr. Elektrochem., Vol. 14, p. 333.

⁸ Elektrotechnische Zeitschr., Vol. 25, p. 359.

⁹ Elektrotechn. Zeitschr., Vol. 21, p. 913.

¹⁰ Ann. der Physik, Vol. 39, p. 976.

¹¹ Zeitschr. Elektrochem., Vol. 14, p. 333.

Two prominent theories have been advanced to explain the action of this cell. The earlier theory, known as *the solid film theory*, ascribes this action to the electrolytic deposition and decomposition of a solid film of some oxide or hydroxide of aluminum on the aluminum anode. The deposition takes place while the current flows in at the aluminum and, being a high resistance material, the film soon grows to a thickness which shuts off the current in that direction. The decomposition takes place when the current is in the opposite direction and permits the current to flow unimpeded from the electrolyte to the electrode.

In 1902 Guthe¹ first gave us the later theory, known as *the gas film theory*. This theory ascribes the action to a film of oxygen gas which is spread over the solid layer. The free electrons of the metal are forced through the gas film by the very high potential gradient with very little difficulty, when the aluminum is the cathode, but when the current reverses, and the aluminum is the anode, no such thing can take place because there are no free electrons in the electrolyte. Instead, the current must be carried through the film by the ions of the electrolyte and these being relatively large as compared to the electrons are with difficulty forced through.

It has been known for a great many years that the aluminum cell acts to a certain extent like a condenser. Schulze² states that a cell 40 × 40 × 40 cm. with both plates of aluminum had a capacity of 5,000 mfd. on 160 volts alternating current of a frequency of 50 cycles per second. It was possible, he states, to take an alternating current of 250 amperes through this cell. But one must not go too far in likening this cell to an ordinary leaking condenser as Greene³ has shown.

This investigation was undertaken to determine if a more careful study of the counter electromotive force, which is produced when current enters at the aluminum, would throw some light on the action of the cell as a condenser and also on the theories advanced.

The cell used was composed of a lead plate with an area of approximately 90 sq. cm. and an aluminum wire .258 cm. in diameter and 10 cm. in length, immersed in a saturated solution of sodium phosphate. The aluminum wire was tested and found to contain .27 per cent. iron and some silicon in the form of silicates. It is about 99 per cent. pure. The lead is the same grade as that used in the chemistry department in qualitative experiments and is believed to be as pure as the aluminum. Both electrodes are heavily coated with a good grade of sealing wax where

¹ PHYS. REV., Vol. 15, p. 327.

² Zeitschr. Elektrochem., Vol. 14, p. 333.

³ PHYS. REV., Vol. 3, series 2, p. 264.

they emerge from the solution to eliminate surface effects which were found to be present when the electrodes were not coated.

II. EXPERIMENTAL WORK.

As a preliminary study, a potentiometer method was devised for the measurement of the counter electromotive force. The cell was placed directly across the storage battery terminals for a time with the aluminum as anode and then the counter electromotive force, after a certain period of open circuit, was compared directly with the storage battery voltage. This period of open circuit was adjusted and measured by means of the disk described in the following method. Although this potentiometer method gives very accurate measurements of time of open circuit and of counter electromotive force, it is too slow to give the desired results.

Since the time for taking the readings by the former method could not be decreased, the oscillographic method was devised. This method enables one to take a complete set of readings in about one second. This eliminates to a large degree the objection to the former method and also enables one to get readings for much shorter periods of closed circuit.

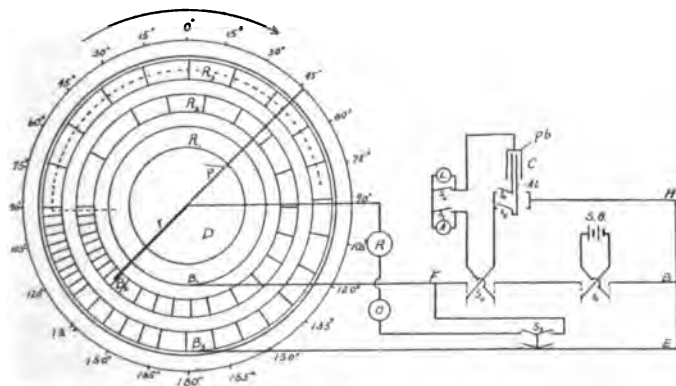


Fig. 1.

The diagram for the arrangement of apparatus is shown in Fig. 1. In this method the same disk was used as before with the same brush contacts and the same electrical connections on the disk. This disk is shown as D . It was designed for this work and made in the engineering shops of the University of Michigan. It is a solid, hard rubber disk of approximately 31 cm. radius. Firmly screwed to this disk are three concentric rings of brass. The inner ring R_1 has an inner radius of 12 cm. and an outer radius of 16 cm. It is made in one solid piece. The second ring R_2 has an inner radius of 19 cm. and an outer radius of 23 cm. It is divided into sectors ranging in magnitude from 5 degrees to

35 degrees. The outer ring R_2 has an inner radius of 26 cm. and an outer radius of 30 cm. It is divided into sectors ranging from 3.75 degrees to 28 degrees. Each sector on each ring is connected electrically to a binding post on the back of the disk. On the circumference of the disk is a steel tire. The disk is mounted on the shaft of a Roth motor, designed to run at 2,000 R.P.M. on 220 volts direct current. This disk is enclosed in a wooden box supported on sliding arms so that the box can be brought as near as desired to the disk and clamped fast. The front of the box holds the brush contacts B_1 , B_2 , B_3 made of spring steel with brushes of sheet copper. B_1 and B_3 are bolted directly to the box and allow the brushes fastened to them to bear on rings R_1 and R_2 . B_2 is a movable brush contact supported on a brass rod bent at right angles and passing through the box front at a point directly in line with the center of the motor shaft. A long brass pointer P is fastened to this brush holder by heavy brass nuts and plays over a protractor of 8 inches radius divided into quarter degrees. On the back of the disk, the sectors are connected as shown by the dotted line. In the back of the box face, a large screw was placed to stop the rotating arm r at the same place each time.

As before, it was arranged to have the cell placed directly across the storage battery when readings were not being taken. S_1 and S_2 make an eight-pole double-throw switch. S_4 and S_5 are double-pole double-throw switches. C is the rectifier. L is a lamp for resistance. A is a Weston ammeter with .01 ampere divisions. L and A may be cut out of the circuit by switches S_6 and S_7 . R is a Leeds & Northrup resistance box with a total resistance of 1,111 ohms and O is the vibrator of the Siemens & Halske Oscillograph. The oscillographic chamber was filled with oil to produce the proper damping effect. The cylinder of the oscillograph is connected directly to the motor shaft which rotates the disk. This makes the cylinder and disk rotate in exact synchronism.

The motor is connected with its field over the 220-volt direct current power line and its armature over the storage battery. By this arrangement, any speed up to 2,000 R.P.M. is attainable by varying the number of cells of the storage battery.

ELECTRICAL CONNECTIONS.

It is evident from the figure that there are two paths for the current from the storage battery S.B. to the cell C if S_3 is thrown to the right. It can flow from S.B. to B and H to the cell through either of the switches S_4 or S_5 , or it can flow by the path B , E , S_3 , F , S_2 to C through either of the switches S_4 or S_5 thrown to the left. This arrangement enables one

to use two anodes of different metals in the same cell and still have them electrically independent of each other. However in this work but the one switch was used and the one anode. If S_3 , S_4 , and S_5 are thrown to the left, current can flow from S.B. to B , E , S_3 , B_3 , along the wire on the back of the disk to R_1 , and B_1 , then to S_2 , through the cell and back to the storage battery, while the brush B_3 is on the sectors connected by the wire. When the last connected sector has passed B_3 , the cell is on open circuit until the sector on R_2 connected to R_1 passes under B_2 . Current then flows from S.B. to B , E , S_3 , O , R , r , B_2 , through the connecting wire to R_1 , and B_1 , then to S_2 and through the cell to the storage battery. It is to be noticed that no matter by which path the current comes to the cell it is in the same direction through it, *i. e.*, from the aluminum to the lead. If there is a counter electromotive force in the cell, the fall of potential over the oscillograph and its resistance must then be equal to the difference between the storage battery voltage and this counter electromotive force. This difference of potential will be registered on the photographic film on the cylinder of the oscillograph.

A flash is registered on the film whenever the small sector on R_2 , which is connected to R_1 and R_3 , passes under B_2 . Then if we rotate B_2 about its axis the resulting flashes on the film will register at exactly corresponding points of the film. Knowing the speed of the motor, we can compute the length of time between any two flashes by measuring the distance between them and comparing with the total film length. One mm. distance between flashes corresponds to 0.00037 sec. when the motor is running at 750 R.P.M. In most cases the flashes are very distinct so that one can measure accurately to 0.0002 sec. If still greater accuracy is desired one has but to increase the speed of the motor.

A calibration curve for the measurement of the voltage recorded by the flashes was made by sending current from a known voltage through the oscillograph and its accompanying resistance when the disk was in rotation. Since the deflections of the oscillograph are proportional to the current through it, they are also proportional to the difference of potential across its terminals. One has but to measure each ordinate and compare it with the deflection from the known source. The value of this ordinate subtracted from the applied voltage gives the counter electromotive force of the cell for that period of open circuit.

If one plots the counter electromotive force as ordinates and the time of open circuit as abscissæ, a curve is obtained which in general appearance resembles the curve of decay of electromotive force in a leaking condenser. But if the equation $V = V_0 e^{-ct}$ is assumed and the value of c computed for several points of the curve, it is found that c is not a

constant throughout the same curve nor the same value in successive curves for the same period of open circuit. It is evident then that this c depends on the time of open circuit and also on the time of closed circuit. But the above formula was derived on the assumption that c is a constant, it is therefore not proper merely to replace c by a function of the times of open and closed circuits. One must go back to initial conditions and construct a formula which will embrace the fact that the counter electromotive force depends both on the time of open circuit and on the time of closed circuit.

THEORY PROPOSED.

Let us assume with Guthe and Schulze that the aluminum anode becomes coated with a layer of some oxide or hydroxide of aluminum and that this solid layer is covered with a very thin film of oxygen gas whose thickness depends on the applied voltage and on the time of open circuit. If this layer of oxygen gas attains a definite thickness for each applied voltage, the thickness of the solid layer must increase with increasing time of closed circuit, for if one places a milliammeter in the circuit with the cell and storage battery, it will be seen that while the current does attain a very low value in a very short time after the circuit is closed, it does not remain at that value but is constantly dropping lower and lower. Even after a whole day on closed circuit, it can be seen that this current is growing smaller, although at a very slow rate. This solid layer is deposited electrolytically, and therefore its thickness must be proportional to the quantity of electricity passing through the cell, which in turn depends on the length of time the cell is left on closed circuit. The drop in counter electromotive force after a period of open circuit is due to the leakage of electricity through the combined thickness of solid layer and oxygen. It has been shown by Schulze that this oxygen film decreases in thickness with increasing time of open circuit. We have here then a clear case of a condenser with a double dielectric, both parts of which leak slightly and change in thickness, the solid layer increasing in thickness with the time of closed circuit and the gas film decreasing in thickness with the time of open circuit.

From the theory of condensers we have

$$i = \frac{-dq}{dt} = \frac{-d(CV)}{dt},$$

where i is the current leaking through the dielectrics, q is the quantity of electricity stored in the condenser, C is the capacity of the condenser at that instant and V is the difference of potential between the two plates. With a double dielectric, we have the formula

$$C = \frac{Sc'c''}{4\pi(c''d' + c'd'')},$$

in which S is the area of the plates, c' is the dielectric constant of the substance whose thickness is d' , c'' is the dielectric constant of the substance of thickness d'' , and C is the capacity. Dividing both numerator and denominator by c'' and placing $Sc'/4\pi = K$ we have

$$C = \frac{K}{d' + \frac{c'}{c''}d''}.$$

Now let d' , c' and d'' , c'' apply to the solid layer and gas film, respectively, and call the maximum thickness of the gas film d_0 . If we assume the rate of change in thickness of the gas film proportional to the thickness, we have

$$\frac{dd''}{dt} = -cd''; \frac{dd''}{d''} = -c dt; \text{integrating } [\log d'']_{d_0}^d = -c[t]_{t_0}^t.$$

Therefore $\log d''/d_0 = -ct$ or $d'' = d_0e^{-ct}$. By substitution of this in the equation for i , we get

$$i = \frac{-d}{dt} \frac{KV}{d' + he^{-ct}},$$

where $h = (c'/c'')d_0$.

But the current leaking through a high resistance is equal to the quotient of the electromotive force over the resistance divided by the resistance.

$$i = \frac{V}{r + se^{-ct}},$$

where r equals the resistance of the solid layer and s equals the maximum resistance of the gas film. Equating these two values for i we have

$$\frac{-d}{dt} \frac{KV}{d' + he^{-ct}} = \frac{V}{r + se^{-ct}}.$$

If we carry out the indicated differentiation, separate our variables and integrate from $V = V_0$ when $t = 0$ to $V = V$ when $t = t$ we obtain

$$V = V_0e^{-at}(A + Be^{-ct})(C + De^{-ct}),$$

where

$$A = \frac{r}{r+s}, \quad B = \frac{s}{r+s}, \quad C = \frac{d'}{d'+h}, \quad D = \frac{h}{d'+h}, \quad a = \frac{d'}{rK},$$

and

$$f = \frac{\frac{d'}{rc} - \frac{h}{cs}}{-K}.$$

Then

$$a = \frac{d'}{rK} = \frac{d'}{\frac{s'd'}{S} \frac{Sc'}{4\pi}} = \frac{4\pi}{s'c'}$$

and

$$f = \frac{\frac{d'}{rc} - \frac{h}{cs}}{-K} = \frac{\frac{d'}{s'd'c/S} - \frac{h}{s''c'ch/c'S}}{-\frac{Sc'}{4\pi}} = \frac{-4\pi}{s'c'c} + \frac{4\pi c''}{s''c(c')},$$

where s' and s'' are the specific resistances of the solid layer and gas layer, respectively. This s'' must be far greater than s' owing to the nature of the substances. It thus appears that a is very small and that f is negative and smaller even than a . Schulze has given us values for r and s , and also values for d and h . These values substituted in the equations above give $A = 1/51$, $B = 50/51$, $C = 100/101$, and $D = 1/101$. Now assuming that these values are at least of the correct order of magnitude, let us expand $(A + Be^{-ct})^f$ by the binomial theorem. We find

$$(Be^{-ct} + A)^f = B^f e^{-fct} + fB^{f-1} e^{-ct(f-1)} A + \frac{f(f-1)}{2} B^{f-2} e^{-2ct(f-2)} A^2 + \dots$$

Since f is so extremely small and A is also small, we do not introduce any great error if we neglect all terms after the second. Factoring we have $(Be^{-ct} + A)^f = e^{-fct}(B^f + fB^{f-1}e^{ct}A) = (X - Ye^{ct})$, where $X = B^f$, $Y = -fB^{f-1}A$ and we put $e^{-fct} = 1$ since f is so very small. This simplifies our formula to

$$V = V_0 e^{-at}(X - Ye^{ct})(C + De^{-ct}).$$

Expanding and substituting $e^{-at} = 1$, since a is small, we obtain

$$\begin{aligned} V &= V_0(CX - CYe^{ct} + DXe^{-ct} - YD) \\ &= V_0(H - Ge^{ct} + Ke^{-ct}), \end{aligned}$$

which will be shown to fit the curves very closely.

In the above formula c is a constant which must fit all the curves obtained from this cell while H , G , and K are constants for each curve but vary from curve to curve.

To determine these constants we write $v_1 = V_1/V_0$, $v_2 = V_2/V_0$, etc. Then we have

$$\begin{aligned} v_0 &= H - G + K = 1, \\ v_1 &= H - Ge^{ct'} + Ke^{-ct'}, \\ v_2 &= H - Ge^{2ct'} + Ke^{-2ct'}, \\ v_3 &= H - Ge^{3ct'} + Ke^{-3ct'}, \end{aligned}$$

where t' is taken as a certain definite interval of time and we begin to count time from $t = 0$. These equations solved simultaneously give us

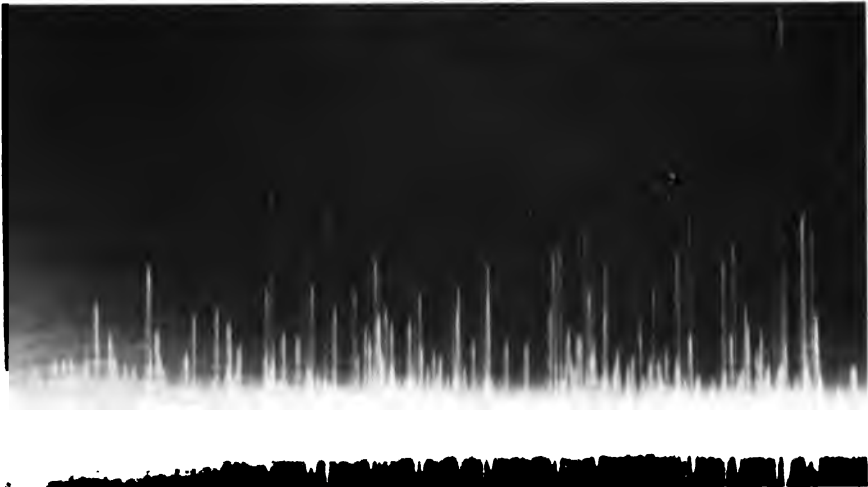


FIG 2.

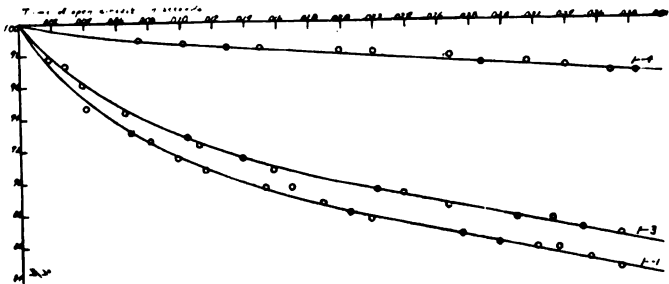


Fig. 3.

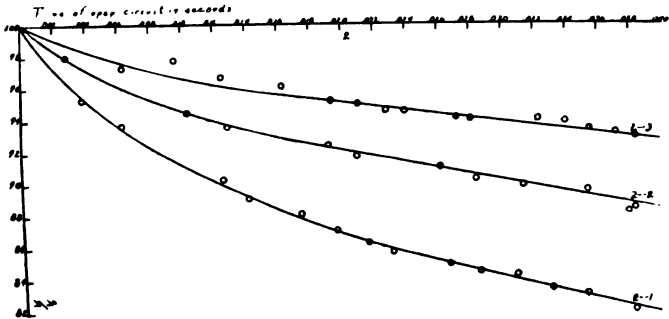


Fig. 4.

ALBERT LEWIS FITCH.

$$K = \frac{(v_3 - v_2) - (v_2 - v_1)e^{ct'}}{(e^{-ct'} - e^{ct'})(e^{-ct'} - 1)},$$

$$G = \frac{(v_3 - v_2) - (v_2 - v_1)e^{-ct'}}{(e^{ct'} - e^{-ct'})(1 - e^{ct'})},$$

$$H = 1 + G - K$$

and

$$\frac{(v_4 - v_3) + (v_2 - v_1)}{(v_3 - v_2)} = e^{-ct'} + e^{ct'} = \text{a constant, } M.$$

Then $e^{2ct'} - Me^{ct'} + 1 = 0$. The solution of this last equation gives us the value of c when we know the values of t' and M . It also shows that M must be a constant for all the curves provided only that we take t' the same. Using t' equal to .0125 sec., the following values are derived for M from six representative curves 3.53, 3.36, 2.96, 3.11, 3.31, and 3.10. The mean value is 3.23 which gives $c = 84$. Likewise using t' equal to .010 sec. we obtain M from the same curves equal to 3.18, 2.87, 2.58, 2.63, 2.75, and 2.93. The mean of these is 2.82 which gives a value of c equal to 88. The mean of these two values of c , 86, was used in all the computation which follows.

1—1.

On closed circuit 16 min.

Applied E.M.F., $V_0 = 99$ volts.

Counter E.M.F., V .

On open circuit t sec.

750 R.P.M.

$K = .138, H = .8629, G = .00094$.

1—3.

On closed circuit 3 hrs.

Applied E.M.F., $V_0 = 99$ volts.

Counter E.M.F., V .

On open circuit t sec.

750 R.P.M.

$K = .121, H = .8798, G = .00087$.

t .	V .	V/V_0 Obs.	V/V_0 Comp.	Error.	t .	V .	V/V_0 Obs.	V/V_0 Comp.	Error.
.0018	96.9	.978	.981	+3	.0028	96.4	.974	.974	0
.0041	93.8	.947	.959	+12	.0039	95.1	.961	.966	+5
.0069	92.2	.931	.938	+7	.0065	93.5	.944	.948	+4
.0081	91.7	.926	.930	+4	.0104	91.9	.928	.927	-1
.0098	90.6	.915	.920	+5	.0111	91.4	.923	.924	+1
.0115	89.8	.907	.912	+5	.0138	90.6	.915	.914	-1
.0152	88.7	.896	.897	+1	.0157	89.8	.907	.908	+1
.0169	88.7	.896	.891	-5	.0222	88.5	.894	.892	-2
.0188	87.7	.886	.886	-0	.0238	88.3	.892	.889	-3
.0205	87.1	.880	.881	+1	.0266	87.5	.884	.883	-1
.0218	86.7	.876	.878	+2	.0309	86.7	.876	.877	+1
.0275	85.7	.866	.863	-3	.0331	86.6	.875	.872	-3
.0298	85.1	.860	.862	+2	.0350	86.1	.869	.869	-0
.0322	84.8	.857	.857	-0	.0374	85.6	.865	.864	-1
.0335	84.8	.857	.854	-3					
.0355	84.2	.850	.849	-1					
.0374	83.6	.844	.845	+1					

DATA FROM PHOTOGRAPHS.

Since the actual figures are difficult to reproduce but one sample figure will be given. However the measured data will be given for several curves and curves drawn from the data which show the general form of the curves and the rise with time of open circuit.

1—4.

On closed circuit 24 hrs. 15 min.

Applied E.M.F., $V_0 = 99$ volts.Counter E.M.F., V .On open circuit t sec.

725 R.P.M.

 $K = .0215$, $H = .979$, $G = .00046$.

2—1.

On closed circuit 2 min. 30 sec.

Applied E.M.F., $V_0 = 49$ volts.Counter E.M.F., V .On open circuit t sec.

725 R.P.M.

 $K = .150$, $H = .8515$, $G = .00145$.

t .	V .	V/V_0 Obs.	V/V_0 Comp.	Error.	t .	V .	V/V_0 Obs.	V/V_0 Comp.	Error.
.0074	97.9	.989	.989	0	.0038	46.7	.953	.957	+4
.0102	97.7	.987	.987	0	.0063	45.9	.937	.937	0
.0129	97.5	.985	.985	0	.0126	44.2	.902	.899	-3
.0150	97.4	.984	.983	-1	.0142	43.6	.890	.891	+1
.0156	97.2	.982	.983	+1	.0175	43.1	.880	.878	-2
.0199	97.2	.982	.980	-2	.0197	42.6	.870	.870	0
.0220	97.1	.981	.979	-2	.0217	42.3	.863	.866	+3
.0268	96.9	.979	.977	-2	.0232	42.0	.857	.860	+3
.0287	96.4	.974	.975	+1	.0268	41.6	.850	.852	+2
.0316	96.4	.974	.973	-1	.0287	41.4	.845	.847	+2
.0340	96.1	.971	.972	+1	.0310	41.3	.843	.841	-2
.0368	95.8	.967	.969	+2	.0332	40.8	.834	.835	+1
.0384	95.8	.967	.967	0	.0354	40.6	.830	.829	-1
					.0384	40.1	.820	.818	-2

2—2.

On closed circuit 9 min. 30 sec.

Applied E.M.F., $V_0 = 49$ volts.Counter E.M.F., V .On open circuit t sec.

725 R.P.M.

 $K = .0945$, $H = .9064$, $G = .00087$.

2—3.

On closed circuit 1 hr. 32 min.

Applied E.M.F., $V_0 = 49$ volts.Counter E.M.F., V .On open circuit t sec.

725 R.P.M.

 $K = .0568$, $H = .9438$, $G = .000645$.

t .	V .	V/V_0 Obs.	V/V_0 Comp.	Error.	t .	V .	V/V_0 Obs.	V/V_0 Comp.	Error.
.0028	48.0	.980	.980	-0	.0096	47.85	.978	.967	-11
.0063	47.6	.973	.960	-13	.0125	47.36	.967	.961	-6
.0104	46.4	.945	.943	-2	.0163	47.04	.961	.955	-6
.0129	45.9	.936	.935	-1	.0194	46.70	.952	.951	-1
.0192	45.23	.924	.920	-4	.0211	46.54	.950	.948	-2
.0210	44.9	.917	.917	-0	.0228	46.38	.946	.947	+1
.0262	44.57	.910	.908	-2	.0240	46.38	.946	.946	0
.0284	44.2	.902	.905	+3	.0272	46.13	.941	.943	+2
.0314	44.0	.898	.900	+2	.0281	46.05	.940	.942	+2
.0354	43.86	.895	.893	-2	.0323	46.05	.940	.937	-3
.0380	43.18	.881	.887	+6	.0340	45.88	.938	.936	-2
.0384	43.26	.883	.886	+3	.0355	45.72	.933	.933	0
					.0372	45.56	.931	.930	-1
					.0384	45.48	.928	.928	0

4—1.

On closed circuit 15 min.

Applied E.M.F., $V_0 = 100$ volts.Counter E.M.F., V .On open circuit t sec.

750 R.P.M.

 $K = .101, H = .9002, G = .00124.$

4—2.

On closed circuit on 100 volts 15 min. and
on 75 volts 5 min.Applied E.M.F., $V_0 = 75$ volts.Counter E.M.F., V .On open circuit t sec.

750 R.P.M.

 $K = .0329, H = .9675, G = .000422.$

t .	V .	V/V_0 Obs.	V/V_0 Comp.	Error.	t .	V .	V/V_0 Obs.	V/V_0 Comp.	Error.
.0058	95.6	.956	.960	+4	.0122	73.3	.978	.978	0
.0098	94.2	.942	.941	-1	.0166	73.1	.975	.974	-1
.0133	92.9	.929	.929	0	.0190	73.1	.975	.972	-3
.0174	91.9	.919	.917	-2	.0219	72.8	.971	.970	-1
.0200	90.8	.908	.911	+3	.0273	72.4	.965	.966	+1
.0233	90.2	.902	.905	+3	.0308	72.0	.960	.964	+4
.0270	89.5	.895	.897	+2	.0332	72.0	.960	.962	+2
.0302	88.8	.888	.891	+3	.0365	72.0	.960	.960	0
.0338	87.8	.878	.883	+5	.0374	72.0	.960	.958	-2
.0374	87.4	.874	.973	-1					

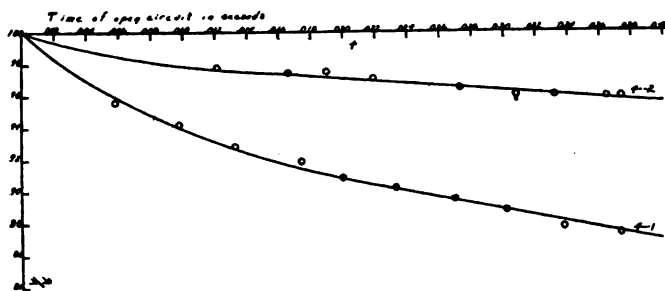


Fig. 5.

5—1.

On closed circuit 1 min.

Applied E.M.F., $V_0 = 10$ volts.Counter E.M.F., V .Time of open circuit t sec.

750 R.P.M.

 $K = .265, H = .735, G = .00003.$

5—2.

On closed circuit 10 min.

Applied E.M.F., $V_0 = 10$ volts.Counter E.M.F., V .On open circuit t sec.

750 R.P.M.

 $K = .0615, H = .9392, G = .000695.$

t .	V .	V/V_0 Obs.	V/V_0 Comp.	Error.	t .	V .	V/V_0 Obs.	V/V_0 Comp.	Error.
.0022	9.16	.916	.955	+37	.0076	9.70	.970	.970	0
.0107	8.20	.820	.840	+20	.0126	9.55	.955	.958	+3
.0142	8.08	.808	.813	+5	.0188	9.52	.952	.948	-4
.0196	7.85	.785	.784	-1	.0218	9.41	.941	.944	+3
.0246	7.78	.778	.767	-11	.0283	9.40	.940	.938	-2
.0307	7.48	.748	.754	+6	.0320	9.34	.934	.932	-2
.0342	7.48	.748	.748	0	.0336	9.34	.934	.930	-4
.0374	6.40	.640	.745	+105	.0365	9.28	.928	.926	-2
					.0374			.934	

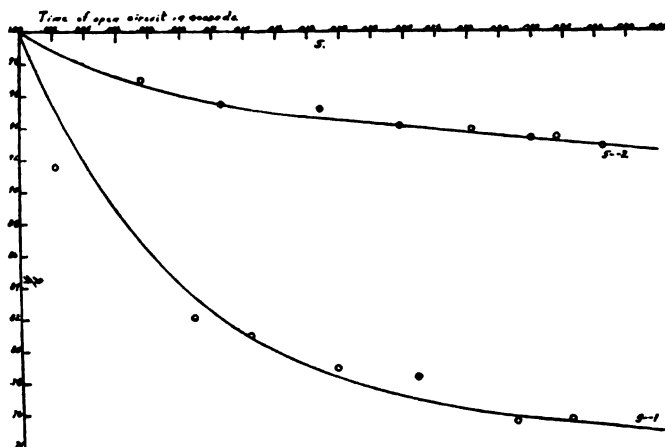


Fig. 6.

DISCUSSION OF FORMULA AND CURVES.

In the foregoing data but a portion of the data taken is given. The curves taken for longer periods of closed circuit lay so close to the axis that accurate measurement is impossible; however they do show the rise of the curve with time of closed circuit.

If one expresses the values of H , G , and K in terms of the thicknesses and resistances of the solid layer and gas film one will find that H should increase, K should decrease, and G should increase with time of closed circuit. The data given show that the H and K fulfill these requirements but that the G actually decreases. This apparent failure of the formula is explained however if we take account of one more term in the binomial expansion.

In every case, the curve of counter electromotive force plotted against time of open circuit rises with time of closed circuit. It seems evident that this counter electromotive force is not due entirely to a gas film as Guthe supposed. It is evident also that a permanent change must take place in the cell with time of closed circuit, for the current which leaks through the cell does not attain a minimum value but is constantly dropping lower and lower. Both of these effects are explained if we assume the cell's action depends upon the thickness of the solid layer which increases in direct proportion to the quantity of electricity passing through the cell, and also upon the thickness of the gas film, which quickly reaches a maximum value for each applied electromotive force and then gradually decreases in thickness with the time of open circuit. The increase in thickness of the solid layer results in an increase in the resistance of the combined layer, which accounts for the gradually

decreasing value of the leakage current on closed circuit and also for the rise of the curves with time of closed circuit. The decreasing thickness of the gas film results in a decrease in the resistance of the combined layer which in turn accounts for the drooping curve.

This way of looking at the cell explains why it acts differently from the ordinary electrolytic cell with both electrodes of some inactive substance, such as platinum. With inactive electrodes, there will be no solid layer deposited; for the electrolytically produced oxygen can not combine with the electrode. In such a cell, the current will drop to a very small value and remain at that value provided the applied voltage is low enough, since the electrodes are unaffected by the passage of the current. But with an aluminum anode there is always deposited some of this solid layer and a drop in current results.

This way of looking at the cell is in accord with the results of Graetz and Pollak, that any electrolyte which will liberate oxygen on electrolysis may be used. It is also in accord with the results of Schulze, that other metals than aluminum may be used, since it only requires that the anode form a compound with the liberated oxygen which has a rather high resistance.

When the cell is used to rectify the alternating current, there is a slight deposit of this solid layer every time the aluminum is the anode. However when the current reverses, this deposit is not decomposed for to decompose the compounds of aluminum by electrolysis a rather high temperature is required. As this process goes on, we may reach a stage where the heat produced in the resistance of this solid film is sufficiently great to raise the temperature of the whole cell to a point where this solid film begins to decompose on the reversal. When this temperature is reached the cell fails to work satisfactorily. This is in accord with the results of others who state that the cell works best at the lower temperatures.

SUMMARY.

1. This investigation has shown that any method of experimentation in which an appreciable length of time must elapse during the taking of data, the cell being on closed circuit with the battery at least a part of this time, will introduce serious errors due to the change taking place in the cell itself.

2. So far as the author is aware, no attempt has ever been made to explain the action of the cell on the theory that the action is due to a double dielectric, one part of which changes with the time of open circuit and the other with time of closed circuit. This investigation has shown such a theory is plausible and fully explains all the cases investigated.

3. The cell does not attain a steady state after a short time of closed circuit, nor does the single dielectric theory seem plausible.

4. The theory leads to the conclusion that any electrolyte which liberates oxygen on electrolysis may be used in the rectifier. It also shows that other metals than aluminum should be available as anodes, since it only requires the metal to form a compound with the liberated oxygen and that this compound have a rather high resistance.

The author is indebted to the University of Michigan for the apparatus provided, to Prof. N. H. Williams for many helpful suggestions, and to Mrs. Fitch for help in the computations.

ANN ARBOR, MICH.,

June 24, 1916.

THE INTENSITY OF X-RAY REFLECTION, AND THE DISTRIBUTION OF THE ELECTRONS IN ATOMS.

BY ARTHUR H. COMPTON.

IN the study of the spectra of X-rays as analyzed by crystal gratings, the remarkably low intensity of the higher orders of reflection has from the first attracted a considerable amount of attention. Preliminary measurements by Mr. W. L. Bragg¹ showed that, when corrected for temperature effects, the intensities of the different orders of reflection of a given X-ray spectrum line are approximately proportional to the inverse square of the order. A more detailed experimental investigation by Professor W. H. Bragg² showed that if X-rays of a definite wave-length are reflected at a glancing angle θ by a crystal in which the successive layers of atoms are similar and are similarly spaced, the energy in the reflected beam can be expressed with considerable accuracy by the formula

$$(1) \quad E_r = \frac{C(1 + \cos^2 2\theta)}{\sin^2 \theta} e^{-B \sin^2 \theta}.$$

In this expression C is a constant depending upon the energy in the incident beam, the wave-length of the X-rays, and the nature of the crystal. The factor $e^{-B \sin^2 \theta}$ accounts for the effect of the thermal motion of the atoms. The constant B can be determined experimentally² or may be calculated from certain thermal properties of the crystal.^{3, 4} The polarization factor $(1 + \cos^2 2\theta)$ was originally deduced by J. J. Thomson⁵ for any case of the scattering of X-rays by electrons, and was first introduced into the formula for X-ray reflection by Darwin.⁴ Professor Bragg was not able to verify this polarization factor, since in his experiments θ was always small. Using rays of longer wave-length, however, the writer has been able to measure⁶ the reflection at sufficiently large glancing angles to obtain an appreciable effect due to this factor, and thus

¹ W. L. Bragg, Proc. Roy. Soc., A, 89, 468 (1914).

² W. H. Bragg, Phil. Mag., 27, 881 (1914), also W. H. Bragg and W. L. Bragg, X-rays and Crystal Structure, p. 195.

³ P. Debye, Ann. d. Phys., 43, 49 (1914).

⁴ C. G. Darwin, Phil. Mag., 27, 325 (1914).

⁵ J. J. Thomson, Conduction of Electricity through Gases, p. 326.

⁶ A. H. Compton, Phys. Rev., 7, 658 (1916).

to verify its existence. Since the sine of the glancing angle is proportional to the order of reflection, this formula includes the result found by W. L. Bragg, but is more general, as it expresses the intensity of the reflection from all possible planes in the crystal.

The theory of the intensity of X-ray reflection has been examined in considerable detail by Mr. C. G. Darwin,^{4, 7} who finds that if all the electrons which are effective in scattering the X-rays are close to the centers of the atoms, the energy in the beam reflected at an angle θ should be proportional to

$$\frac{1 + \cos^2 2\theta e^{-B \sin^2 \theta}}{\sin \theta \cos \theta}.$$

This expression differs from Bragg's experimental formula by the factor $\tan \theta$, which must be explained, as Darwin pointed out, by the fact that the electrons are not all concentrated near the centers of the atoms, but that at least some of the electrons are at distances from the atomic centers which are of the same order of magnitude as the distance between the atoms. Since the relative intensity of the different orders of X-ray reflection is thus a function of the distribution of the electrons in the atoms of the crystal, it should be possible, knowing the relative intensity of the different orders, to obtain some definite idea of the manner in which these electrons are arranged.

The possibility of finding an arrangement of the electrons which will account in a satisfactory manner for the observed intensity of X-ray reflection at different angles was suggested first by Professor Bragg⁸ and independently soon after by the writer.⁹ Both of us were able to show the nature of the effect on the intensity of reflection due to certain different distributions of the electrons in the atoms of a crystal grating, but we both neglected to consider certain important factors that must seriously modify the conclusions at which we arrived. We based our arguments on the assumption that the reflected energy would be the same for all orders if all the scattering occurred at the centers of the atoms. This is indeed true for the intensity in the middle of the reflected line, if the crystal acts as a perfect grating, but since the effective width of the spectrum line can be shown to be proportional to $1/\sin \theta \cos \theta$, the reflected energy is reduced in the same ratio. Thus instead of a factor $1/\sin^2 \theta$ there is really, as pointed out above, a factor of only $1/\tan \theta$ to be accounted for by the assumed structure of the atom. In the present paper a more complete theory will be obtained of the dependence

⁷ C. G. Darwin, *Phil. Mag.*, 27, 675 (1914).

⁸ W. H. Bragg, Bakerian Lecture, March 18, 1915; *Phil. Trans., A*, 215, 253 (July 13, 1915).

⁹ A. H. Compton, *Nature*, May 27, 1915.

mass of the electron, and C is the velocity of light. If the electrons are held very rigidly in position, this expression might have to be slightly modified, but it seems probable that such effects are inappreciable for waves of the frequency of X-rays. The scattering due to the positive nucleus of the atom may be neglected on account of its comparatively great mass.

If we write $\phi = e^2/mC^2$, the amplitude of the reflected ray at the point P ($R, \epsilon_2, \epsilon_4$) due to a single electron near the surface of the crystal is therefore $(\phi/R) \cdot dA_i$. If $f(z)$ is the volume density of distribution of the electrons at a distance z from the middle plane of a layer of atoms in the crystal, the electric displacement at P at the time t due to a volume element of an atomic layer near the surface of the crystal is therefore

$$f(z)dx dy dz \cdot \frac{\phi}{R} dA_i \cdot \cos 2\pi \left(\frac{t}{T} - \frac{\delta + r}{\lambda} \right),$$

where T is the period of vibration, $2\pi\delta/\lambda$ is the phase angle at P due to a crystal element at (x, y, O) at the time $t = 0$, and r is the distance $2z \{ \sin \theta + (\epsilon_1 + \epsilon_2) \cos \theta \}$. Neglecting the small term $(\epsilon_1 + \epsilon_2) \cos \theta$, and writing,

$$\beta = 2\pi \left(\frac{t}{T} - \frac{\delta}{\lambda} \right),$$

we find for the displacement at P due to an element of area of a layer of atoms near the surface of the crystal,

$$\frac{\phi}{R} dA_i \cdot dx dy \cdot \int_a^b f(z) \cos \left(\beta - \frac{4\pi z \sin \theta}{\lambda} \right) dz,$$

where $b - a$ is the thickness of the atomic layer, and represents the diameter of the atoms. If N is the number of electrons per unit volume of the crystal, and D is the distance between two successive atomic layers, the number of electrons in unit area of such a layer is ND . The function $f(z)$ may therefore be written $ND F(z)$, where $F(z)$ is the probability that a given electron shall be at a distance z from the mid-plane of the atomic layer to which it belongs. Since the function $F(z)$ can nowhere be greater than at the plane of symmetry, *i. e.*, at $z = 0$, the displacement becomes a maximum when $\beta = 0$. The integral factor of the above expression then becomes,

$$\psi = \int_a^b F(z) \cos \left(\frac{4\pi z \sin \theta}{\lambda} \right) dz,$$

and the amplitude at P due to an element of area of the atomic layer considered may be written

$$\frac{\phi}{R} dA_i \cdot ND \cdot \psi \cdot dx dy.$$

If we consider an element of the crystal so small that the reflection from all of its atomic layers may be considered to be in the same phase, yet containing a large number dn of such layers, the amplitude of the reflected beam at P due to this element is evidently

$$\frac{\phi}{R} dA_i N D \psi dx dy \cdot dn.$$

Or since $dn = dz/D$, the amplitude due to a volume element of the crystal is

$$\frac{\phi}{R} dA_i N \psi dx dy dz = dA_0 \cdot dx dy dz,$$

where dA_0 is the amplitude of the reflected ray at P per unit volume of the crystal near its surface due to an incident beam of solid angle $d\sigma$.* When the rays are reflected from a depth z below the surface of the crystal, they travel a distance $2z/\sin \theta$ through the crystal before they emerge, so that the intensity is reduced by a factor $e^{2\mu z/\sin \theta}$, where μ is the absorption coefficient. The amplitude is thus reduced by a factor $e^{\mu z/\sin \theta}$, so the amplitude of the beam reflected from any part of the crystal is, per unit volume of the crystal,

$$(2) \quad dA_r = dA_0 e^{-\frac{\mu z}{\sin \theta}} = dA_i N \frac{\phi}{R} \psi e^{-\frac{\mu z}{\sin \theta}}.$$

Perfect Crystal and Long Trains of Waves.—We shall first determine the energy in the reflected beam on the assumptions (1) that the crystal has no faults, but acts as a perfect grating, and (2) that the X-rays come in trains of waves which are long compared with the depth to which they penetrate the crystal. These are the assumptions on which Darwin's theory of X-ray reflection is based, although he considers also the general nature of the modification to be expected if the crystal is imperfect. On these assumptions reinforcement will occur from all parts of the crystal struck by the rays if all the ϵ 's are zero, but when the ϵ 's differ from zero the reflections from different parts of the crystal will in general be in different phases. The difference in the path of the ray reflected from an element of the crystal at $p(x, y,$

* It will be noted that this expression seems to make the amplitude of the reflected beam proportional to the number of electrons and hence approximately proportional to the atomic weight of the atoms of the crystal. Though this result is confirmed by some early experiments by Mr. W. L. Bragg,¹ later experiments give different results. In the case of calcite Professor Bragg finds² that the *intensity* of the beam reflected by an atom is more nearly proportional to the atomic weight than is the *amplitude*. He writes me that some of his results point one way and some the other. It seems very difficult to obtain a consistent reflection formula on the assumption that it is the intensity which is proportional to the number of electrons. The difficulty is probably to be explained by the nature of the function ψ , for if many of the electrons are at an appreciable distance from the center, the amplitude of the ray scattered by an atom will be considerably less than the sum of the amplitudes due to each electron.

z) and that reflected from an element at O (Fig. 2) is

$$x(\epsilon_2 - \epsilon_1) \sin \theta + y(\epsilon_4 + \epsilon_3) + z\{2 \sin \theta + (\epsilon_1 + \epsilon_2) \cos \theta\},$$

where second order terms in the ϵ 's are neglected. The reflected rays

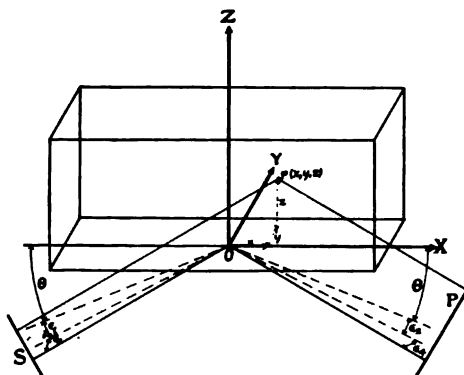


Fig. 2.

from all parts of the crystal are in the same phase when the difference in their paths is $2z \sin \theta$, so the difference in phase between the two reflected beams is

$$\delta = \frac{2\pi}{\lambda} \{x(\epsilon_2 - \epsilon_1) \sin \theta + y(\epsilon_3 + \epsilon_4) + z(\epsilon_1 + \epsilon_2) \cos \theta\}.$$

If the phase angle at P of the ray reflected from the point $(0, 0, 0)$ is $2\pi\gamma/\lambda$, the displacement due to an element of volume of the crystal at p is, by equation (2),

$$dA_0 e^{-\frac{\mu z}{\sin \theta}} \cdot \cos \frac{2\pi}{\lambda} \{ \gamma - x(\epsilon_2 - \epsilon_1) \sin \theta - y(\epsilon_3 + \epsilon_4) - z(\epsilon_1 + \epsilon_2) \cos \theta \} dx dy dz.$$

The displacement at P due to the reflection from the whole crystal is therefore

$$dA_0 \int_0^\infty dz \int_{-(k/2)}^{k/2} dy \int_{z \cot \theta - l/\sin \theta}^{z \cot \theta + l/\sin \theta} dx \cdot e^{-\frac{\mu z}{\sin \theta}} \cos \frac{2\pi}{\lambda} \{ \gamma - x(\epsilon_2 - \epsilon_1) \sin \theta - y(\epsilon_3 + \epsilon_4) - z(\epsilon_1 + \epsilon_2) \cos \theta \},$$

where l is the width and k the length of the slit through which the primary beam passes just before it strikes the crystal. This becomes when integrated,

$$\frac{4dA_0 BC}{gh} \cdot \frac{m \cos \frac{2\pi\gamma}{\lambda} - b \sin \frac{2\pi\gamma}{\lambda}}{m^2 + b^2},$$

where

$$B = \sin \left\{ \frac{\pi l}{\lambda} (\epsilon_2 - \epsilon_1) \right\}, \quad C = \sin \left\{ \frac{\pi k}{\lambda} (\epsilon_3 + \epsilon_4) \right\},$$

$$g = \frac{2\pi}{\lambda} (\epsilon_2 - \epsilon_1) \sin \theta, \quad h = \frac{2\pi}{\lambda} (\epsilon_3 + \epsilon_4),$$

$$m = \frac{\mu}{\sin \theta}, \quad b = \frac{4\pi}{\lambda} \epsilon_2 \cos \theta.$$

If this expression is differentiated with respect to γ and equated to zero, it is found that the maximum displacement occurs when

$$\frac{2\pi\gamma}{\lambda} = \tan^{-1} \left(\frac{b}{m} \right),$$

i. e.,

$$\sin \frac{2\pi\gamma}{\lambda} = \frac{b}{\sqrt{m^2 + b^2}}, \quad \cos \frac{2\pi\gamma}{\lambda} = \frac{m}{\sqrt{m^2 + b^2}}.$$

Substituting these values in the above equation, we find for the amplitude of the reflected beam,

$$(3) \quad dA_r = \frac{4dA_0BC}{gh\sqrt{m^2 + b^2}}.$$

The corresponding intensity of reflection is therefore,

$$dJ_r = c(dA_r)^2 = \frac{16c(dA_0)^2 B^2 C^2}{g^2 h^2 (m^2 + b^2)},$$

where c has the value assigned above. This may be expanded into the form

$$J_r d\epsilon_1 d\epsilon_3 = \frac{c l^2 k^2}{\sin^2 \theta} \cdot \frac{\sin^2 \{n(\epsilon_2 - \epsilon_1)\}}{\{n(\epsilon_2 - \epsilon_1)\}^2} \cdot \frac{\sin^2 \{p(\epsilon_3 + \epsilon_4)\}}{\{p(\epsilon_3 + \epsilon_4)\}^2} \cdot \frac{A_0^2 d\epsilon_1 d\epsilon_3}{m^2 + g\epsilon_2^2},$$

where $n = \pi l/\lambda$, $p = \pi k/\lambda$, and $q = (4\pi/\lambda)\cos \theta$, and dJ_r and dA_0 are written as $J_r d\epsilon_1 d\epsilon_3$ and $A_0 \sqrt{d\epsilon_1 d\epsilon_3}$ respectively. dJ_r has its greatest value when all the ϵ 's are zero, and is then

$$dJ_{(\max)} = \frac{c(dA_0)^2 l^2 k^2}{\mu^2}.$$

The maximum intensity of the reflected beam is thus independent of θ except for the function dA_0 , a result which is in accord with that found by Webster¹⁰ on similar assumptions.

It is not, however, this maximum intensity which is measured in the experiments. It is customary^{2, 6} rather to have the opening into the ionization chamber which measures the reflected X-ray beam large enough to receive the rays reflected at all angles ϵ_2 and ϵ_4 . If the X-rays come from a point source, the angle ϵ_3 has all values between $-k/2r$ and

¹⁰ D. L. Webster, *Phys. Rev.*, 5, 241 (1915).

+ $k/2r$, where r is the distance from the anticathode to the slit, and ϵ_1 has a range of values $\delta\epsilon_1 = l/r$. The energy per unit time in the reflected beam which is due to the rays incident at angles between ϵ_1 and $\epsilon_1 + d\epsilon_1$ is therefore,

$$E_{\epsilon_1} d\epsilon_1 = \int_{-\infty}^{\infty} R d\epsilon_4 \int_{-k/2r}^{+k/2r} d\epsilon_3 \int_{-\infty}^{\infty} R d\epsilon_2 \cdot J_{\epsilon} d\epsilon_1.$$

If the crystal is rotated with a constant angular velocity $\omega = d\epsilon_1/dt$, it is exposed to rays incident at the angle ϵ_1 for a time $\delta\epsilon_1/\omega = \delta t$, and the total reflected energy due to the rays incident at the angle ϵ_1 is $E_{\epsilon_1} d\epsilon_1 \cdot \delta t$. If in the time Δt the crystal moves through all angles ϵ_1 at which any appreciable radiation of wave-length λ is reflected, the whole reflected energy is thus,

$$E_r = \delta t \int_{-\infty}^{\infty} E_{\epsilon_1} d\epsilon_1 = R^2 \delta t \int_{-\infty}^{\infty} d\epsilon_4 \int_{-k/2r}^{+k/2r} d\epsilon_3 \int_{-\infty}^{\infty} d\epsilon_2 \int_{-\infty}^{\infty} d\epsilon_1 \cdot J_{\epsilon}.$$

Substituting the value of J_{ϵ} given above, and integrating, this becomes,

$$E_r = \frac{cA_0^2 R^2 l^2 k^3}{r \sin^2 \theta} \cdot \frac{\pi^3}{mn pq} \delta t,$$

or

$$(4) \quad E_r = \frac{c l^2 k^2 \lambda^3 \cdot N^2 A_i^2 \phi^2 \psi^2}{2 r^2 \omega \mu \sin(2\theta)}.$$

The amplitude dA_i of the beam incident in the solid angle $d\sigma$ has been defined by the relation $c(dA_i)^2 = cA_i^2 d\sigma = I_i d\sigma$, where I_i is the intensity. The energy of the radiation in the solid angle $d\sigma = d\epsilon_1 d\epsilon_3$ which passes through the slit in unit time is therefore $klI_i d\sigma = ck l A_i^2 d\epsilon_1 d\epsilon_3$, and as above, putting $\delta\epsilon_1 = l/r$, $\delta\epsilon_3 = k/r$, the total energy which passes through the slit in the time Δt is

$$E_i = klI_i \cdot \delta\epsilon_1 \delta\epsilon_3 \Delta t = \frac{k^2 l^2}{r^2} \cdot c A_i^2 \Delta t.$$

Writing $\Delta\theta = \omega\Delta t$ we find for the ratio of the reflected to the incident energy,

$$(5) \quad R = \frac{E_r}{E_i} = \frac{N^2 \lambda^3 \phi^2 \psi^2}{2 \mu \sin 2\theta} \cdot \frac{1}{\Delta\theta},$$

where $\Delta\theta$ is the angle through which the crystal is turned while making the observations. This result is in accord with that obtained by Darwin, but is worked out for somewhat different experimental conditions.

Imperfect Crystal.—It remains to determine the effect of changing our assumptions. Let us consider the case in which the crystal is not perfect, but is made up of a large number of small crystals, each of which acts as a separate unit. We shall have to determine the energy in the beam reflected by each little crystal, and sum up for all the component crystals.

Each component crystal will for convenience be considered to have the form of a rectangular parallelopiped of dimensions δx , δy , δz . If one of these little crystals is so small that the rays are not appreciably absorbed in passing through it, the phase of the beam reflected from the whole little crystal is the same as that of the ray reflected from its center. In a crystal whose center is at x_1 , y_1 , z_1 the amplitude contributed by a crystal element at that point is, by equation (2), $e^{-\mu z_1/\sin \theta} dA_0 dx dy dz$. If δ is the phase difference between this ray and the one reflected from the point x , y , z , the amplitude contributed by a crystal element at x , y , z is therefore $e^{-\mu z_1/\sin \theta} dA_0 \cos \delta \cdot dx dy dz$. Substituting the value of δ similar to that used above, we find for the amplitude at P of the beam reflected by the whole little crystal,

$$\begin{aligned} dA_e' &= dA_0 e^{-\frac{\mu z_1}{\sin \theta}} \int_{x_1-(\delta x/2)}^{x_1+(\delta x/2)} dx \int_{y_1-(\delta y/2)}^{y_1+(\delta y/2)} dy \int_{z_1-(\delta z/2)}^{z_1+(\delta z/2)} dz \cdot \cos \frac{2\pi}{\lambda} \\ &\quad \times \{ (x-x_1)(\epsilon_3-\epsilon_1) \sin \theta + (y-y_1)(\epsilon_3+\epsilon_4) + (z-z_1)(\epsilon_1+\epsilon_2) \cos \theta \} \\ &= dA_0 e^{-\frac{\mu z_1}{\sin \theta}} \cdot \frac{\sin \left\{ \frac{\pi \delta x}{\lambda} (\epsilon_3 - \epsilon_1) \sin \theta \right\}}{\frac{\pi \delta x}{\lambda} (\epsilon_3 - \epsilon_1) \sin \theta} \cdot \frac{\sin \left\{ \frac{\pi \delta y}{\lambda} (\epsilon_3 + \epsilon_4) \right\}}{\frac{\pi \delta y}{\lambda} (\epsilon_3 + \epsilon_4)} \\ &\quad \times \frac{\sin \left\{ \frac{\pi \delta z}{\lambda} (\epsilon_1 + \epsilon_2) \cos \theta \right\}}{\frac{\pi \delta z}{\lambda} (\epsilon_1 + \epsilon_2) \cos \theta} \cdot \delta x \delta y \delta z. \end{aligned}$$

As before, the whole energy reflected by this crystal as it is turned through an angle $\Delta \theta$ at an angular velocity ω is given by

$$E_r' = R^2 \delta t \int_{-\infty}^{\infty} d\epsilon_4 \int_{-(k/2r)}^{+(k/2r)} d\epsilon_3 \int_{-\infty}^{\infty} d\epsilon_2 \int_{-\infty}^{\infty} d\epsilon_1 \cdot c(A_e')^2.$$

Substituting the above value of $A_e' = dA_e'/d\epsilon_1 d\epsilon_3$ and integrating, we obtain

$$E_r' = \frac{cA_0^2 R^2 k l \lambda^3 e^{-\frac{2\mu z_1}{\sin \theta}}}{2r^2 \omega \sin \theta \cos \theta} \delta x \delta y \delta z.$$

The energy reflected from the whole crystal is of course the sum of the energies reflected from all the component crystals, *i. e.*, $E_r = \Sigma E_r'$, or replacing the summation sign by an integral,

$$\begin{aligned} (4a) \quad E_r &= \int_0^{\infty} dz \int_{-(k/2)}^{+(k/2)} dy \int_{z \cot \theta - (l/2 \sin \theta)}^{z \cot \theta + (l/2 \sin \theta)} dx \cdot \frac{cA_0^2 R^2 k l \lambda^3 e^{-\frac{2\mu z}{\sin \theta}}}{2r^2 \omega \sin \theta \cos \theta} \\ &= \frac{c l^2 k^2 \lambda^3 N^2 A_0^2 \phi^2 \psi^2}{2r^2 \omega \mu \sin 2\theta}, \end{aligned}$$

which is the same as was found in the case of a perfect crystal. Although this result has been obtained from the consideration of one particular type of imperfect crystal, it is evident that the same formula will hold in whatever manner the crystal is divided into its components.

Short Trains of Waves.—It has been pointed out by Webster¹⁰ that if X-rays do not come in long trains like light waves, but come rather in trains which are short compared with the depth to which they penetrate the crystal, the intensity of the reflected beam at the angle of maximum reflection is proportional to $1/\sin^2 \theta$. He has since noted, however,¹¹ that the breadth of the reflected line increases with θ so that the total reflected energy does not obey the same law. The following analysis will show that the total energy reflected in this case is the same as with long trains of waves. If we consider the primary beam to be made up of trains M waves in length, the total number of reenforcing layers is M/n , where n is the order of the reflection. The thickness of the crystal which is effective in reenforcement at any instant is therefore

$$\delta z = \frac{M}{n} D,$$

where D is the distance between two adjacent layers of atoms. If the point x_1, y_1, z_1 is the center of the part of the crystal which has reflected the ray reaching P ($R, \epsilon_2, \epsilon_4$) at any particular instant, the amplitude of the reflected wave at P at that instant is

$$\begin{aligned} dA_0' e^{-\frac{\mu z_1}{\sin \theta}} \int_{z_1 - (MD/2n)}^{z_1 + (MD/2n)} dz \int_{-(k/2)}^{k/2} dy \int_{z \cot \theta - (l/2 \sin \theta)}^{z \cot \theta + (l/2 \sin \theta)} dx \\ \times \cos \frac{2\pi}{\lambda} \left\{ \begin{aligned} &(x - x_1)(\epsilon_2 - \epsilon_1) \sin \theta \\ &+ (y - y_1)(\epsilon_4 + \epsilon_3) \\ &+ (z - z_1)(\epsilon_1 + \epsilon_2) \cos \theta \end{aligned} \right\} \\ = dA_0' e^{-\frac{\mu z_1}{\sin \theta}} \cdot \frac{k l M D}{n \sin \theta} \cdot \frac{\sin u}{u} \cdot \frac{\sin v}{v} \cdot \frac{\sin w}{w}, \end{aligned}$$

where

$$u = \frac{\pi l}{\lambda} (\epsilon_2 - \epsilon_1), \quad v = \frac{\pi k}{\lambda} (\epsilon_4 + \epsilon_3), \quad w = \frac{2\pi M D}{n \lambda} \epsilon_2 \cos \theta,$$

and the corresponding intensity is

$$c(dA_0')^2 e^{-\frac{2\mu z_1}{\sin \theta}} \frac{k^2 l^2 M^2 D^2}{n^2 \sin^2 \theta} \cdot \frac{\sin^2 u}{u^2} \cdot \frac{\sin^2 v}{v^2} \cdot \frac{\sin^2 w}{w^2}.$$

Since by hypothesis the primary train of waves is short compared with the depth to which it penetrates the crystal, the length of the reflected train will depend, not on that of the incident one, but only upon the depth

of penetration. In order to find the mean intensity of the reflected beam at this angle, we must therefore integrate this intensity over the whole length of the reflected train, and divide by the average distance between successive trains of waves. If this distance is ρ , the mean intensity of the reflected beam at this angle is thus,

$$\frac{c}{\rho} (dA_0')^2 \frac{k^2 P^2 M^2 D^2}{n^2 \sin^2 \theta} \cdot \frac{\sin^2 u}{u^2} \cdot \frac{\sin^2 v}{v^2} \cdot \frac{\sin^2 w}{w^2} \int_0^\infty e^{-\frac{2\mu z}{\sin \theta}} \cdot 2 \sin \theta \, dz$$

$$= \frac{c(dA_0')^2 P^2 k^2 M^2 D^2}{\rho n^2 \mu} \cdot \frac{\sin^2 u}{u^2} \cdot \frac{\sin^2 v}{v^2} \cdot \frac{\sin^2 w}{w^2}.$$

Since $n\lambda = 2D \sin \theta$, this may be written:

$$J_e d\sigma = \frac{cM^2 \lambda^2 k^2 P^2 (dA_0')^2}{4\rho \mu \sin^2 \theta} \cdot \frac{\sin^2 u \sin^2 v \sin^2 w}{u^2 v^2 w^2}.$$

When $u = v = w = 0$, this intensity is a maximum, and the resulting expression agrees with that obtained by Webster. To obtain the whole energy of the reflected beam, however, we must have as before,

$$E_r = R^2 \delta t \int_{-\infty}^{\infty} d\epsilon_4 \int_{-(k/2r)}^{+(k/2r)} d\epsilon_3 \int_{-\infty}^{\infty} d\epsilon_2 \int_{-\infty}^{\infty} d\epsilon_1 \cdot J_e,$$

and substituting the above value of J_e this becomes

$$E_r = \frac{cM\lambda^4 k^2 P^2 (A_i')^2 N^2 \phi^2 \psi^2}{2r^2 \omega \rho \mu \sin 2\theta}.$$

The mean value of the square of the amplitude of the incident beam is evidently $A_i'^2 = A_i'^2 \cdot M\lambda/\rho$, so the energy of the reflected beam is as formerly,

$$(4b) \quad E_r = \frac{cP^2 k^2 \lambda^3 N^2 A_i'^2 \phi^2 \psi^2}{2r^2 \omega \mu \sin 2\theta},$$

where $cA_i'^2$ now represents the mean intensity of the incident beam instead of its intensity at a given instant.

If there is any difference in the reflecting power of a crystal according to its degree of perfection or the nature of the incident rays, it must therefore be accounted for by a difference in the value of the absorption coefficient μ , since all the other factors in equation (4) have definitely defined values. It has been found by Darwin⁷ that the value of this absorption coefficient does differ according to the degree of perfection of the crystal. This results from the fact that a sort of selective absorption occurs near the angle of maximum reflection, which is great only in the case of a crystal that is nearly perfect. Darwin concludes, however, that this effect may be accounted for by inserting a constant factor into

the reflection formula.* We may therefore consider the expression here derived for the energy of the reflected beam of X-rays to hold for any crystal and any beam of X-rays if μ is taken as the *effective* absorption coefficient.

If a beam of X-rays of wave-length λ , polarized so that its electric vector is perpendicular to the plane of incidence, is reflected at an angle θ from a crystal in which all the atomic layers are equally spaced and whose atoms have no thermal motion, the energy in the reflected beam is therefore given by equation (5) as

$$E_r = \frac{E_i N^2 \lambda^3 \phi^2 \psi^2}{\Delta\theta \, 2\mu \sin 2\theta},$$

where E_i is the whole energy of the radiation of wave-length λ which strikes the crystal as it is turned with uniform angular velocity through an angle $\Delta\theta$, and $\Delta\theta$ is large enough to include all the angles at which rays of wave-length λ are reflected. N is the number of electrons per unit volume of the crystal, μ is the effective absorption coefficient of the X-rays in the crystal at the angle of reflection, and may differ by a constant factor from the coefficient of absorption at other angles. ϕ is the amplitude at unit distance of the ray scattered by a single electron, and has the value e^2/mC^2 . ψ is a factor depending upon the arrangement of the electrons in the atomic layers, and has the value

$$\int_a^b F(z) \cos\left(\frac{4\pi \sin \theta}{\lambda} z\right) dz,$$

where $b - a$ is the diameter of an atom, and $F(z)$ is the probability that

* The statement that this correction for selective absorption at the angle of maximum reflection can be accounted for by the insertion of a constant factor into the reflection formula implies that this absorption is equally strong in all orders. Darwin has shown (*loc. cit.*) that this means a high degree of imperfection of the reflecting crystal. If the crystal is more perfect, the selective absorption must be large compared to the normal absorption and be proportional to $1/\sin 2\theta$. This would make the reflected energy E independent of the angle except for the factor ψ . Darwin considers the fact that the reflected energy at the larger angles is known to fall off rapidly to be sufficient proof that the reflecting crystals cannot be so perfect. It is to be noted, however, that since it is possible, as will be shown below, completely to explain the low intensity of the higher orders by assuming the proper distributions of the electrons in the atoms, the existence of this diminution cannot properly be used to prove the imperfection of the crystal. The fact that the absorption coefficient μ which is to be used in the reflection equation is found by Darwin to be of the same order of magnitude as the normal absorption coefficient indicates, however, that the selective absorption is not large, and hence that the crystal must be very imperfect. The writer has recently made a series of experiments comparing the rate of falling off of the higher orders when tungsten and rhodium rays are reflected from the same crystal. These experiments, which will be published in the near future, indicate rather definitely the existence of the term $\sin 2\theta$ in the denominator of the reflection expression, which would not occur if the crystal used were not sufficiently imperfect to make the absorption coefficient approximately constant.

a given electron will be at a distance z from the mid-plane of the atomic layer to which it belongs. This factor corresponds to the "excess scattering" factor introduced by Darwin.⁴ As pointed out above, if the primary beam of X-rays is unpolarized, the factor $\frac{1}{2}(1 + \cos^2 2\theta)$ will have to be introduced, and to account for the thermal motion of the atoms the factor $e^{-B \sin^2 \theta}$ must also be included. Introducing these factors, writing $\sin 2\theta = 2 \sin \theta \cos \theta$, and expanding the terms ϕ and ψ , the complete expression for the energy of the reflected beam becomes:

$$(6) \quad E_r = \frac{E_i}{\Delta\theta} \cdot \frac{N^2 \lambda^3}{2\mu} \cdot \frac{1 + \cos^2 2\theta}{4 \sin \theta \cos \theta} \left(\frac{e^2}{mC^2} \right)^2 \\ \times \left\{ \int_a^b F(z) \cos \frac{4\pi z \sin \theta}{\lambda} dz \right\}^2 \cdot e^{-B \sin^2 \theta}.$$

It may be remarked that this same equation holds when the crystal remains stationary if the angular aperture $\Delta\theta$ of the slit as observed from the anticathode includes all the angles at which X-rays of wave-length λ are reflected.

THE DISTRIBUTION OF THE ELECTRONS IN ATOMS.

By comparing this equation with Bragg's experimental formula (1) it should be possible to determine the form of the function $F(z)$, and hence to find the mean distances of the electrons from the centers of their respective atoms. We shall first attempt a direct solution of the problem, and shall find that there is no possible distribution of the electrons in atoms which will give Bragg's formula as it stands; but a slightly modified form of his law will be found to lead to a definite value of $F(z)$, thus determining the probable distance of an electron from the center of its atom. On comparison with experiment it will be shown, however, that the differences in the intensities of reflection by different crystals are such that no single distribution of the electrons can explain the reflection from all crystals. Arbitrarily chosen distributions which will explain satisfactorily the observed differences between the reflections from certain crystals will then be described, and these will be of considerable interest in connection with certain hypotheses concerning the structure of the atom.

Direct Method.—According to equation (6), if X-rays of wave-length λ are reflected by a certain crystal at two different angles, θ and θ_1 , the ratio of the energy in the reflected beam in the two cases is

$$(7) \quad \frac{E}{E_1} = \frac{\frac{1 + \cos^2 2\theta}{\sin \theta \cos \theta} \left\{ \int_a^b F(z) \cos \left(\frac{4\pi z \sin \theta}{\lambda} \right) dz \right\}^2 e^{-B \sin^2 \theta}}{\frac{1 + \cos^2 2\theta_1}{\sin \theta_1 \cos \theta_1} \left\{ \int_a^b F(z) \cos \left(\frac{4\pi z \sin \theta_1}{\lambda} \right) dz \right\}^2 e^{-B \sin^2 \theta_1}}.$$

Bragg's experimental formula (1) gives this ratio as

$$(8) \quad \frac{E}{E_1} = \frac{\frac{1 + \cos^2 2\theta}{\sin^2 \theta} e^{-B \sin^2 \theta}}{\frac{1 + \cos^2 2\theta_1}{\sin^2 \theta_1} e^{-B \sin^2 \theta_1}}$$

and on comparing these two expressions, we find

$$(9) \quad \frac{\left\{ \int_a^b F(z) \cos \left(\frac{4\pi z \sin \theta}{\lambda} \right) dz \right\}^2}{\left\{ \int_a^b F(z) \cos \left(\frac{4\pi z \sin \theta_1}{\lambda} \right) dz \right\}^2} = \frac{\tan \theta_1}{\tan \theta}.$$

Extracting the square root, and substituting

$$\begin{aligned} x &= \frac{4\pi \sin \theta_1}{\lambda} z, & \phi(x) &= F\left(\frac{\lambda x}{4\pi \sin \theta_1}\right) = F(z), \\ \alpha^2 &= \frac{\tan \theta}{\tan \theta_1}, & \beta &= \frac{\sin \theta}{\sin \theta_1}, \\ r &= \frac{4\pi \sin \theta_1}{\lambda} a, & s &= \frac{4\pi \sin \theta_1}{\lambda} b, \end{aligned}$$

this equation may be reduced to the form

$$(10) \quad \int_r^s \phi(x) \{ \alpha \cos(\beta x) \pm \cos x \} dx = 0.$$

From this equation we have to evaluate the function $\phi(x)$ and the limits of integration, r and s .

This seems to be a difficult form of integral equation to attack by direct mathematical methods. It is possible by graphical methods to show that there are an infinite number of solutions of the form $\phi(x) = c \sin px$, if $r = -s$, where the constants c and p can have any value. These solutions, however, imply that the probability function ϕ becomes negative at certain points, which is meaningless, so that they do not apply to this problem. There are also an infinite number of solutions of the type

$$\phi(x) = c_1 \sin^n p_1 x + c_2 \cos^m p_2 x + c_3,$$

if p_1 and p_2 are integers, r and s are integral multiples of π , β is an integer, c_1 , c_2 , n , m , may have any value, and c_3 is large enough to prevent the expression from becoming negative. Experiment shows, however, that β is not necessarily an integer, so that these solutions also do not apply. Other possible solutions are not obvious.

The necessary form of the function $F(z)$, and hence also of the function

$\phi(x)$ can be found, however, by certain physical considerations. If we consider the primary beam of X-rays to be plane polarized, and the reflecting crystal to be at a very low temperature, the experimental law expressed in equation (8) becomes $E/E_1 = \sin^2 \theta_1 / \sin^2 \theta$. The ratio of the intensity of the reflected beam at two different angles thus depends only upon the angles at which reflection occurs, and is independent of the wave-length of the reflected beam. Thus with the layers of atoms at a distance a apart and with a beam of X-rays of wave-length λ , the ratio of the reflected energy at the two angles θ and θ_1 is the same as when the layers are at a distance ma apart using X-rays of wave-length $m\lambda$. Since, however, the ratio of these intensities is determined by the distribution of the electrons in the atomic layers, this means that this distribution shall be the same on the scale ma as it is on the scale a . That is, if the density of distribution is p times as great at $z = (1/q)a$ as at $z = (n/q)a$, it must be also p times as great at $z = (1/q)(ma)$ as at $z = (n/q)(ma)$, or

$$p = \frac{F\left(\frac{1}{q}a\right)}{F\left(\frac{n}{q}a\right)} = \frac{F\left(\frac{1}{q}ma\right)}{F\left(\frac{n}{q}ma\right)},$$

and in general

$$(II) \quad \frac{F(z)}{F(nz)} = \frac{F(mz)}{F(nmz)},$$

where the constants n and m may have any value. The only type of function which will satisfy this relation may be shown to be

$$\begin{aligned} F(z) &= b_1 z^{-p} & [z > 0] \\ &= b_1 (-z)^{-p} & [z < 0], \end{aligned}$$

where b_1 and p are arbitrary constants. Since

$$(12) \quad \phi(x) = F(z) = b_1 \left\{ \frac{\lambda x}{4\pi \sin \theta} \right\}^{-p} = bx^{-p} \quad [x > 0].$$

Equation (II) shows that the density of distribution of the electrons can vanish nowhere unless it is everywhere zero, so the limits of integration of equation (10) must be $-\infty$ and ∞ . This of course corresponds to an atom with an infinite radius.

Substituting this value for $\phi(x)$, and remembering that the function under the integral sign is symmetrical on both sides of the plane $z = 0$,

equation (10) may be written

$$(13) \quad \int_0^{\infty} x^{-p} \{ \alpha \cos \beta x \pm \cos x \} dx = 0$$

The integral is a known one,¹² the solution being

$$\begin{aligned} & (\alpha \beta^{p-1} \pm 1) \frac{\pi}{2 \Gamma(p)} \sec \left(\frac{p\pi}{2} \right) \text{ for } [0 < p < 1] \\ & \infty \quad \quad \quad \text{for } [p \geq 1] \\ & \text{indeterminate} \quad \quad \quad \text{for } [p \leq 0]. \end{aligned}$$

This integral can vanish only when

$$\alpha \beta^{p-1} \pm 1 = 0$$

or

$$\frac{\cos \theta}{\cos \theta_1} = \left\{ \frac{\sin \theta}{\sin \theta_1} \right\}^{2p-1}.$$

Since θ and θ_1 can have any values, it is obvious that there is no constant value of p which will satisfy this equation. Thus there is no solution of the integral equation (10) which meets the conditions. We must conclude, therefore, that there is no possible distribution of the electrons in the atomic layers which will give Bragg's experimental law as it stands.*

If equation (1) is modified by inserting the factor $\cos \theta$ in the denominator, its value is not greatly changed, since θ is usually small. This modification changes the quantity on the right-hand side of equation (9) to $\sin \theta_1 / \sin \theta$, and the coefficient α in equations (10) and (13) becomes $(\sin \theta / \sin \theta_1)^{1/2}$, the other quantities remaining unchanged. In this case the left hand side of equation (13) vanishes only when

$$\frac{\sin^{1/2} \theta}{\sin^{1/2} \theta_1} \cdot \left\{ \frac{\sin \theta}{\sin \theta_1} \right\}^{p-1} = 1,$$

or

$$p = \frac{1}{2}$$

(14) whence

$$\begin{aligned} F(z) &= b_1 z^{-1/2} \quad [z > 0] \\ &= b_1 (-z)^{-1/2} \quad [z < 0]. \end{aligned}$$

Thus although Bragg's law in its original form is not given by any possible

¹¹ D. L. Webster, *Phys. Rev.*, 7, 696 (1916).

¹² Bierens de Haan, *Nouv. Tables d'integrales definies*.

* On somewhat different assumptions from those used here, Professor Bragg has found a distribution of the scattering material which leads to the law $E/E_1 = n_1^2/n^2$, where n is the order of reflection.⁸ The distribution he finds is, however, a function of the distance between the atomic layers, and so cannot truly represent the arrangement of the electrons in the atoms. In fact it is possible to show by reasoning similar to that used above that on his assumptions also there is no distribution of the electrons which will give his law for the intensity of reflection.

distribution of the electrons, it can be obtained in a slightly modified form if the density of distribution of the electrons is inversely proportional to the square root of the distance from the mid-plane of the atomic layer to which they belong.

The quantity $F(z)$ represents the probability that a given electron shall be at a distance z from the middle of its layer of atoms at any instant. It is evident, therefore, that the relation $\int_{-\infty}^{\infty} F(z)dz = 1$ or $\int_0^{\infty} F(z)dz = \frac{1}{2}$ must be satisfied. The constant b_1 in equation (12), representing the probability $F(z)$ at unit distance from the mid-plane is thus defined by the expression

$$b_1 \int_0^{\infty} z^{-p} dz = \frac{1}{2},$$

or

$$b_1 = 0,$$

since

$$\int_0^{\infty} z^{-p} dz = \infty$$

for all values of p . This means that there can be no appreciable density of distribution of the electrons in the atomic layer unless there are an infinite number of electrons in it. It is evident therefore that the function $F(z)$ cannot have the form $b \cdot z^{-p}$. According to the argument above, this means that the relative intensity of the beam of X-rays reflected at different angles must depend not only upon the angle of reflection, but also upon the wave-length of the incident beam of X-rays.

Empirical Method.—Since there is no possible distribution of the electrons in the atoms which will give a law of reflection of the form of Bragg's empirical expression, let us see if it is not possible to find some arbitrary distribution which will give a reflection formula fitting the experimental data better than his law. In order conveniently to compare data obtained with different crystals and with X-rays of different wave-lengths, let us remember, according to equation (7), that the part of the ratio E/E_1 which is due to the diffuseness of the atomic planes is

$$U^2 = \frac{\left\{ \int_a^b F(z) \cos \left(\frac{4\pi z \sin \theta}{\lambda} \right) dz \right\}^2}{\left\{ \int_a^b F(z) \cos \left(\frac{4\pi z \sin \theta_1}{\lambda} \right) dz \right\}^2} = \frac{\psi^2}{\psi_1^2} = \frac{E}{E_1} \cdot \frac{1 + \cos^2 2\theta_1}{1 + \cos^2 2\theta} \cdot \frac{e^{-B \sin^2 \theta_1}}{e^{-B \sin^2 \theta}}.$$

From any assumed distribution of electrons $F(z)$ the theoretical values of

$$(15) \quad U = \frac{\psi}{\psi_1}$$

may be calculated for different angles of reflection, and by comparison with the corresponding experimental values as given by the equation

$$(16) \quad U = \left\{ \frac{E}{E_1} \cdot \frac{\frac{1 + \cos^2 2\theta_1}{\sin \theta_1 \cos \theta_1}}{\frac{1 + \cos^2 2\theta}{\sin \theta \cos \theta}} \cdot \frac{e^{-B \sin^2 \theta_1}}{e^{-B \sin^2 \theta}} \right\}^{1/2},$$

the accuracy of the assumed distribution may be tested.

For the cases of rock-salt and calcite the values of E/E_1 have been determined experimentally by Professor Bragg.^{2, 8} His results are given in Tables I. and II., columns 3. In the case of calcite the reflection at

TABLE I.

Rock-Salt.

Crystal Face.	$\frac{\sin \theta}{\sin \theta_1}$	$\frac{E}{E_1}$	U_{obs}	U_{calc_1}	U_{calc_2}
100	1.00	1.00	1.00	1.00	1.00
110	1.41	.41	.79	.83	.81
111	1.73	.244	.69	.75	.695
100	2.00	.187	.67	.70	.63
110	2.83	.0705	.55	.58	.56
100	3.00	.0625	.55	.56	.55
111	3.46	.042	.54	.51	.50

TABLE II.

Calcite.

Crystal Face.	$\frac{\sin \theta}{\sin \theta_1}$	$\frac{E}{E_1}$	U_{obs}	U_{calc_1}	U_{calc_2}
$\bar{1}\bar{1}\bar{1}$	.80	1.44	1.05	1.12	1.03
100	1.00	1.12	1.03	1.00	1.00
111	1.07	.95	.98	.97	.98
$\bar{1}\bar{1}\bar{0}$	1.24	.67	.88	.90	.95
110	1.59	.46	.86	.78	.87
$\bar{1}\bar{1}\bar{1}$	1.60	.48	.88	.78	.87
100	2.00	.257	.72	.70	.79
211	2.12	.286	.78	.68	.76
111	2.14	.297	.80	.68	.76
$\bar{1}\bar{1}\bar{0}$	2.46	.164	.65	.62	.69
100	3.00	.113	.61	.56	.60
110	3.22	.091	.57	.54	.58
$\bar{1}\bar{1}\bar{1}$	3.21	.108	.62	.54	.58
100	4.00	.034	.41	.48	.44
111	4.28	.036	.42	.46	.42
110	4.78	.021	.37	.43	.36
$\bar{1}\bar{1}\bar{0}$	4.91	.013	.30	.42	.35

different angles is produced by layers of different kinds of atoms, but the values here given have been corrected according to the method explained by Professor Bragg³ for the effect of the differences in the atomic layers, so these figures may be taken as due to a crystal all of whose atoms are similar and whose atomic layers are uniformly spaced. In these tables θ_1 is taken as the angle of the first order reflection from the (1, 0, 0) face of the crystal. The values of U calculated from the observed values of E/E_1 are given in columns 4, and are plotted in Fig. 3 against the corresponding values of $\sin \theta / \sin \theta_1$.^{*} The open circles in this figure represent the experimental values of U for rock-salt, and the solid ones for calcite. The agreement between the results of successive experiments indicates that these data may be taken as accurate within a probable error of about one or two per cent., although consistent errors of considerably greater magnitude may occur due to various causes.

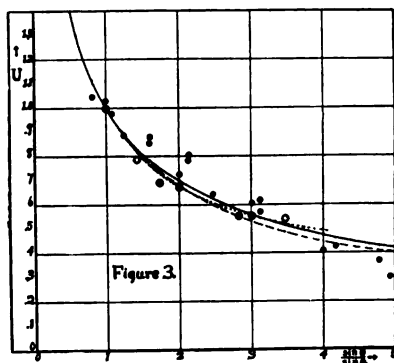


Fig. 3.

The values of U calculated from Bragg's law, equation (1), are given in columns 5 of the tables and are shown on Fig. 3 by the solid line. If the function $F(z)$ is assumed to have the form bz^{-p} , U has the values observed for the first and second order reflections from rock-salt when

* In calculating the values of U here given, the constant B in equation (16) has been evaluated by Debye's formula,³

$$B = \frac{1.142 \times 10^{-12} \phi(x)}{A\Theta\lambda^3 x},$$

where A is the atomic weight, Θ is Debye's "characteristic temperature," $x = \Theta/T$, where T is the absolute temperature, and $\phi(x)$ is a function which Debye evaluates. The value of B thus calculated has been found by Bragg³ to account with considerable accuracy for the observed changes of the intensity of reflection with temperature. For rock-salt $\Theta = 260^\circ \text{ K.}$, $A = 29$, $x = .89$, $\phi = .80$, and using rhodium rays of wave-length $\lambda = .614 \times 10^{-8}$ we obtain $B = 3.6$. In the case of calcite $\Theta = 910^\circ$ (determined from the specific heat at 298° K.), and taking $A = 20$, $x = 3.10$ and $\phi = .473$ we find $B = 0.25$. It is these values which have been used in calculating U . If a "Nullpunktsenergie" is assumed, Debye shows that the value of B is

$$B = \frac{1.142 \times 10^{-12}}{A\Theta\lambda^3} \left(\frac{\phi(x)}{x} + \frac{1}{4} \right).$$

In this case for rock-salt $B = 4.6$ and for calcite $B = .67$. This makes no very large difference in the values of U . In any case the effect of the temperature factor is not large at the angles at which these measurements are made, so that any slight error in the value of B will not greatly affect the results.

$p = 0.425$, which gives $U = (\sin \theta_1 / \sin \theta)^{.575}$. This expression is plotted in the broken line of Fig. 3, and is about as accurate an expression for the reflected energy as can be obtained with this type of function. Professor Bragg has suggested⁸ the use of a function of the form $F(z) = be^{-cz}$ to express the distribution of the electrons. The coefficient b in this case has a finite value, so although this function also implies an atom of infinite radius it is not *a priori* impossible. It can be shown, however, that the values of U obtained with such a distribution are considerably too large at moderately small angles, and decrease much too rapidly at large angles. Better agreement is obtained if a distribution of this type is combined with a certain concentration $F(0) = a$ of the electrons at the middle of the reflecting layers. In this case

$$U = \frac{a/b + c / \left\{ c^2 + \left(\frac{4\pi \sin \theta}{\lambda} \right)^2 \right\}}{a/b + c / \left\{ c^2 + \left(\frac{4\pi \sin \theta_1}{\lambda} \right)^2 \right\}}.$$

In order to make the values of U thus calculated agree with the observed values for the second and third orders of reflection from rock-salt we must put $a/b = 0.274 \lambda$ and $c = 1.51/\lambda$. The values of U thus obtained are plotted in the dotted curve of Fig. 3, and are seen to be in better agreement with the experiments on rock-salt than is the empirical law expressed by the solid curve. A comparison of the experimental data for rock-salt and calcite as shown in Fig. 3 shows at once that a curve which corresponds to the observed values of U for rock-salt cannot fit accurately the data for calcite. Professor Bragg has suggested⁸ the possibility of the existence of such a difference, but it was a difficult thing to detect on account of the many disturbing factors. Since by equation (15) U is a function only of the distribution of the electrons in the atomic layers and of $\sin \theta/\lambda$, any observed differences in U for a given value of $\sin \theta/\lambda$ can be explained only by differences in the distributions of the electrons in the layers of atoms which do the reflecting. Thus we may conclude that the distribution is not the same in the atoms of calcite as it is in the atoms of rock-salt.* It appears, therefore, that

* Note added December 9, 1916: A more striking example of the difference in the value of U for different crystals is afforded by Vegard's recent determination of the intensity of X-ray reflection from silver (Phil. Mag., 32, 94, July, 1916). He finds the intensity of the first three orders of reflection from the (111) plane to be in the ratio of 1.00 : 0.45 : 0.11, respectively. The corresponding values of U are 1.00 : 1.03 : 0.73. In the case of rock-salt we have found the values of U for the first three orders of reflection from the (100) plane to be 1.00, 0.67 and 0.55. The difference is too great to be accounted for by experimental error. Since, as pointed out above, U depends only upon the distribution of the electrons in the atoms of the crystal, these differences must be taken as a strong confirmation of our

instead of considering the electrons to be distributed according to some general law such as we have been discussing, one should rather consider every atom to possess a finite number of electrons, each placed at a definite distance from its center.

Atoms with a Finite Number of Electrons.—If each atom of a certain kind has an electron at a distance a from its center, the average effect from a large number of such atoms will be the same as that due to a uniform distribution of the electrons over the surface of a sphere of radius a . The center of this equivalent spherical shell will be in the mid-plane of the atomic layer, and the probability that a given electron in the shell will be at a distance z from the middle of the layer may be shown to be

$$F(z) = c \quad [-a < z < a],$$

or, in virtue of the relation

$$\int_{-a}^a c dz = 1,$$

$$F(z) = \frac{1}{2a} [-a < z < a].$$

The value of ψ due to an electron in such a shell is therefore

$$(17) \quad \psi' = \frac{1}{2a} \int_{-a}^a \cos\left(\frac{4\pi z \sin \theta}{\lambda}\right) dz = \frac{\sin\left(\frac{4\pi a \sin \theta}{\lambda}\right)}{\frac{4\pi a \sin \theta}{\lambda}},$$

and the value of ψ for a whole atom is

$$(18) \quad \psi = \frac{1}{\nu} \sum_r \psi_r' = \frac{1}{\nu} \sum_r \sin\left(\frac{4\pi a_r \sin \theta}{\lambda}\right) \bigg/ \left(\frac{4\pi a_r \sin \theta}{\lambda}\right),$$

where ν is the number of electrons in the atom, and the summation extends over all the r 's from 1 to ν . In any actual case this summation is most readily performed by plotting ψ' according to equation (17) and taking off its values corresponding to the desired values of $a \sin \theta / \lambda$. From the value of ψ thus obtained, the quantity U can be calculated as before by equation (15).

Attempts to obtain a suitable formula for U by thus adding the effects of a number of electrons placed at arbitrary distances a from the centers

fundamental assumption that the intensity of X-ray reflection depends upon the distribution of the electrons in the atoms of the reflecting crystal. If the values of U for silver are calculated on the basis of the assumptions used below in the case of calcite, giving to each electron an angular momentum of h/π , we obtain the values of U for the first three orders of reflection as 1.00 : 0.93 : 0.82. Though the agreement is far from perfect, it is much better than that obtained with the values 1.00 : 0.70 : 0.56, assigned by Bragg's law.

of the atoms soon convince one that the form of the resulting curve is very sensitive to changes in the assumed values of a . The results obtained above indicate that there must be a rather strong concentration of the electrons near the centers of the atoms, but it is difficult to select a distribution of the outer electrons which will give a reflection formula that agrees with the experimental data.

In the case of calcite it was found that a fairly acceptable expression could be obtained if the atoms of calcium, carbon and oxygen were assumed to have a number of electrons equal to their positive valence, *i. e.*, 2, 4 and 6, respectively, placed at distances from the centers of the atoms inversely proportional to the valence, together with an arbitrary number of electrons placed at the centers of the atoms. The use of this number of electrons in the outer part of each atom was suggested by well-known theories of valence, while some such spacing as this seemed necessary in order to obtain a suitable formula for U . This assumption concerning the spacing is, however, very nearly what is to be expected if these valency electrons revolve in rings about the centers of their respective atoms, all the electrons having the same angular momentum. For according to classical mechanics,¹³ the centrifugal force $m\omega^2a$, where m is the mass and ω the angular velocity of the electron and a is the radius of its orbit, must be balanced by the centripetal force $(M - \sigma_n)e^2/a^2$ where e is the electronic charge, M is the total charge in electronic units on the part of the atom inside the ring considered, and σ_n is a term which depends upon the mutual repulsion between the electrons in the same ring. With one electron in the ring $\sigma_1 = 0$, for two $\sigma_2 = .25$, and similarly $\sigma_4 = .96$, $\sigma_6 = 1.83$, $\sigma_7 = 2.31$ and $\sigma_8 = 2.81$. But if the angular momentum is constant, *i. e.*, $ma^2\omega = c$, we may write

$$ma \cdot \frac{c^2}{m^2a^4} = \frac{e^2}{a^2} (M - \sigma_n),$$

or

$$(19) \quad a = \frac{c^2}{me^2} \frac{1}{(M - \sigma_n)}.$$

Since for the outer ring of a neutral atom M is equal to the number of electrons in the ring, a is thus approximately inversely proportional to the number of valency electrons. This suggested, therefore, the assumption of the constancy of the angular momentum of the electrons in the atoms as a working basis.

It is of course possible to find any number of satisfactory arrangements unless the number of electrons in the atoms is defined. Barkla has shown¹³ that the number of electrons in an atom which are effective in

¹³ H. G. J. Moseley, *Phil. Mag.*, 26, 1032 (1913).

scattering X-rays is approximately equal to half the atomic weight. His work in connection with that of Rutherford on the scattering of alpha particles¹³ and of Moseley on the high frequency spectra of the elements¹⁴ makes it very probable that if N is the atomic number, there are N electrons distributed about the atomic nucleus, and that all these electrons are effective in scattering X-rays. This, together with the assumption used above concerning the valency electrons, determines the number of electrons in both rings of the first row of elements in the periodic table, including carbon and oxygen. For the second and third rows it has been assumed, in accordance with Moseley's interpretation of his X-ray spectra,¹⁵ that the inner ring contains 4 electrons. This leaves 6 electrons in the middle ring of the second row of atoms. There remain 8 electrons to be placed in the calcium atom, and it seemed reasonable to put these in a ring just inside of the valency electrons.

With the number of electrons in each ring thus determined, the relative distance of each electron from the nucleus can immediately be

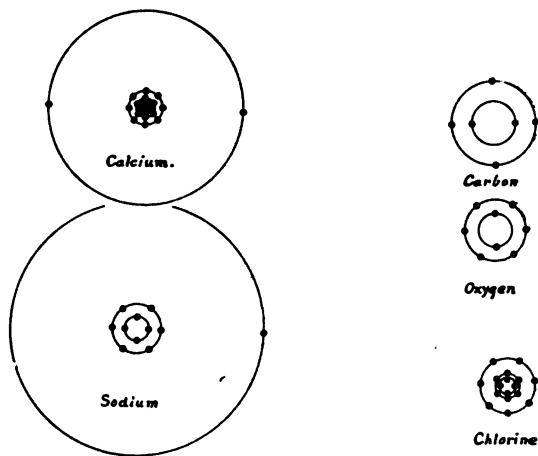


Fig. 4.

calculated by equation (19), and the corresponding values of U can be determined by equations (15) and (18). If the angular momentum of all the electrons is the same, their relative spacing in the atoms here used is as shown in Fig. 4. The value of U obtained for calcite on the basis of these assumptions is shown in the dotted curve of Fig. 5. Although the agreement is fair, it is evident that the theoretical values are too high in the low orders, and fall off too rapidly in the higher orders. It is to be noted that as there is but a single arbitrary constant, the

¹³ C. G. Barkla, *Phil. Mag.*, 14, 408 (1907).

¹⁴ E. Rutherford, *Phil. Mag.*, 27, 488 (1914).

angular momentum of an electron, in the equation for U , even this general agreement indicates that we are working along the right line. If the assumptions are modified to the extent of giving the electrons in the inner rings and in the second calcium ring half the angular momentum of those in the outer rings, the values of U given in column 6 of Table II. and shown in the solid curve of Fig. 5 are obtained. The equation for ψ is in this case,

$$\psi = \frac{1}{50} \left\{ \begin{aligned} &[4 \sin (.013k \sin \theta) / .013k \sin \theta + 6 \sin (.018k \sin \theta) / .018k \sin \theta] \\ &+ 8 \sin (.139k \sin \theta) / .139k \sin \theta + 2 \sin (.571k \sin \theta) / .571k \sin \theta] \\ &+ [2 \sin (.043k \sin \theta) / .043k \sin \theta + 4 \sin (.329k \sin \theta) / .329k \sin \theta] \\ &+ 3[2 \sin (.032k \sin \theta) / .032k \sin \theta + 6 \sin (.240k \sin \theta) / .240k \sin \theta] \end{aligned} \right\}.$$

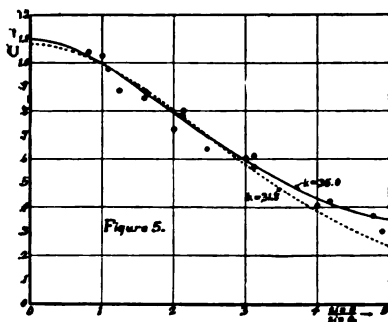


Fig. 5.

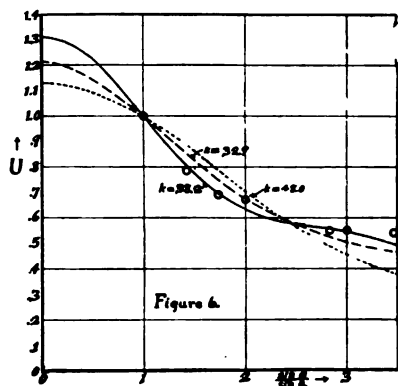


Fig. 6.

50 is the number of electrons in a molecule of calcium carbonate, and k is the constant $(4\pi/\lambda) \cdot (c^2/me^2)$ where c is the angular momentum of an electron in one of the outer rings. The agreement of the theoretical curve with the observed values is well within the probable experimental error, and is evidently better than that obtained with Bragg's empirical formula.

If the assumption that all the electrons have the same angular momentum is used in the case of rock-salt, a result is obtained which does not at all correspond with the observed intensities of reflection, as is shown in the dotted curve of Fig. 6. In order to account for the low value of U around the second order, it seems necessary that there shall be a ring of electrons about 1.00×10^{-8} cm. in radius which contains a considerable number of electrons, and that there shall be no other heavy ring of more than half as great diameter. This arrangement is obtained if we consider the electrons in the outer rings of chlorine to have $3/2$ as great angular momentum as those of sodium, and if, as before, we assume the

inner ring of each atom to have $\frac{1}{2}$ the normal angular momentum. According to the work of J. J. Thomson¹⁶ it appears probable that the valency electron of sodium goes over to the chlorine atom and becomes a part of its outer ring. This would give 8 electrons in the outer ring of chlorine and none in that of sodium. The values of U given in Table I. and shown in the solid curve of Fig. 6 have been calculated on this basis. If the valency electron of sodium is not thus transferred, the broken curve in this figure is obtained on the same assumptions concerning angular momentum. It will be seen that the former hypothesis yields the better results. Although the accuracy of the experimental values of U is not great enough at present to warrant any definite conclusions concerning such details, the difference in the curves obtained by thus shifting the position of a single electron indicates the extreme sensitiveness of this method for determining the distribution of the electrons in atoms.

CONCLUSIONS AND DISCUSSION.

Thus we see that if certain definite distributions of the electrons in atoms are assumed, it is possible to explain in a satisfactory manner the rapid decrease in the intensity of the higher orders of reflection of X-rays by crystals. Although it is not possible thus to derive Bragg's empirical formula, expressions for the intensity of reflection can be derived which agree better with experiment than does his law. We have shown from a theoretical standpoint that any equation which is to express adequately the relative intensity of the beam of X-rays reflected at different angles must be a function not only of the angle of reflection, as is Bragg's law, but must involve also the wave-length of the incident beam of X-rays. Professor Bragg has suggested that the observed values of the intensities of the reflection from different crystals actually show differences which do not depend upon the angle alone. These differences are usually masked by the large general variations of the intensity with the angle of reflection which is to be expected with any crystal grating; in the cases of rock-salt and calcite, however, these general variations can be accounted for, and it has been found above that there remain certain decided differences between the spectra from these crystals. It has been shown theoretically that any such differences must be due to differences in the distribution of the electrons in the atoms of the respective crystals, and the arrangements of the electrons which have been assigned to the different atoms have been found capable of accounting for these variations. Thus our fundamental hypothesis that the intensity of the reflected beam is a function of this distribution is strongly confirmed.

¹⁶ J. J. Thomson, *Phil. Mag.*, 27, 757 (1914).

The scarcity and comparative uncertainty of the experimental data make it premature to arrive at any positive conclusions concerning the details of the distribution of the electrons in the atoms considered. In general, however, it may be said that a comparison of the different distributions of the electrons which have been found to give satisfactory results show that there is usually a rather strong concentration of the electrons near the centers of the atoms, but with a considerable number at appreciable distances from the atomic centers. The expression $F(z) = bz^{-\frac{1}{2}}$, which was found to give Bragg's law in a slightly modified form, implies that the volume of density of distribution of electrons is inversely proportional to the $5/2$ power of the distance from the center of the atom, and the satisfactory distributions in the case of a limited number of electrons have followed this rule approximately.

From the value of the constant k which is determined by the experimental data, it is possible to calculate the radii of the different rings of electrons according to the formula

$$a = \frac{k\lambda}{4\pi(M + \sigma_n)}.$$

Using the values corresponding to the solid curve of Fig. 5 we thus find for the outer ring of calcium, $a = 0.97 \times 10^{-8}$ cm., for carbon, 0.56×10^{-8} cm., and for oxygen, 0.41×10^{-8} cm. The absolute magnitudes here given are of course subject to revision by further experiment, but if the number of electrons in the outer ring is taken equal to the positive valence of the atom, a number of unsatisfactory attempts to obtain expressions for U with other arrangements make it seem necessary that the ratio of the diameters of the rings shall be about that here given. This result is interesting in connection with the fact that while the carbon and oxygen atoms in calcite are only about 1.07×10^{-8} cm. apart, the calcium atom is about 2.95×10^{-8} cm. from its nearest neighbor. In the case of rock-salt, if the valency electron of sodium is in the outer chlorine ring, the radius of the next sodium ring is 0.36×10^{-8} cm., and of the outer chlorine ring is 1.00×10^{-8} cm. If the valency electron remains in the sodium atom, it is, according to our assumptions, 1.86×10^{-8} cm. from the center of the atom, and the chlorine ring is reduced to 0.89×10^{-8} cm. This is a possible arrangement, since the atoms are 2.81×10^{-8} cm. apart. As mentioned above, the diameter of the chlorine ring is probably determined with a fair degree of accuracy. The arrangement of the inner rings is by no means so definitely determined. A number of attempts were made, however, to fit the data for rock-salt assuming only 2 instead of 4 electrons in the inner rings, and this seemingly small change made the curve for U depart too far from the experi-

mental values to be acceptable. Results such as these make it reasonable to suppose that in the case of the lighter elements it may be possible, with a sufficient amount of accurate experimental data, to determine positively the distance of each individual electron from the center of its atom.

Bearing on Theories of Atomic Structure.—While it is difficult by any such "cut and try" method to find the only possible arrangement of the electrons in atoms, it is evident that a study of the intensity of X-ray spectra thus affords an extremely sensitive test of any theory which assigns a definite distribution to the electrons in atoms. From the experimental data now available it may be said, for example, that unless some important factor has been neglected in our formula for X-ray reflection, it seems impossible to account for the rapid diminution of the intensity of the higher orders on any theory, such as Crehore's,¹⁷ which would confine the electrons of an atom within a distance less than 10^{-10} cm. from its center. Such a distribution would make the quantity ψ of equation (6) approximately equal to unity, and this equation would then differ from the experimental equation (1) by the factor $\tan \theta$, which is much greater than the experimental error. Even if such a factor were introduced into the theoretical equation, it would still be unable to account for the variations in intensity which are characteristic of the individual crystal used as a grating. It seems necessary, therefore, to reject Crehore's theory of the atom as an impossible hypothesis.

On the other hand, it seems possible to explain all the X-ray intensities on the basis of the type of atom suggested by Bohr.¹⁸ The working assumption that we have used, the constancy of angular momentum, is the fundamental hypothesis of Bohr's theory. We may calculate the angular momentum of the electrons in the atoms here considered according to the relation

$$c = \sqrt{k \frac{\lambda m e^2}{4\pi}}.$$

Using the values $\lambda = 0.614 \times 10^{-8}$ cm., $m = 9.0 \times 10^{-28}$ gm. and $e = 4.77 \times 10^{-10}$ gm.^{1/2}cm.^{3/2}sec.⁻¹, for the outer rings of the atoms in calcite $k = 35.0$ and c becomes 1.87×10^{-27} gm. cm.² sec.⁻¹. For the inner rings our hypotheses make c half this value. According to Bohr the angular momentum of an electron in an atom is an integral multiple of $h/2\pi$, where h is Planck's constant. Using the value $h = 6.57 \times 10^{-27}$

¹⁷ A. C. Crehore, *Phil. Mag.*, 26, 25 (1913) and elsewhere.

¹⁸ N. Bohr, *Phil. Mag.*, 26, pp. 1, 476, 857 (1913), 27, p. 506 (1914).

gm. cm.² sec.⁻¹, we find $c = 2(1 - .10) \cdot h/2\pi$. In the case of rock-salt it is the outer chlorine ring whose diameter is most definitely defined. In this case $k = 9/4 \cdot 38.2$, and $c = 2.93 \times 10^{-27} = 3(1 - .06) \cdot h/2\pi$. It may be only an accident that the angular momentum in these cases works out so nearly in accord with Bohr's hypothesis, but it is at least a most interesting accident. The difference between the coefficients of $h/2\pi$ and integers is probably too great to be due to experimental error in the determination of the radii of the electronic rings, but there are good reasons to believe that there may be forces acting on the electrons which have not been considered, and which would be sufficient to account for the difference.

SUMMARY.

In the first part of this paper an expression is derived for the energy of a beam of X-rays of definite wave-length which is reflected from a crystal. The result is in accord with that previously derived by Darwin, and shows that the intensity of the reflected beam depends not only upon the angle of reflection but also upon the arrangement of the electrons within the atoms of the reflecting crystal.

The form of the equation for the energy in the reflected beam is shown to be independent of the degree of perfection of the crystal and of the length of the wave-trains of which the X-rays consist.

In the latter part of the paper a study is made of the possible distributions of the electrons in atoms which will account for the observed energy in the reflected beam of X-rays. It is shown that there is no possible distribution of the electrons which will lead to Bragg's empirical law for the intensity of reflection; it is found rather that any formula for this intensity must depend not only upon the angle of reflection but also upon the wave-length of the incident rays.

Attention is called to the fact that Bragg's experimental data indicate differences in the reflection from certain crystals which can arise only from differences in the distribution of the electrons in the atoms of which the crystals are composed.

Assuming a number of electrons in each crystal equal to the atomic number, and making certain plausible assumptions concerning the arrangement of these electrons in rings, it is found possible to account in a satisfactory manner for the observed intensities of the X-ray spectra.

Although the particular distributions assigned to the electrons in the atoms of calcite and rock-salt may not be the only ones which will account for the observed intensities of the X-ray spectra, it seems probable that these distributions are not far from correct.

The results of this investigation seem to show conclusively that the

electrons are not concentrated within a very small distance from the center of the atom, as is assumed in Crehore's theory of atomic structure. On the other hand, the conclusions arrived at are in good accord with the theory of the atom due to Bohr.

My thanks are due to Professor H. L. Cooke for his helpful interest in this research.

PALMER PHYSICAL LABORATORY,
PRINCETON UNIVERSITY,
June 19, 1916.

AN EFFECT OF LIGHT UPON THE CONTACT POTENTIAL
OF SELENIUM AND CUPROUS OXIDE.

BY E. H. KENNARD AND E. O. DIETERICH.

THE increased electrical conductivity of selenium under illumination has been explained in terms of the electron theory by assuming that the light produced an increase in the number of free electrons, and this explanation is strongly supported by observations on the rate of decay of the conductivity, which suggest the recombination of ions according to familiar laws. It occurred to the authors that the increase in the number of electrons ought to result in a change of the contact potential under illumination, because the electrons would tend to diffuse away from the illuminated region and would therefore leave the selenium more positive than before. A search for this effect was accordingly made; the work was interrupted by the departure of one of the authors, but it seems worth reporting as a quick survey of a new field.

METHODS OF OBSERVATION.

Two methods of observation were used. The first was a condenser method for the measurement of quick changes in contact potential. The plate of selenium, which was about 5 cm. square and 0.2 to 0.4 mm. thick, was mounted between a sheet of gauze above and a copper plate below (cf. Fig. 1), the plates being insulated from each other and when

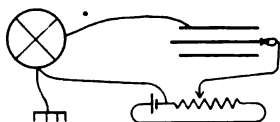


Fig. 1.

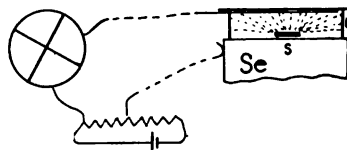


Fig. 2.

desired from earth by means of bits of amber or by supports tipped with sealing-wax. Any one of the three plates could be connected to an electrometer giving 3,000 mm./volt or to an improvised potentiometer, contact with the selenium being made by means of a wire clip; thus any deflection of the electrometer due to a change in the potential of the selenium surface could be nullified by adjusting the potentiometer and the change was thus determined. Readings were usually taken in pairs,

the light being thrown on and then off; the two readings generally differed by 10–20 per cent. but the means of successive pairs agreed mostly within a few per cent.

The second method was an adaptation of the ionization method of measuring contact potentials. An "ion contact" capsule (Fig. 2) was made by coating a copper strip *s* on top with polonium and mounting it in a fiber ring *r* so that it would clear when the ring lay upon a plane surface. The ring was closed on top by a sheet of mica, and just below this was mounted a sheet of brass gauze, one strand of which protruded through the ring and was connected to the electrometer. The mica was of sufficient thickness to stop α -particles and all cracks were closed with soft wax, while the arrangement of the parts prevented α -particles from striking the selenium. The ring was about 12 mm. in diameter and 3 mm. high.

A null method was employed as suggested by Fig. 2. The equalization of potential between gauze and specimen was only about half effected in a minute, corresponding to a resistance of some 10^{13} ohms, but when the sensitiveness of the electrometer was increased to 18,000 mm./volt by the use of a concave lens in front of the scale the adjustment of the potentiometer within a millivolt could be made in less than a minute.

The specimen was illuminated by a 400-watt tungsten lamp placed some 30 cm. away and focused roughly upon it by means of a large lens, a glass dish containing 2 cm. of water being usually interposed to minimize heating effects. Electrostatic screening was carefully attended to, the light being admitted through a sheet of coarse gauze.

PHOTO-ELECTROMOTIVE EFFECT IN SELENIUM.

Various connections of the plates in the condenser arrangement were employed, the selenium being connected either to the electrometer or to the potentiometer or left insulated; while the ion contact capsule was used chiefly on a 12-mm. cube of selenium. All the results consistently indicated that illumination suddenly made the surface of the selenium more *negative*, instead of more positive as anticipated. The effect on the plate was usually not far from -0.1 volt, the deflections in the absence of compensation being of the order of 20 mm. Shading the contact on the selenium had no appreciable effect. On the cube mean values of -0.152 , -0.176 volt were obtained for a certain face on different days, while later, after washing thoroughly with chloroform, the effect was -0.129 on the same face and -0.125 volt on the opposite face. The latter observations indicate a variation with the state of the surface, which we should rather expect if the phenomenon is a change in the contact potential.

An E.M.F. in a closed circuit containing selenium was looked for repeatedly but without appreciable success. In one case two brass clips were placed on the selenium plate, each touching the plate on one (but not the same) side and insulated from the other surface by a bit of paper. When one clip was now connected to the electrometer and the other to earth, illumination produced a minute deflection that was consistent as to sign but amounted only to .3 mm. (0.1 mv.); the sign was reversed by shading the upper contact. These minute residual deflections may very well be due to a heating effect, for the thermo-electric power of selenium is quite large. It is to be noted however that the actual point of contact between the selenium and the metal was not here appreciably illuminated.

A natural suspicion is that the effect might be due to photo-electric emission from the gauze above the selenium. But aside from the question of quantitative sufficiency, this explanation seems to be ruled out completely by these two facts: first, the effect is sudden and yet no steady current persisted under continuous illumination even when all potentials were kept the same by adjusting the potentiometer; secondly, the effect at once reverses when the light is removed.

The conclusion seems therefore a safe one that the observed effect is due to a sudden change produced by illumination in the contact potential of the selenium surface. In other words, between more and less strongly illuminated selenium there exists what might be called a "photo-electromotive force" tending to drive positive electricity away from the illuminated region.

VARIATION WITH INTENSITY OF THE LIGHT.

The variation of the effect with the intensity of illumination was investigated briefly by varying the voltage on the lamp, using the polonium capsule and the selenium cube. An approximate calibration of the lamp was made in terms of an arbitrary standard.

A high degree of sensitiveness to faint light was at once apparent, for a change in potential of about 3 mv. was produced with only 13 volts on the lamp (normal, 110); the illumination on the selenium must have been only about 0.01 candle-foot. Yet at 25 volts, or 2-3 candle-feet, the effect had increased only to 35 mv. From this point on a series of readings was taken, the light being thrown on and off at 2 min. intervals while its intensity was varied to full value and back. The photo-electromotive potentials are plotted against lamp voltages in Fig. 4, curve *I*, while in Fig. 3 the corresponding mean values are plotted on logarithmic scale against the intensity of the light. Something like an

approach to saturation is evident, for the last light intensity is 2,000 times the first while the photo-electromotive effect is only increased by a factor of 3.6; but the logarithmic curve shows a persistent slow increase proportional to about the fifth or sixth root of the intensity.

Closely similar results were obtained when the light was kept on continuously (except for a short break while changing connections). In this case five minutes was allowed between readings, as a previous test had indicated that a fairly steady equilibrium was thus attained. The results are shown in Fig. 4, curve *II*, the potential of the selenium relative to the gauze in the capsule being plotted against lamp voltage.

FATIGUE, DRIFT AND TIME LAG.

Curve *I* in Fig. 4 shows clear evidence of fatigue with respect to repetition, for the later readings taken after the lapse of 4-28 minutes are all 2-5 mv. smaller. A similar difference between readings taken in immediate succession was frequently noticed. Fatigue with respect to

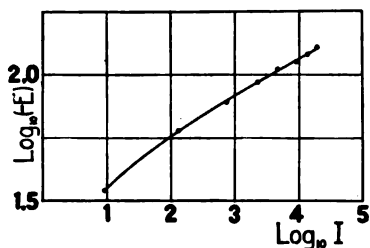


Fig. 3.

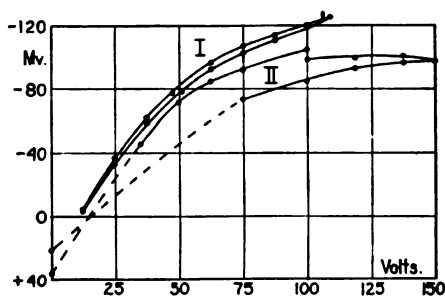


Fig. 4.

continuation, also, is indicated by curve *II*, Fig. 4. The difference in the initial and final dark readings points to a drift of the dark potential toward the negative, yet during the strong illumination the potential was evidently drifting toward the positive: the most natural explanation is that the negative displacement due to illumination tended to decrease with time. Such a drift of the dark potential was frequently noticed; it probably accounted for an apparent hysteresis of 10-20 per cent. that was often observed between the on- and off-readings.

If any time lag occurred when the light was thrown on, it was a matter of seconds only and escaped detection. On the other hand, the change in potential seemed usually to overshoot and then to settle back in the course of several minutes to an equilibrium value.

THE CONTACT POTENTIAL OF SELENIUM.

Several comparisons of the contact potential of selenium with that of freshly sandpapered copper were made.

Measurements on two different plates by the usual variable condenser method gave, in the dark, -0.34 , -0.45 volt, resp. Two faces of the cube were found to be at $-.27$ and $-.36$, resp., relative to the gauze in the polonium capsule, while on the same day the latter was observed to be $-.099$ relative to copper, giving for the selenium -0.37 and -0.46 , resp. Thus the contact potential of selenium crystallized in mass is not far from -0.4 volt in the dark relative to freshly sandpapered copper, changing under moderate to intense illumination by about -0.1 volt.

PHOTO-ELECTROMOTIVE EFFECT IN CUPROUS OXIDE.

Interesting results were obtained on a plate of cuprous oxide, which is also light-sensitive as regards resistance. The plate was about 2.5 cm. square and 1 mm. thick, and was prepared by heating a copper plate at a bright red heat for 48 hours and then dissolving off the surface layer of cupric oxide with aqua regia. The time allowed proved insufficient, however, so that the plate contained a layer of metallic copper down the middle.

The polonium capsule was placed on top of the plate and two clips were attached, one insulated with paper above and the other below. By connecting one clip to the electrometer and the other to the potentiometer it was found that an E.M.F. (voltaic?) of some 20 mv. existed between these two points; under illumination this E.M.F. increased by

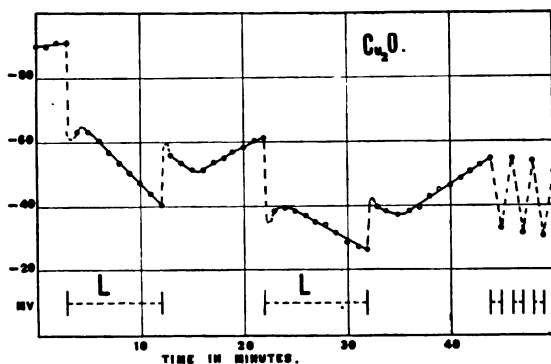


Fig. 5.

7 mv. when the top contact was unshaded but decreased by the same amount when the contact was shaded. When the capsule was connected to the electrometer and one of the clips to the potentiometer, illumination made the surface under the capsule become more positive by 20 to 28 mv.

As in selenium, the initial effect seemed to overshoot and to be followed by a moderate reaction. There was also a decided drift of the potential

in the dark, sometimes in one direction, sometimes in the other, but a true hysteresis was not apparent. A series of readings of the potential at intervals of a minute is shown in Fig. 5, the dotted line marked *L* showing the periods of illumination. Apparently illumination produces, besides the sudden effect, a slow drift of potential followed by recovery in the dark but lagging by 2-3 minutes; this may be a temperature effect. The curve shows decided fatigue with respect to continuation of illumination, the return being practically complete if made at once but amounting to only some 60 per cent. after ten minutes of illumination. Fatigue with respect to repetition does not appear here, but in some of the other readings it was evident to the extent of some 10 per cent.

CONCLUSION.

The negative sign of the effect in selenium disposes at once of the diffusion of electrons as a possible cause. A tempting hypothesis is that the light modifies the molecules in such a way as to diminish the potential energy of the electrons in the intermolecular spaces, and that this results in an increase of the number of free electrons, partly at the expense of the surrounding atoms of selenium and partly at the expense of other bodies with which it is in contact. This would explain both the increased conductivity and the change toward the negative in the contact potential. But the lack of proportionality between the two effects and especially the apparent occurrence of saturation in the latter seem to be fatal objections to this hypothesis. Apparently the two effects are due to independent causes.

The experimental results may be summarized as follows:

1. The contact potential of selenium changes under illumination by an amount exceeding a millivolt for 0.01 candle-foot and tending to show saturation at a value of about -0.1 volt under strong illumination. The phenomenon is complicated by fatigue effects and a slow drift of the potential with time.
2. No E.M.F. is produced however in a closed circuit.
3. The contact potential of selenium relative to freshly sandpapered copper is about -0.4 volt in the dark.
4. The surface potential of a plate of cuprous oxide changed under illumination by about 25 mv.; but the plate also contained a small voltaic (?) E.M.F. which varied under illumination.

The authors take pleasure in acknowledging a debt to Professor A. F. Kovarik for his kindness in preparing the polonium deposit for the ion contact capsule.

WAVE-LENGTH ENERGY DISTRIBUTION IN THE CONTINUOUS X-RAY SPECTRUM.

BY BERGEN DAVIS.

PROFESSOR DUANE and F. L. Hunt¹ have described some very interesting and important experiments on the energy radiated at various wave-lengths from a Coolidge X-ray tube.

The Coolidge X-ray tube was operated by a steady high voltage obtained from a large number of small storage cells. This voltage, however, was less than that required to produce the characteristic rays of tungsten. The radiation, after passing through a slit, was reflected from a cleavage surface of a calcite crystal. The wave-length was determined in the usual manner by the angle of reflection and the intensity of the radiation was measured by the ionization produced in the ionization chamber of the spectrometer. In this way an experimental relation was obtained between the energy and the wave-length of the radiation from a Coolidge X-ray tube.

The results of one experiment are shown by the full line in Fig. 3. This curve is reproduced from the curve published by Duane and Hunt. The results have not been corrected for variation of absorption with wave-length nor for the dependence of reflectivity of the crystal upon wave-length.

There is no radiation emitted at a wave-length less than λ_0 . The curve rapidly rises to a maximum and then decreases with increasing wave-length. A point of great interest is that the shortest wave-length λ_0 corresponds to that required by the quantum relation.

$$P_0 e = \frac{1}{2} m v_0^2 = h \nu_0 = \frac{hc}{\lambda_0}.$$

The significance of these results is to be found in the fact that although the electrons of the cathode stream all have practically the same velocity, the radiation has a wide range of frequencies less than the frequency (ν_0) required by the Planck equation. Also as the voltage applied to the tube is varied the wave-length (λ_0) at which the curve starts is shifted, the curve still retaining the same form. Dr. A. W. Hull² has recently

¹ Proc. Amer. Phys. Soc., PHYS. REV., August, 1915.

² Proc. Amer. Phys. Soc., PHYS. REV., Jan., 1916.

shown that the wave-length (λ_0) of the end radiation fulfills the condition:

$$P_0 e = h\nu_0 = \frac{hc}{\lambda_0},$$

for all voltages applied to the X-ray tube, at least for all voltages up to 100,000 volts, which was the upper limit of the experiments.

It is proposed here to develop a theory of this continuous or "white" X-ray spectrum.

Two distinct hypotheses will be made as to the origin of the radiation.

A. A PULSE HYPOTHESIS.

The radiation will be considered as consisting of the pulses arising from the accelerations of the electrons of the cathode stream upon their impacts with the atoms of the target.

B. A QUANTUM HYPOTHESIS.

The radiation will be considered as arising from electrons of the atom, excited in some manner by the transfer of energy from the electrons of the cathode stream. The quantity of energy radiated will fulfill the condition:

$$\epsilon = h\nu.$$

The frequency also is controlled by the condition:

$$\frac{1}{2}mv^2 = h\nu.$$

In this hypothesis, only a small fraction of the electrons of the cathode stream make impacts that are effective in exciting radiation.

A. THE PULSE HYPOTHESIS.

The following fundamental assumptions are made:

(a) The "white" or continuous X-radiation are the electro-magnetic pulses produced by the accelerations of the electrons of the cathode stream upon their collisions with the atoms of the target. (The Stokes pulse theory of X-rays.)

(b) The energy radiated by an impinging electron is given by the equation:

$$E = \frac{2}{3} \frac{e^2}{c} \int a^2 dt,$$

where a is the acceleration of the electron, e is the charge and c is the velocity of propagation.

(c) The frequency (ν) of the emitted radiation fulfills the condition:

$$\frac{1}{2}mv_n^2 = h\nu,$$

where v_n is the component of the velocity of the electron normal to the atomic surface.

(d) The atoms of the target are considered to possess centers of force that strongly repel the colliding electrons. The repulsive force varies inversely as the cube of the distance from the center of force, and has a spherical distribution about this center.

(e) The electrons do not lose all of their kinetic energy at a single collision, but as they thread their way down through the surface of the target they lose their energy (velocity) by successive collisions with the atoms. These successive collisions continue until the translational energy of the electrons is exhausted.

A distinction is here made as to the nature of the continuous X-radiation and the characteristic radiation. The continuous radiation which is due to the accelerations of the impinging electrons is probably independent of the nature of the atoms of the target, except in so far as the repulsive force of the atoms for the electrons may depend on atomic properties. The characteristic radiation on the other hand depends directly on the atomic number of the element composing the target. This radiation arises from the electrons within the atom while the continuous radiation originates in the accelerations of the electrons of the cathode stream.

The difference between the continuous X-ray spectrum and the characteristic radiation is exhibited in a striking manner by the curves in Fig. 1, which Dr. A. W. Hull, of the Research Laboratory of the General Electric Company, has kindly allowed me to reproduce here. Curve 1 is the spectrum of the radiation from a tungsten target, and curve 2 is that from a molybdenum target. They were both taken at the same voltage of 43,000 volts. This voltage is less than that required to produce the characteristic radiation of tungsten, and is similar to that obtained for tungsten by Duane and Hunt (see Fig. 3). The voltage, however, is greater than that required to produce the characteristic rays of molybdenum. The curve 2 shows the characteristic (line) radiation superimposed on the continuous spectrum. The intensity of the characteristic radiation is great compared to that of the continuous spectrum.

The way in which the characteristic (line) spectrum is added to the continuous indicates that the two radiations may have different origins, and suggests the view advanced here, that the one arises from the electrons of the atoms and the other from the electrons of the cathode stream. These two curves (Fig. 1) were obtained under identical conditions.

When an electron impinges on the atomic surface (region of repulsive force), it is accelerated and a pulse is emitted. In two papers on a

pulse theory of temperature radiation, Sir. J. J. Thomson¹ has described the kind of impact required by the laws of such temperature radiation. For the sake of brevity the results obtained by Professor Thomson will be applied to the present problem.

(a) It is shown that in order that the radiation may satisfy the second

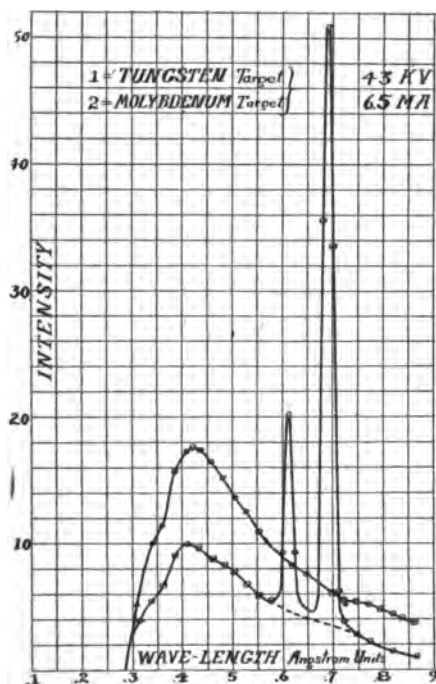


Fig. 1.

law of thermodynamics, the product of the kinetic energy of the colliding electron and the time of impact with the atom must be equal to a constant. This condition may be represented by:

$$\frac{1}{2}mv_n^2t = H = \text{a constant.} \quad (1)$$

This is similar to Planck's quantum relation since a frequency is a reciprocal of a time.

(b) It is shown that the inverse cube law of force between the colliding electron and the atom gives approximately the above relation between the impact time and the kinetic energy of the electron.

(c) It is shown that approximately the rate of emission of energy during the impact of an electron is:

$$E = \frac{2}{3} \frac{e^2}{\pi e} \frac{v_n^2}{t}, \quad (2)$$

¹ Phil. Mag., August, 1907, and July, 1910.

whose v_n is the normal component of velocity at beginning of the collision and t is the duration of the impact.

The radiation (2) may be expressed in terms of the normal velocity v_n by means of equation (1)

$$E = \frac{1}{3} \frac{m}{H} \frac{e^2}{\pi c} v_n^4. \quad (3)$$

The energy is proportional to the fourth power of the normal component of the velocity of the colliding electron.

It is assumed that there is some constant relation between the pulse length and the wave-length at which a greater part of the energy is radiated. That is, the constant H of equation (1) has such a value that the frequency of the radiation approximately fulfills the conditions of the quantum hypothesis:

$$\frac{1}{2} m v_n^2 = h\nu.$$

The energy radiated is:

$$E = \frac{4}{3} \frac{e^2}{m} \frac{h^2}{c\pi H} \nu^2.$$

or

$$E = k\nu^2. \quad (4)$$

The *energy radiated* is *proportional* to the *square* of the *frequency*.

B. QUANTUM HYPOTHESIS.

There are serious objections to a pulse theory of the origin of X-radiation. It is doubtful if any pulse theory can satisfy all the results of experiment.

The diffraction effects obtained by means of the crystal grating indicates interference of many successive waves. There can be no true interference by the reflection of a single pulse from the successive molecular planes of the crystal. The pulse may however be resolved into its harmonic constituents and the interference may occur between the successive waves of each frequency. The crystal may perhaps possess the power of making such analysis of the pulse. But this would not lessen the difficulty. The resolution of a pulse into its harmonic constituents would in general give a series of harmonic terms extending to infinite frequency, whose coefficients might have definite magnitudes. The experiments however show that there is no measureable energy present at a frequency greater than the end frequency ν_0 . In order to apply the pulse theory one must assume all the coefficients of the higher terms to be approximately zero. The investigation of Professor Thomson, however, previously referred to, shows that this is not the case.

It is quite possible that the mere acceleration of an electron may not necessarily cause radiation. The pulse hypothesis is of course founded on continuous mechanics. The ability of an electron to emit radiation may be in some way dependent on its position with reference to an atom. The success of the Bohr model of the atom strongly suggests that this may be so.

If a system of discontinuous mechanics is required, then of course a pulse hypothesis of radiation is not possible, and one must adopt some form of quantum theory to account for the radiation.

There are a number of physical phenomena that indicate that exchanges of energy probably take place in "quanta." In addition to temperature radiation one might mention the photo-electric effect, the excitation of characteristic X-radiation by impact of cathode rays, the secondary characteristic X-rays and other phenomena. All of these phenomena apparently involve exchanges of energy in definite quanta.

The following assumptions are made:

(a) If the impinging electron excites X-rays at all, it transfers all of its normal kinetic energy (that part of its energy due to the normal component of its velocity) into radiant energy.

(b) Only a small fraction of the collisions of the electrons result in the production of radiation. This small fraction of the electrons that is effective in producing radiation may be expressed as the probability p that any one electron makes an effective collision.

(c) The probability p that any one electron will produce X-radiation is taken proportional to the kinetic energy due to the normal component of its velocity

$$p = C\frac{1}{2}mv_n^2,$$

where C is a constant. Actual experiment of course shows that but a small proportion of the energy of the cathode stream appears as X-radiation. The greater part of the energy appears as heat in the target.

The energy emitted by the effective electron will be

$$\epsilon = h\nu.$$

The probability that any one electron in an encounter with an atom will emit radiation is p . The energy emitted by any one electron at an effective impact will be:

$$E = p h \nu.$$

This probability p is placed proportional to the kinetic energy due to the normal component of the velocity.

From which

$$E = C\frac{1}{2}mv_n^2 h \nu.$$

But

$$\frac{1}{2}mv_n^2 = h \nu.$$

The probable energy emitted by one electron at a collision now becomes:

$$E = Ch^2\nu^2. \quad (5)$$

This expression is of the same form as equation (4).

The energy emitted is proportional to the square of the frequency.

The foregoing derivation of course, does not attempt to explain the process, or state the conditions that must obtain in order that an occasional electron shall be effective in producing radiation.

THE RESULTANT RADIATION FROM THE TARGET.

A general expression for the radiated energy in terms of the frequency having been obtained, it remains to derive an equation for the resultant radiation produced by all of the electrons of the cathode stream upon their collisions with the atoms of the target. The electrons of the cathode stream all have the same original velocity v_0 upon impact with the atoms at the surface of the target. The velocity at which they collide with the atoms progressively decreases.

All frequencies below the end frequency ν_0 may be emitted by the impact of the electrons against the atomic "surface." The forces acting are considered to be radial forces and hence only the normal component

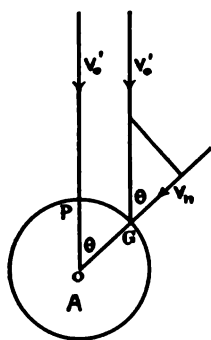


Fig. 2.

of the velocity will be affected, and this component only will determine the frequency. Thus electrons making impact at the pole p of an atom will emit the highest frequency, while those making impact at an angle will emit a lower frequency corresponding to that component of the velocity which is along the radius. This process will take place for all the impacts taking place with the atoms at various depths within the target. The total radiation at any frequency will be found by summing the radiations emitted at the impacts of the electrons that have a normal velocity component corresponding to that frequency

as they make successive collisions with the atoms of the target.

Let n' be the number of electrons that impinge on a zonal element of the surface of an atom at an angle (see Fig. 2). The radiation produced by these n' electrons will be by equation (4) or (5).

$$E = kn'\nu^2.$$

The frequency of the radiations may be expressed by:

$$\frac{1}{2}mv_n^2 = h\nu. \quad (6)$$

The electrons possess a velocity ν_0 at any depth within the target,

and those that make impact at poles p of the atoms will emit radiation of frequency ν_0' .

$$\frac{1}{2}m(v')^2 = h\nu_0'. \quad (7)$$

Since

$$v_n = v_0' \cos \theta,$$

the frequency emitted at any angle θ may be expressed in terms of the polar frequency ν_0' .

$$\nu = \nu_0' \cos^2 \theta. \quad (8)$$

If I is the number of electrons that strike the atomic surfaces in an element of time (number of impacts), then

$$n' = I 2 \sin \theta \cos \theta d\theta. \quad (9)$$

The energy radiated by all those electrons having a velocity v_0' , that make impact on a zonal element of surface of an atom will be

$$dw = kI(\nu_0')^2 \cos^4 \theta (2 \sin \theta \cos \theta d\theta).$$

By differentiation of equation (8) and substituting in above

$$dw = -kI\nu_0' \left(\frac{\nu}{\nu_0'} \right)^2 d\nu. \quad (10)$$

This expresses the energy lying between a frequency ν and $\nu + d\nu$ emitted by those electrons having a velocity v_0' .

This velocity v_0' is any velocity at any depth x within the surface of the target.

In order to derive the total radiation it is necessary to express the number of impacts at any velocity at any depth of penetration.

Let v_0 be the velocity with which N electrons enter the surface of the target. The number n_1 found in a layer dx deep will be

$$n_1 = N \frac{dx}{v_0}.$$

The number n present in any layer dx at any depth will be

$$n = n_1 \frac{v_0}{v_0'},$$

where v_0' is the velocity at any depth.

From these two expressions we obtain

$$n = N \frac{dx}{v_0'}.$$

The number of impacts will be proportional to the velocity and to the number having that velocity. The number of impacts in any region dx thick is

$$I = nv_0' = Ndx. \quad (11)$$

It is difficult to formulate an expression for the distribution of velocities of the electrons as they pass down through the surface of the target. One may imagine that the electrons will be differently affected, and that although they all enter the surface with the same velocity, yet at a depth within the surface, after a number of chance collisions there must be a wide range of velocities.

Widdington¹ has investigated the decrease of velocity of electrons on passing through thin films of various thickness. The decrease of velocity observed could be very well represented by:

$$v^4 = v_0^4 - ax.$$

Since $V = 0$ at a thickness corresponding to the range R , this may be written:

$$v^2 = v_0^2 \sqrt{1 - \frac{x}{R}}. \quad (12)$$

But this represents the law of velocity decrease for those few electrons only that passed directly through the film without deviation. The greater part of the entering electrons must have been scattered about, many issuing at various angles and velocities and some of them absorbed in the film itself.

The following consideration will show that Widdington's law cannot represent the velocity distribution completely. In the case of a solid target the electrons all strike the surface at the same velocity v_0 . At some depth all of the electrons must have lost all of this initial velocity and have acquired the probability velocity distribution required by Maxwell's law for the temperature of the target. At intermediate depths there must be a gradual transition from the velocity v_0 to the Maxwell distribution. The true expression of the velocity distribution at various depths below the surface would be some equation which would gradually change from a velocity v_0 at the surface ($x = 0$) to an approach to the probability distribution as the depth x became greater and greater. It is manifestly a difficult matter to write such an equation empirically.

I have assumed as an *approximation* for the decrease of velocity for small depths of penetration a modification of Widdington's law of the following form:

$$(v_0')^2 = v_0^2 \left[1 - \left(\frac{x}{R} \right)^a \right]. \quad (13)$$

R is the maximum range of the electrons. The exponent a is a constant whose value may depend on the nature of the target.

¹ Proc. Royal Soc., April, 1912.

From equation (13),

$$\nu_0' = \nu_0 \left[1 - \left(\frac{x}{R} \right)^a \right],$$

$$d\nu_0' = -\nu_0 \frac{a}{R} x^{a-1} dx.$$

By substitution these two give

$$dx = -\frac{R}{a\nu_0} \left(1 - \frac{\nu_0'}{\nu_0} \right)^{\frac{1-a}{a}} d\nu_0'. \quad (13)$$

By means of equations (11) and (13), equation (10) becomes

$$w = kN\nu^2 d\nu \frac{R}{a\nu_0} \int_{\nu_0'}^{\nu} \frac{d\nu_0'}{\nu_0'} \left(1 - \frac{\nu_0'}{\nu_0} \right)^{\frac{1-a}{a}}.$$

The indefinite integral of this cannot be obtained, but by expanding and taking four terms:

$$\begin{aligned} w = kN \frac{R}{a} \nu_0 \left(\frac{\nu}{\nu_0} \right)^2 d\nu \left\{ \log \frac{\nu_0}{\nu} - \frac{1-a}{a} \left[1 - \frac{\nu}{\nu_0} \right] \right. \\ + \frac{(1-a)(1-2a)}{4a^2} \left[1 - \left(\frac{\nu}{\nu_0} \right)^2 \right] \\ - \frac{(1-a)(1-2a)(1-3a)}{18a^3} \left[1 - \left(\frac{\nu}{\nu_0} \right)^3 \right] \\ \left. + \frac{(1-a)(1-2a)(1-3a)(1-4a)}{96a^4} \left[1 - \left(\frac{\nu}{\nu_0} \right)^4 \right] \right\}. \quad (14) \end{aligned}$$

The above expresses the energy, lying between a frequency interval ν and $\nu + d\nu$, radiated by N electrons upon their impact with the target of the X-ray tube. The frequency ν_0 is the end frequency or the highest frequency, and is that emitted by those electrons that make impact at the poles (or normal impact) with the first layer of atoms in the surface of the target.

It is more convenient for comparison with experimental results to express equation (14) in terms of the wave-lengths.

$$\begin{aligned} w = kN \frac{R}{a} \frac{1}{\lambda_0^3} \left(\frac{\lambda_0}{\lambda} \right)^4 d\lambda \left\{ \log \frac{\lambda}{\lambda_0} - \frac{1-a}{a} \left[1 - \frac{\lambda_0}{\lambda} \right] \right. \\ + \frac{(1-a)(1-2a)}{4a^2} \left[1 - \left(\frac{\lambda_0}{\lambda} \right)^2 \right] \\ - \frac{(1-a)(1-2a)(1-3a)}{18a^3} \left[1 - \left(\frac{\lambda_0}{\lambda} \right)^3 \right] \\ \left. + \frac{(1-a)(1-2a)(1-3a)(1-4a)}{96a^4} \left[1 - \left(\frac{\lambda_0}{\lambda} \right)^4 \right] \right\}, \quad (15) \end{aligned}$$

Equation (15) is compared to the experimental results in Figs. 3, 4 and 5 where the broken line represents the plot of the equation. Fig. 3

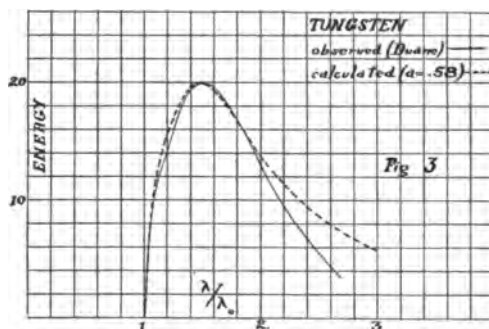


Fig. 3.

(the full line) represents the experimental results obtained by Duane and Hunt for a Coolidge tube with a tungsten target.

Figs. 4 and 5 represent the results for tungsten and molybdenum tar-

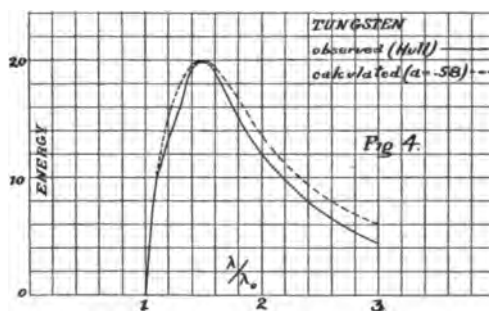


Fig. 4.

gets, kindly furnished me by Dr. A. W. Hull. In all of these curves the abscissæ represent the ratio of any wave-length to the shortest or end wave-length. I believe that none of these curves have been corrected

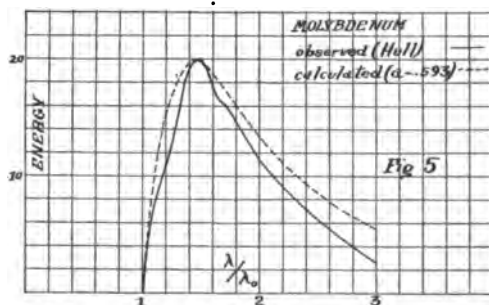


Fig. 5.

for the variation of absorption with the wave-length nor for the dependence of the reflectivity of the crystal upon wave-length.

In calculating equation (15) such a value of a was chosen as would make the maximum of the calculated curve coincide with that of the observed curve. It will be observed in Figs. 1, 4 and 5 that the maximum does not occur at the same ratio of λ/λ_0 for tungsten and molybdenum, but that this ratio is less for the element of smaller atomic number. The constant a also possesses the property of controlling the position of the maximum of the calculated curve. The agreement between theory and experiment is perhaps as good as might be expected, since the assumed law of loss of velocity of the electrons as they penetrate the target can only approximately represent the actual state of the case.

RELATION BETWEEN THE ENERGY RADIATED AT CONSTANT WAVE-LENGTH AND THE VOLTAGE APPLIED TO THE X-RAY TUBE.

Professor Duane and F. L. Hunt¹ also investigated the radiation emitted at a *constant wave-length* when the voltage applied to the X-ray tube was varied. The angle of reflection from the crystal was kept constant and the applied voltage was increased by steps. No radiation was observed at voltages less than that given by the relation

$$Pe = h\nu = \frac{hc}{\lambda},$$

but the radiation increases very rapidly with increase of voltage.

The theory here proposed (equations 14 and 15) also expresses a relation between the voltage and the radiation at constant wave-length. The variable in the energy equation in this case is ν_0 while ν remains constant. The constant frequency ν may be expressed by

$$Pe = h\nu,$$

while the variable frequency ν_0 is connected to the variable voltage P_0 by the relation

$$P_0e = h\nu_0.$$

From these we obtain

$$\frac{\nu}{\nu_0} = \frac{P}{P_0}.$$

If P/P_0 is substituted for ν/ν_0 in equation (14) and for ν_0^2 we write $(e/h)^2 P_0^2$, the equation expresses the energy emitted at constant frequency as a function of the applied voltage. A plot of equation (14) so modified is shown in Fig. 6. The theory indicates that the energy in-

¹ Proc. Amer. Phys. Society, PHYS. REV., August, 1915.

creases rapidly with increasing voltage, but attains a maximum value. This maximum was not observed by Duane and Hunt since their experiments did not extend over a sufficient range of voltage.

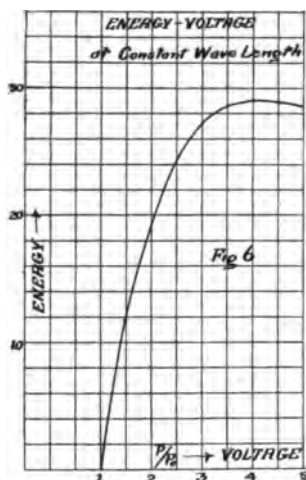


Fig. 6.

THE TOTAL ENERGY RADIATED AT AN APPLIED VOLTAGE P_0 .

The total energy radiated at all wavelengths by an X-ray tube when operated at a voltage P_0 may be found by taking the total area under the curves (Figs. 3, 4, 5). This may be readily accomplished by the integration of equation (14) between the limits $\nu = 0$ and $\nu = \nu_0$.

For a given value of the constant a the terms within the bracket contribute only a numerical constant to the valuation of the integral.

Let J represent this numerical constant.

The integration of (14) gives

$$W = JkN \frac{R}{a} \nu_0^2. \quad (18)$$

Since

$$P_0 e = h \nu_0^2,$$

this becomes

$$W = \frac{k}{h^2} (JNe^2) \frac{R}{a} P_0^2. \quad (19)$$

The total energy radiated in the continuous spectrum (not including the characteristic) is proportional to the *square of the applied voltage*.

This result agrees with the experimental results of Beatty¹ who found that the total energy radiated was proportional to the atomic weight of the target and to the square of the voltage operating the X-ray tube.

In the development on the basis of the quantum hypothesis

$$k = Ch^2.$$

Let the constants JNe^2 be represented by K .

Equation (19) becomes

$$W = KC \frac{R}{a} P_0^2. \quad (20)$$

The range R , that is, the number of collisions of the electrons before losing their energy may be expected to depend on the atomic weight of the element composing the target.

¹ Proc. Roy. Soc., vol. 89, 1913.

Since the constant a varies but little with the atomic weight, one may place a/R proportional to the atomic weight of the target. Also the constant C expresses the probability of an electron making the kind of impact that is effective in producing radiation. This probability may be expected to depend in some way on the atomic weight. It will be assumed that C is proportional to the square of the atomic weight. Equation (20) becomes

$$W_1 = KAP_0^2. \quad (21)$$

where A represents the atomic weight of the element composing the target. This interpretation of the constants brings the theoretical equation in agreement with the results obtained by Beatty for the dependence of the energy upon the atomic weight.

The above result is obtained from the equation developed on the quantum hypothesis. The equation derived on the pulse hypothesis does not contain a natural constant that one may reasonably set proportional to the square of the atomic weight.

I wish to express my acknowledgment to my colleague, Professor A. P. Wills, for valuable assistance and discussion.

PHENIX PHYSICAL LABORATORY,
COLUMBIA UNIVERSITY,
November, 1915.

TEMPERATURE COEFFICIENT OF CONTACT POTENTIAL.
A REJOINDER.

BY K. T. COMPTON.

PROFESSOR SANFORD has recently published in this journal¹ a criticism of work reported by the writer in an earlier issue.²

Professor Sanford first suggests that, in the attempt to measure the contact difference of potential between "the opposing nickel and iron faces, respectively, of two hollow metal boxes," the metal of which the remaining parts of the boxes are constructed would influence the results through the formation of electrolytic cells when the boxes are filled with water. He neglects the fact, however, that the internal resistance of such a cell so far exceeds its external resistance that the resulting potential difference between metal parts of a box is entirely inappreciable. A simple calculation shows that in our apparatus the error introduced would not exceed $4(10)^{-9}$ volt, which is about a million times smaller than the variation in contact difference of potential which we measured.

The second criticism is based on the assumption that we allowed the boxes to remain connected, through tubes, with the water reservoir while making measurements, and that we consequently measured the temperature variation of the electrolytic contact potential. "If this were so it were a grievous fault." I had not thought it necessary to state that both boxes were disconnected from the reservoir while measurements were in progress, and the diagram shows plainly that the earthing connections were made directly to the metal boxes and not to the water reservoir.

Finally, Professor Sanford states that, in deriving Professor Richardson's formula $V_m - V_s = (1/e)(\phi_m - \phi_s) + P$, the constancy of the term $(\phi_m - \phi_s)$ as the temperature varies is *assumed*, contrary to apparently reliable experimental evidence regarding the temperature variation of contact potentials. This is, of course, part of the very theory being tested; but the sufficient constancy of the term $(\phi_m - \phi_s)$ is not assumed, but is a logical consequence, according to the theory, of experi-

¹ PHYS. REV., N. S., 8, p. 95, 1916.

² PHYS. REV., N. S., 7, p. 209, 1916.

mental evidence regarding the magnitude of the Thomson coefficient for metals.¹

It appears, therefore, that Professor Sanford's criticisms are without foundation. The essential similarity of results obtained with different experimental conditions, and direct evidence cited in the writer's original paper, indicate that the discrepancy between theory and experiment is probably due to the presence on the metal of surface films of gas or oxide. To make a valid test it is therefore necessary to devise experimental arrangements to eliminate the films or to extend the theory to include the effect of the films.

PALMER PHYSICAL LABORATORY,
PRINCETON, N. J.

¹ Richardson, "Electron Theory of Matter," p. 452 et seq.

NOTE ON THE DETERMINATION BY JUDGMENT OF THE CONSTANTS OF LINEAR EMPIRICAL FORMULAS.

BY HARRY M. ROESER.

ALTHOUGH physicists are agreed that a true linear variation between two characters is quite uncommon in physical relations, yet there are a great number of cases where a straight line suffices to represent a relation over a limited region well within the errors of observation. In such cases the data may be submitted to computation and the constants of the "most probable" line evaluated by the method of least squares. Such a line is the best that can be had by any means to represent the data. However, many times experimenters feel that on account of the accuracy of the data, or, their implicit faith in their ability to locate a line by eye, that the necessary though rather moderate amount of computation involved in the prim rigor of a least square solution is unjustified. Then the observations are plotted to an appropriate scale and a stretched thread or transparent straightedge shifted about among the points until a position is found that seems to best satisfy the relation between the variables. The "criterion" for location is that the sum of the deviations from the line of the points above should be equal to the sum of the deviations of those below.¹

In handling the problem according to the latter scheme, evidently a point, fixed in the distribution, through which it be known that the best representative line must pass, and about which the straightedge might be rotated until the best location was found, would be of great practical value. Herein is shown how by a very natural and simple calculation *one* point on the most probable line may be rigorously determined in any assumed linear distribution whatever.

Let $x_1, y_1; x_2, y_2; \dots x_i, y_i; \dots x_n, y_n$, be n observations on two quantities x and y which vary according to the relation $y = a + bx$. Let x_m be the mean of the observed values of x , and y_m be the mean of the observed values of y . Then the line that would be given by a least square solution for the constants, a and b , passes through the point $x, y \equiv x_m, y_m$. A proof follows.

¹ Goodwin, Precision of Measurements and Graphical Methods, p. 44.

Consider the "normal equations,"¹ the solution of which give the constants, a and b , by least squares.

$$n \cdot a + b \cdot \Sigma x = \Sigma y. \quad (A)$$

$$a \cdot \Sigma x + b \cdot \Sigma x^2 = \Sigma xy. \quad (B)$$

The relation between x and y being $y = a + bx$.

Solve for a in equation (A),

$$a = \frac{\Sigma y}{n} - b \cdot \frac{\Sigma x}{n} = y_m - b \cdot x_m.$$

Substitute this value of a in $y = a + bx$ (the equation of the line to be satisfied),

$$y = y_m - bx_m + bx = y_m + b(x - x_m). \quad (C)$$

Hence when $x = x_m$, then $y = y_m$, which was to be shown.

This idea admits of generalization to the case where the observations are of different weight. Let $p_1, p_2 \dots p_n$, be respectively the weights of the above n observations on x and y .

The normal equation (A) then becomes,

$$a \cdot \Sigma p + b \cdot \Sigma px = \Sigma py,$$

whence,

$$a = \frac{\Sigma py}{\Sigma p} - b \cdot \frac{\Sigma px}{\Sigma p},$$

which substituted in $y = a + bx$ gives,

$$y = \frac{\Sigma py}{\Sigma p} + b \left(x - \frac{\Sigma px}{\Sigma p} \right),$$

whence,

$$y = \frac{\Sigma py}{\Sigma p} = y_m', \quad \text{when} \quad x = \frac{\Sigma px}{\Sigma p} = x_m'.$$

In this case x_m' and y_m' are what are termed "general"² means of the observed values of x and y respectively.

The point $x, y \equiv x_m, y_m$, or $x, y \equiv x_m', y_m'$, being located near the middle of the range, it serves very advantageously in assisting to locate the line without other arithmetical labor than computing two averages. After the line is located the constants, a and b , are picked from the plot, a being the intercept on the Y -axis and b the slope.

A word of warning is in order here. If, according to the very nature of things, it be known beforehand that regardless of the data the line

¹ It is not considered necessary to enter into any detail concerning the intricate theory of the formation of the normal equations. See any current work on least squares.

² See Merriman's, *Least Squares*, 7th edition, 1913, p. 42, Bartlett, *Least Squares*, p. 7.

must pass through some known point,¹ x_0, y_0 , the above scheme does not give a second point and thus completely determine the line. In this case the known point, x_0, y_0 , will be a point in the distribution having infinite weight and consideration of this fact in taking the general mean gives $x_m' \equiv x_0, y_m' \equiv y_0$. However should the practical conditions surrounding the experiments be such as to vitiate the absoluteness of the theoretical position of the line, *e. g.*; calibration curves of certain types of instruments having on account of friction more or less indefinite zero error. The point x_0, y_0 loses its weight and is no longer truly an observational value. In cases of this kind the above scheme may properly be applied to locating a point near the middle of the range, the theoretical point, x_0, y_0 , being neglected.

The usual "practical" test for location of a line, *i. e.*, that the algebraic sum of the deviations of the observed points from the line should be zero, is satisfied by any line drawn in any direction through the point $x, y \equiv x_m, y_m$, and no line will satisfy the test if it does not pass through this point. As previously carried out, adjusting a line to suit the above conditions amounts to nothing more than an extremely roundabout method of placing a line through the point, x_m, y_m . Knowing beforehand that the line must pass through x_m, y_m invalidates the test completely. This last fact should save much labor to graphical analysts inasmuch as a great deal of the necessity of a "residual" plot is eliminated. See Goodwin, *Precision of Measurements and Graphical Methods*, p. 60.

U. S. BUREAU OF STANDARDS,
WASHINGTON, D. C.,
September 22, 1916.

¹ As in the case of a uniformly heated rod, $L = L_0 + L_0\beta t$, where L is the length at temperature t and β is the coefficient of expansion. Here the line is obliged to pass through the point $L, t \equiv L_0, 0$, L being the known length at 0° .

PROCEEDINGS
OF THE
AMERICAN PHYSICAL SOCIETY.

MINUTES OF THE CLEVELAND MEETING.

THE eighty-fourth regular meeting of the Physical Society was held at Nela Research Laboratory, National Lamp Works of the General Electric Co., Nela Park, Cleveland, Ohio, October 27-28, 1916.

There were three sessions for the reading of papers, on Friday afternoon, Saturday morning and Saturday afternoon. President Millikan presided. The following twenty-two papers were presented:

The Precision of Photometric Measurements. F. K. RICHTMYER and E. C. CRITTENDEN.

The Emissive Powers of Tungsten for Short Wave-Lengths. E. O. HULBURT.
On Photoelectric Photometry. JACOB KUNZ.

The Reflecting Power of Tungsten in the Infra-red at Incandescent Temperatures. W. WENIGER and A. H. PFUND.

Note on a Comparison of High Temperature Scales. W. E. FORSYTHE.
(By title.)

An Attempt to Detect a Change in the Emissive Properties of Platinum and of Tungsten with a Change in the Method of Heating. A. G. WORTHING.

Photo-electric Potentials in the Schumann Region of the Spectrum. P. E. SABINE.

Electro-chemical Analogies of Photo-chemistry. W. R. MOTT.

Scales for Calculating Watts-per-Candlepower and Candle Power for Tungsten Lamps. HERBERT BELL.

An Ionization Manometer for High Vacuum Measurements. O. E. BUCKLEY.

So-called Ionization Pressure of the Corona Discharge. H. D. ARNOLD.

Experiments on the Production of Metallic Spectra by Cathode Luminescence. ARTHUR S. KING and EDNA CARTER.

The Crystal Structure of Iron. ALBERT W. HULL.

Spontaneous Generation of Heat in Recently Hardened Steel, II. C. F. BRUSH.

Unipolar Induction and the Electron Theory. GEORGE B. PEGRAM.

The Structure of the Surfaces of Liquids and a New Theory of Surface Tension. IRVING LANGMUIR.

A New Thermometer Scale. ALEXANDER MCADIE. (By title.)

Improved Forms of Mercury Vapor Vacuum Pumps: The Condensation Pump. IRVING LANGMUIR.

The Spectra of the B-rays of Radium D E and of Radium and its Products, Determined by the Statistical Method. ALOIS F. KOVARIK and L. W. MCKEEHAN.

The Dielectric Constant of Aqueous Solutions. E. A. HARRINGTON.

An Investigation of the Relative Sensibility of the Average Eye to Light of Different Colors, and some Practical Applications to Radiation Problems. W. W. COBLENTZ and W. B. EMERSON.

Transformation Equations of Tolman and the Photoelectric Effect. JACOB KUNZ.

At a short business session the following resolution was presented by the Council and on motion, adopted.

"WHEREAS, The Natural Academy of Science at the request of the President of the United States has organized the National Research Council for the purpose of bringing into cooperation existing governmental educational, industrial and other research agencies, with the object of encouraging the investigation of natural phenomena, the increased use of scientific research in the development of American industries, the employment of scientific methods in strengthening the national defense and such other applications of science as will promote the national security and welfare, and

"WHEREAS, The American Physical Society is itself organized to promote research in that subject whose methods and results lie at the bottom of well nigh all industries and all sciences.

"Therefore be it resolved that the American Physical Society pledge its active and full and continuous support to the National Research Council, and express its readiness to cooperate with it to the largest possible extent in accomplishing the purposes sought in its appointment."

The registration for the meeting numbered forty-five. Visiting members were the guests of the General Electric Company for lunch on both days of the meeting, and also at a dinner given at the Mayfield Country Club Friday evening. At the dinner an opportunity was given members of the society to meet a number of prominent manufacturers of Cleveland, whose business involves important applications of physical principles. For the success of this and other special features of the meeting the society is much indebted to Director Hyde of the Nela Research Laboratory and to members of his staff.

A. D. COLE,
Secretary.

THE CRYSTAL STRUCTURE OF IRON.¹

BY A. W. HULL.

IN the X-ray analysis of iron a special procedure is necessary on account of the difficulty of obtaining large crystals. The analysis was made in

¹ Abstract of a paper presented at the Cleveland Meeting of the Physical Society, October 27, 28, 1916.

two parts. First, a sample of silicon steel, containing about 3.5 per cent. Si and practically no carbon, was analyzed. This has the advantage over pure iron that the crystals, formed by long annealing at high temp., are so hard that they can be split along cleavage planes without being distorted. Single crystals about 6 mm. square, and 2 mm. thick were isolated and mounted so that a narrow beam from a tungsten target passed through the crystal nearly parallel to one of its principal planes. As the crystal was slowly rotated several complete spectra, formed by this plane and others in the same zone, were spread out on the photographic plate as long straight traces. The angles between these traces give the indices of the respective planes, and the position of the *K* lines on each trace the spacing of the plane. By rotating about different axes the same plane could be observed in different zones, and its identity and spacing checked. The values obtained are given in Table I.

TABLE I.

Indices of Plane.	Spacing. A. U.
100	1.43
110	2.02
111	.82
211	1.15
310	.90
321	.75

The value of $d_{100} = 1.43 \times 10^{-8}$ cm. requires two atoms for a cube of side $2d_{100}$. A lattice having atoms at cube corners and cube centers satisfies this condition, and gives the observed spacing for the other planes.

Pure iron was then investigated in the form of very fine powder, obtained by reduction of the oxide with hydrogen. A narrow beam of rays from a tungsten target passed through the powder and formed on the photographic plate a kind of generalized Lane photograph, in which every possible plane in the crystal structure had an equal opportunity of reflecting, and reflected all wave-lengths present. What was actually observed was the position of the *K* lines, which, with the tube running at 110,000 volts, stood out very clearly on the continuous background. The reflection of these lines in different planes appeared on the plate as concentric, nearly circular, lines, whose distance from the center should be inversely proportional, approximately, to the spacing of the planes. The distance of these lines from the center can be measured and compared with the values calculated for the assumed crystal structure. If the assumed structure is correct, every calculated line must be present, and no more, and the intensity must fall off in the manner predicted.

The observed and calculated values are given in Table II. In the calculation it was first assumed that the atoms were arranged on a centered cubic lattice, and that the scattering electrons in each atom were concentrated at its center. The spacings calculated on this assumption are given in column 3, and the intensities in column 5, under *A*. They agree remarkably well with the observed values, but fail to account for three facts, viz.: (1) That the

TABLE II.

Indices of Plane.	Distance of Line from Center.		Intensity of Line.			
	Obs.	Calc.	Obs. ¹	Calc.		
				A.	B.	C.
110	.703	.71	1.00	1.00	1.00	1.00
100	1.00	1.00	.46	1.00	.60	.54
211	1.23	1.22	.54	1.00	.60	.46
110(2)	1.42	1.41	.24	1.00	.60	.39
310	1.59	1.58	.18	1.00	.40	.13
111	1.74	1.73	.16	1.00	.60	.29
321	1.89	1.87	.22	1.00	.60	.24
100(2)	—	2.00	—	1.00	.20	.0003
411						
110(3)	2.15	2.12	.12	2.00	1.40	.41
210	2.26	2.24	.03	1.00	.60	.12
332	2.38	2.35	.02	1.00	.60	.15
211(2)	2.50	2.45	.02	1.00	.60	.11
431						
510	2.58	2.55	.10	2.00	1.40	.27
521	2.74	2.74	.02	1.00	.60	.04

intensity of the lines falls off continuously with increasing distance from the center. (2) That the first order 100 reflection is much too weak for its position. (3) That the second order 100 reflection is entirely lacking.

It is very difficult to conceive of any arrangement of point atoms which will satisfy these conditions and still give all the observed lines. We are forced, I think, to look for the explanation in the internal structure of the atoms. If it is assumed that eight of the 26 electrons in each atom are arranged along the cube diagonals at a distance from the center equal to one-fourth the distance to the nearest atom, calculation gives the values of intensity shown in column 6 under *B*. It is seen that condition (2) above is satisfied, and (3) is nearly accounted for, but not (1). If all the electrons are displaced from the center of the atom along the cube diagonals in 4 groups of 2, 8, 8, 8, at distances $1/32$, $1/16$, $1/8$ and $1/4$, respectively, of the distance to the nearest atom, all the observed facts are accounted for within the limit of experimental error (*C*, column 7).

This is obviously only a rough approximation to the correct position of the electrons in the atom, and more accurate data will enable us to place them more exactly. The excellent agreement with experiment, however, indicates that it is a step in the right direction, and gives promise that we shall be able, by this method, to determine the positions of all the electrons in the atom.

It may be noted that the above arrangement of electrons and atoms gives to iron the correct valence, corresponding to its position in the periodic table, and suggests a very interesting mechanism for ferromagnetism.

¹ The intensity of the first seven lines was measured with a photometer. The rest were estimated.

The calculation of intensities is based upon four assumptions, which can only be briefly stated here. It is assumed: (1) That the rays, by the time they reach the crystal, consist of long trains of waves, to which the electromagnetic theory is applicable; (2) that the scattering is due entirely to the electrons in the atoms, each of which scatters independently and equally; (3) that the number of electrons in an atom is equal to its atomic number; (4) that the electrons have fixed positions in the atom, about which they move in very small orbits.

A full discussion of these assumptions, and of the experimental data, will be published shortly in the *PHYSICAL REVIEW*.

RESEARCH LABORATORY,
GENERAL ELECTRIC COMPANY,
SCHENECTADY, N. Y.

AN INVESTIGATION OF THE RELATIVE SENSIBILITY OF THE AVERAGE EYE TO LIGHT OF DIFFERENT COLORS, AND SOME PRACTICAL APPLI- CATIONS TO RADIATION PROBLEMS.¹

BY W. W. COBLENTZ AND W. B. EMERSON.

IN the present investigation the methods are practically the same as used by previous experimenters. In the visual measurements, the spectral light was compared with a standard white light by means of a flicker photometer, and also an equality of brightness photometer. The source of white light was a standardized vacuum tungsten lamp. A cylindrical acetylene flame was used as a source of spectral light. The distribution of energy in the spectrum of the acetylene flame was determined with great care in view of the fact that the disagreements in previous work seemed to be due, in part, to uncertainties in radiometrically evaluating the light stimulus.

Sensibility curves were obtained on 130 persons, of which number 5 were color-blind. The visibility curve of the average normal eye, using 125 observers is wider than previously observed. These data are given in Table I., using a flicker photometer. Only a few observers were able to make accurate settings with the equality of brightness photometer.

As was to be expected the visibility curves of no two persons appear to be exactly alike. When a visibility curve does not coincide with the average there is usually a marked departure from the average visibility in a given spectral region. This gives rise to (1) wide visibility curves with the maximum shifted toward the red, *i. e.*, "red sensitive," (2) narrow curves with a sharp maximum in the green and (3) curves with the maximum shifted toward the violet.

The data available indicate that 60 per cent. of the cases examined fall into three quite evenly divided groups (*i. e.*, 20 per cent., roughly estimated

¹ Abstract of a paper presented at the Cleveland meeting of the Physical Society, October 27, 28, 1916.

TABLE I.

Average Visibility Curve for Even Wave-Lengths, 125 Observers.

Wave Length.	Visibility.	Wave Length.	Visibility.	Wave Length.	Visibility.
0.400 μ	0.010	0.520 μ	0.710	0.640 μ	0.194
0.410	0.017	0.530	0.862	0.650	0.116
0.420	0.024	0.540	0.954	0.660	0.0670
0.430	0.029	0.550	0.994	0.670	0.0360
0.440	0.033	0.560	0.998	0.680	0.0194
0.450	0.041	0.570	0.968	0.690	0.0095
0.460	0.056	0.580	0.898	0.700	0.0047
0.470	0.083	0.590	0.800	0.710	0.0024
0.480	0.125	0.600	0.687	0.720	0.0012
0.490	0.194	0.610	0.557	0.730	0.0007
0.500	0.316	0.620	0.427	0.740	0.0004
0.510	0.503	0.630	0.302	0.750	0.0003

in each group) which are either (1) red sensitive (2) blue sensitive or (3) average. Similarly 30 per cent. of the cases examined are quite evenly divided into three groups which fall below the average sensibility in either (1) the red or (2) in the blue or (3) in both the red and the blue, thus giving rise to an apparently high sensitivity in the green. One person in 20 has a very wide visibility curve; while 4 per cent. are color blind, *i. e.*, they confuse colors.

The point of maximum sensibility is very different for different observers, and for the 125 persons the maximum is at $\lambda_m = 0.5576\mu$. The curve of average visibility, when corrected for selective transmission of the ocular media, including the yellow spot, is very symmetrical.

The complete paper gives an empirical equation of the visibility curve determined. Using this visibility equation and Planck's equation of black body radiation, calculations are given of the luminous energy emitted by a black body at various temperatures, also the luminous efficiency, the Crova wave-length and the mechanical equivalent of light. Using the recent measurements of the brightness of a black body as determined by Hyde, Cady and Forsythe and the most probable values of the radiation constants ($C_2 = 14,350$, $\sigma = 5.7 \times 10^{-12}$) the value is 1 lumen = .00161 watt of luminous flux; or 1 watt (of radiation of maximum visibility) = 621 lumens = 49.5 candles. The direct determination of 613.6 lumens for green mercury radiation, $\lambda = .5461\mu$ (made by Ives, Coblentz and Kingsbury, using 61 observers) when corrected for visibility V (at $\lambda = .5461\mu$) = .985 V_m , gives 622.8 lumens per watt, which is in good agreement with the present determination.

BUREAU OF STANDARDS,
WASHINGTON, D. C.,
October 16, 1916.

ELECTROCHEMICAL ANALOGIES OF PHOTOCHEMISTRY.¹

BY WILLIAM ROY MOTT.

HYDRAULIC and mechanical analogies of electrical and magnetic action are well known to be very useful to physicists and electrical engineers.² It is my firm belief that likewise very useful analogies can be shown to exist between electro-chemistry and photochemistry. In line with this, Prof. Bancroft wrote in 1908 an interesting paper, entitled "Electrochemistry of Light"³ in which the theory of Grotthus was revived. The Grotthus theory of the electrochemistry of photochemistry is rather general and is fairly consistent with our modern electron theory of matter and the electromagnetic nature of light.⁴ However, the value of a theory is measured by its power of prediction and in this respect Grotthus's theory has not accomplished much.

As a result of my experiments on dye fading and other photochemical reactions,⁵ I have come to the conclusion that the general theory of the electrochemistry of photochemistry can be revised to a more concrete form in which the two electrochemical factors of current and voltage are respectively related to amount of light and wave-length of light.

Electrochemical Analogy of Current and Amount of Light.—The first law of photochemistry is that the amount of chemical action is nearly proportional to the energy of the light absorbed in the photochemical reaction chamber. The first law of electrochemistry is that the amount of electrolysis is proportional to the current passed through the electrochemical cell. (This law in its complete form with reference to electrochemical equivalents, is known as Faraday's law of electrolysis.) (The law of photochemistry is generally known as Draper's law although Grotthus also proposed it.) The amount of photolysis is proportional to absorbed energy provided the wave-length of light is of a short enough value. This is anticipating our next section, however, but the same limitation exists as regards Faraday's law of electrolysis where sufficient voltage must be supplied to exceed the decomposition voltage of the material being electrolyzed. Further decrease in wave-length or increase in the voltage of a cell does not affect respectively the proportion of photolysis or of electrolysis. The physicist Lasareff⁶ showed that dye fading in respect to spectral distribution depends on amount of absorbed light energy. He says "Within the limits of experimental error, the amount of dye stuff which is destroyed (by the light) is directly proportional to the amount of absorbed

¹ Abstract of a paper presented at the Cleveland meeting of the Physical Society, October 27, 28, 1916.

² See book on Electricity and Magnetism, by W. S. Franklin and B. MacNutt.

³ See Am. Electrochem. Soc., 13, p. 244.

⁴ See Henri Compt. rend., vol. 156, p. 1979.

⁵ See "Use of Flame Arc in Paint and Dye Testing Trans. Am. Electrochem. Soc. vol. 28, p. 371 1915; also "The Flame Arc in Chemical Manufacture," C. W. Bedford and writer, Jour. of Ind. & Eng. Chem., vol. 8, p. 1029, 1916.

⁶ Ann. der Physik, 1907, 24, p. 661.

energy and is independent of the wave-length of the incident ray in an absorption band (of the dye solution).” Also V. Henri and R. Wurmser¹ found that there is a striking similarity between the absorption curve for acetone in ultraviolet and the chemical activity of the different rays. Hence there is shown a working analogy between current in electrolysis and light energy absorbed in photolysis.

The great importance of absorbing media in photochemistry leaves no apparent relation to electrochemical equivalents as in electrochemistry in going from one material to a different material. This is the natural limitation of the analogy. However, with a given material under photolysis the departure from apparent proportionality to amount of absorbed light energy can be explained in general in a way parallel to apparent departures of electrochemical reaction in a given electrolyte from Faraday's law. For instance, at a current of one millionth of an ampere per square centimeter experiments by Dr. H. E. Patton and the writer in 1903 showed extremely little electrochemical polarization,—due to what is electrochemically known as residual currents. In the same way in photochemistry, there is a limiting low exposure beyond which no apparent record of photolysis is obtained.² As the coulombs or the candle power meter seconds exposure are increased, a marked similarity of phenomena can be noted. The region of residual current is generally due to a secondary reaction calling for extremely little energy and according to the law of least work, the easy reaction takes place before the reaction calling for more energy. A marked case of this kind with electrolytic cells is obtained by mixing certain oxidizing and reducing agents—for example ferrous and ferric salts and then large residual currents can be readily passed with little or no polarization—although no doubt chemical analysis at each electrode would show Faraday's law to have been obeyed. This brings us then to the voltage factor of electrolysis versus the intensity factor in photolysis.

Electrochemical Analogy of Voltage and Wave-length.—The intensity factor or voltage analogy is found in light in increased frequency (shorter wave-lengths giving greater energy intensity). As a rule, the shorter wave-lengths can decompose the more stable compounds (not decomposed by longer wave-lengths) calling for a higher decomposition voltage electrochemically. It is well known that infra-red rays are extremely lacking in power to cause chemical change whereas ultraviolet rays affect chemically an enormous number of substances. In photoelectric currents, much physical work has shown that certain limiting longer wave-lengths were not effective. The curves of these effects with reference to wave-length of light are quite similar to decomposition curves in which the critical wave-length would hold analogous place to voltage of decomposition.

Berthelot and Gaudechon³ have shown that high frequency of vibration of light plays the same rôle as high temperatures in heat reaction. I extend this

¹ Compt. rend., 1912, 155, 503.

² See P. Villard, J. Physique, 4, 6, 369-79, 445-57, 1907.

³ Compt. rend., 156, p. 889, p. 1243, 1913.

to say that high frequency of vibration of light plays the same rôle as high decomposition voltages in electrolysis. For instance the halogen compounds illustrate very well the analogy, the fluorides being most stable and resisting light to a marked degree. For example silver fluoride is said to be not sensitive to light. The bromides are more easily decomposed than the chlorides or fluorides, consistent with their decreased voltage of decomposition. The following table illustrates the generalization.

Material.	Formula.	Voltage for Dec. ¹	
Calcium fluoride.....	CaF ₂	4.7	Most transparent to far ultraviolet.
Hydrofluoric acid.....	HF	2.2	Not decomposed by far ultraviolet.
Hydrochloric acid.....	HCl	1.7	Slightly decomposed by far ultraviolet.
Hydrobromic acid.....	HBr	1.2	Easily decomposed by ultraviolet.
Hydriodic acid.....	HI	.6	Easily decomposed by blue light.

A curve could probably be constructed between wave length of light required and decomposition voltage required. It is interesting to note that the extremely short waves are readily absorbed by everything except the *non-reactive* noble gases.

This theory can be further expanded to show a relationship between heat of formation (on electrochemical equivalents) and refractive index and point of probable absorption of light as the curve is carried into ultraviolet region. Also, the theory has fascinating possibilities in predicting the character and probability of light production by chemical change.

RESEARCH LABORATORY,
NATIONAL CARBON COMPANY,
CLEVELAND OHIO.

A NEW THERMOMETER SCALE.²

BY ALEXANDER MCADIE.

IN a previous communication³ the writer urged the importance of adopting scientific units in aerophysics. At Blue Hill Observatory, summaries of air pressure and vapor pressure are published in kilobars or force units, temperature in degrees Absolute centigrade; and wind velocities in meters per second. The latest publication of the British Meteorological Office, the Réseau Mondial, gives world data for 1911 in similar units, a noteworthy advance. Passing for the present, the true conception of a megadyne atmosphere and the proper definition of the *bar* as a basic unit, representing the

¹ Calculated from thermal data and actual electrolysis of solutions of halogen acids. See writer's paper, Am. Electrochem. Soc., V., p. 85, 1904.

² Abstract of a paper presented at the Cleveland meeting of the Physical Society, October 27, 28, 1916.

³ PHYSICAL REVIEW, Vol. VI., No. 6, Dec., 1915.

force which would impart to a gram, an acceleration of one centimeter per second; and not the multiple erroneously used by Bjerknes and other European meteorologists, we may ask, at this stage whether the Absolute scale is best adapted to the needs of meteorologists and climatologists. Physicists will generally concede that the Fahrenheit scale has outlived its usefulness. Many meteorologists however have opposed the introduction of the Centigrade scale (and therefore the Absolute) on the ground that even when read to tenths, the scale division of the latter was too large for meteorological purposes. To meet this objection the writer suggests a new scale to be known as the New Absolute or briefly New. The zero of the new scale will be the same as that of the absolute, approximately 491 degrees below freezing on the Fahrenheit and 273 degrees below on the Centigrade. The other fiducial point is the temperature of melting ice at a pressure of 1,000 kilobars and is marked 1000°. Thus the scale divisions .366° of a Centigrade degree are even smaller than the Fahrenheit and permit of the refinement of reading desired by meteorologists. The new scale has other advantages.

First, the abolition of minus signs. In all upper air work temperatures are as a rule far below freezing. At a height of 10 kilometers we have to deal with temperatures as low or lower than those experienced by Scott in the Antarctic. For example, -66.0°F. , -54.0°C. or 218.4°A. is on the New scale 800; or if we take some of the winter surface temperatures of our own cities (Cleveland for instance) we have -17.5°F. , -27.0°C. or 245.7°A. represented by the reading 900.

Second, the great distinction between *warm* and *cold* as experienced in the every-day affairs of life is plainly marked; thus temperatures below freezing are below the 1000° mark; and temperatures above so-called summer heat are above 1100°.

Third, there is a saving of figures and increase of accuracy in tabulation and computation. The weather bureau insists on readings to the tenth of a degree. A reading, -10.8°F. requires seven pieces of type if printed. The equivalent temperature on the new scale is 915 N. (four pieces).

Fourth, the arrangement of the new scale makes for clearer conceptions of the magnitude of temperature changes. It is astonishing how vague are the conceptions of temperature held not only by the public in general but by those trained in universities and even technical schools. And I may be pardoned for quoting here even before a body of physicists from Professor Ames's little book, "The Constitution of Matter": "There is no word in our language, I think, which is so much used to conceal ignorance as 'heat' and no word about which there is so much confusion of ideas as 'temperature.'"

Finally in thermodynamic problems and also problems of radiation, the New scale would seem to be serviceable, starting as it does from the temperature of no molecular motion and laying emphasis as it does on the one great physical change of state of water, familiar to all.

BLUE HILL OBSERVATORY,
HARVARD UNIVERSITY.

SO-CALLED "IONIZATION" PRESSURE OF THE CORONA DISCHARGE.¹

BY H. D. ARNOLD.

IN articles by S. P. Farwell,² J. Kunz³ and E. H. Warner,⁴ data has been given on the sudden increase in pressure that results when a corona discharge takes place in a closed chamber. Prof. Kunz has developed a theory of the phenomenon assuming the pressure increase to be due to the presence of extra particles produced by ionization. Results by Warner appear to confirm Kunz's theory in that they show the pressure increase to be proportional to the corona current.

If, however, we compute the corona currents that would result from the presence of enough ionized particles to produce the observed pressure changes, the currents calculated are many thousand times greater than those actually obtained. It appears probable therefore that some cause other than ionization contributes largely to the pressure increases which are observed.

In his theory Professor Kunz neglects the thermal expansion which must result from the dissipation within the tube of the power represented by the product of the corona current and the corona voltage. Simple calculations show that the pressures thus generated are of the same magnitude as those observed and that the pressure effects will appear quite suddenly when the corona discharge starts. Experimental verification of this is obtained by electrically heating the central wire in apparatus similar to Farwell's and observing the resulting pressure increase.

When the same power is dissipated in the wire as was used in Farwell's corona discharge the pressure increase observed is somewhat larger than that shown in Farwell's data. This would be expected, for in the corona the power loss is distributed throughout a considerable volume and hence the heat generated can be removed with less temperature (and therefore, pressure) rise than is the case with the hot wire. The agreement between the results is however, such as to show that the magnitude and suddenness of the so-called "Ionization Pressure" can be entirely accounted for by thermal expansion of the gas.

NEW YORK CITY.

PHOTO-ELECTRIC POTENTIALS IN THE SCHUMANN REGION OF THE SPECTRUM.¹

BY P. E. SABINE.

THIS is a preliminary report of work done at the Jefferson Physical Laboratory. A monochromatic illuminator, a modification of Lyman's

¹ Abstract of a paper presented at the Cleveland meeting of the Physical Society, October 27, 28, 1916.

² Proc. A. I. E. E., November, 1914, p. 1631.

³ PHYS. REV., July, 1916, p. 28.

⁴ PHYS. REV., September, 1916, p. 285.

vacuum spectrometer, has been made and calibrated for wave-lengths between $230\mu\mu$. and $125\mu\mu$. The maximum potentials acquired by a zinc plate under the action of light of wave-lengths, between $185\mu\mu$, and $125\mu\mu$ have been measured. The surfaces had been prepared in air. The voltage-frequency relationship was found to be quite independent of the surface condition of the metal when precautions were taken to insure the constancy of these conditions while the measurements for the different wave-lengths were being made. The voltage-frequency relation was found to be strictly linear, but the slope of the line proved to be only 60 per cent. of that called for by the Einstein equation. Following Millikan's suggestion that the small slope obtained by earlier experimenters could be accounted for by the effect of stray light of short wave-lengths, the magnitude of this effect was investigated. With the monochromator set for wave-length $185\mu\mu$, and a source of light in which the longest wave-length was $167\mu\mu$, a decided photo-electric leak was observed. The voltage-current curve for this stray light effect was found to coincide very closely with that due to the strongest group of lines in the source. The stray-light effect would accordingly account for the small slope in this and probably in all other experiments in which it is not eliminated by the use of color screens. Such screens for use in the Schumann region of the spectrum have not yet been obtained.

The measurements for $\lambda 125\mu\mu$, the limit of the transparency of fluorite, were not subject to this source of error. Consequently measurements of the maximum potential attained by plates of zinc, cadmium, and copper under the action of this wave-length were made. Contact potential measurements were made simultaneously. The mean value for zinc was 6.70 volts, for cadmium 6.57 volts, and for copper 6.68 volts. These values for zinc and cadmium, taken with the earlier measurements of Hughes, corrected for contact potential difference, give values of h in the Einstein formula of $6.58 \times 10^{-27} \frac{\text{ergs}}{\text{frequency}}$ and $6.71 \times 10^{-27} \frac{\text{ergs}}{\text{frequency}}$ respectively. The value of 6.68 volts for $\lambda 125\mu\mu$ in the case of copper is not in agreement with the very low value of the slope of the voltage-frequency line for this metal reported by Richardson and Compton.

CASE SCHOOL OF APPLIED SCIENCE,
CLEVELAND, OHIO.

EXPERIMENTS ON THE PRODUCTION OF METALLIC SPECTRA BY CATHODO-LUMINESCENCE.¹

BY ARTHUR S. KING AND EDNA CARTER.

AN apparatus has been constructed in which a substance whose spectrum is to be examined may be vaporized through the heat generated by a pencil of cathode rays. These are concentrated upon it by a concave cathode,

¹ Abstract of a paper presented at the Cleveland meeting of the Physical Society, October 27, 28, 1916.

the anode being so placed as to keep the conduction current away from the anti-cathode. The whole is enclosed in a bell-jar which may be pumped out to a high vacuum.

In the preliminary experiments thus far carried out, the spectra of calcium and iron have been examined in this way, the luminescent vapor in the stream of cathode rays a short distance above the anti-cathode being examined. An electro-luminescence is thus obtained, away from direct heating effects and from the conduction current.

Twenty-six lines of calcium and forty-two lines of iron were photographed. The calcium list includes most of the stronger lines from λ_{3100} to λ_{5100} and shows what may be expected from a discharge of this kind. The spectrum resembles that of the arc more nearly than any other of the usual sources, but is distinctive in the relative intensities of the lines. Thus λ_{4227} is very strong, as in the furnace and flame, but also remarkably sharp. Two series of single lines, to which class λ_{4227} probably belongs, are much strengthened. Two pairs of lines which are enhanced in the spark spectrum are also strong here, while the series of triplets are relatively weak. The general feature of this discharge, as was also noted by Lewis in his experiments on cathodo-luminescence of metals, appears to be an ability to bring out the more fundamental vibrations. According to this view, the series of single lines, of pairs, and of triplets are of increasing complexity in their vibrations and are arranged in this order in the facility with which they are given by the cathode excitation.

The iron lines thus far photographed are relatively few compared with the richness of this spectrum and are the lines most easily brought out in flame, furnace, arc, and spark spectra. The enhanced lines of the spark and those relatively weak in the arc but strong in the furnace did not appear.

The nitrogen spectrum, consisting of both positive and negative pole bands, was given by the residue of air in the chamber. The negative pole bands were much the stronger, which is in harmony with the view that these are excited by cathode rays.

MT. WILSON SOLAR OBSERVATORY,
October 10, 1916.

THE PRECISION OF PHOTOMETRIC MEASUREMENTS.¹

BY F. K. RICHTMYER AND E. C. CRITTENDEN.

PROGRESS in any department of physical science, as was pointed out by Professor Zeleny, is dependent in large measure upon our ability to make more and more precise measurements. It is of interest, therefore, to inquire into the present available precision in the various lines of experimental work in order to determine what factors make greater precision possible. The present paper is the result of an analysis of 20,000 observations showing the precision attainable in a number of different kinds of photometric measure-

¹ Abstract of a paper presented at the Cleveland meeting of the Physical Society, October 27-28, 1916.

ments: with a Lummer-Brodhun photometer for both a color match and a color difference; also with the flicker photometer for a large range of color differences. The precision attainable with the Lummer-Brodhun was approximately 0.3 per cent. for color match; increasing color difference caused a marked decrease in sensibility. With the flicker photometer the precision 0.75 per cent., approximately independent of color difference, was obtained. By precision is here meant average deviation of a single observation from the mean of its set.

NOTE ON THE AMOUNT OF ERROR IN APPLYING TO NON-PARALLEL PLATES
THE FORMULA FOR ELECTRICAL CAPACITY OF PARALLEL PLATES.¹

BY L. E. DODD.

EXPERIMENTAL work by the author in which simultaneous optical and electrical measurements were made of the distance between conducting flats in air required the application of a formula for distance in terms of electrical capacity. While the particular formula for the case of a limited area in extensive parallel plates was employed, the apparatus used prevented parallelism of the flats, and the question arose as to the amount of error due to application of the formula for parallel plates. The application of this formula neglects edge corrections. An expression has been derived for a limited area in extensive non-parallel plates which in its application to the apparatus must also neglect edge corrections. If edge corrections are neglected the lines of force may be considered arcs of circles, which is consistent with a result derived by others from a special transformation with conjugate functions.² The present derived expression for electrical capacity of non-parallel plates is,

$$C = W/4\pi \cdot \log (r_2/r_1) \cdot 1/\theta,$$

where W is width of plates (supposed of like dimensions and symmetrically situated), r_1 and r_2 the distances to the vertex from lower and upper edges (parallel to vertex) respectively of a plate, and θ is the angle between the plates. This formula is seen to be of a form similar to that for parallel plates, *i. e.*, a constant multiplied into the reciprocal of a distance. Further comparison of the two formulas reveals that the error in applying the latter for the former (neglecting error due to edge corrections) depends alone on the ratio (r_2/r_1) , and is negligible over the range of values, 1 to 1.5, particularly in the author's experimental work,³ and also in work of F. C. Brown,⁴ and in unpublished work of G. W. Stewart.

STATE UNIVERSITY OF IOWA,
IOWA CITY, IA.

¹ Abstract of a paper presented at the Chicago meeting of the Physical Society, December 2, 1916.

² See Jeans, *Elec. and Mag.*, 2nd ed., sec. 318, p. 268.

³ Dodd, *PHYS. REV.*, Vol. V., No. 1, Jan., 1915, p. 78.

⁴ Brown, *PHYS. REV.*, Vol. II., No. 4, Oct., 1913, p. 314.

THE PHYSICAL REVIEW.

POLARIZATION AT THE CATHODE IN A HIGHLY IONIZED GAS.

BY C. A. SKINNER.

SOME time ago the writer published a theory of the cathode fall in gases,¹ together with some applications, in which potential measurements in "the Crookes dark space" were made by introducing a wire and observing its potential. The measurements proved to be consistent with the theory.

Basing his conclusions on some experiments of Ashton,² H. A. Wilson has recently published a theory of the cathode fall,³ under the assumption that there is no appreciable difference of potential between the cathode and the adjacent gas—although wire sound measurements indicate it to be about one half the total cathode fall. If Ashton's contention be true my theory is only true in part, and the derived magnitudes very much in error. On the other hand, if the wire sound measurements are reliable, Wilson's theory covers but half the observed cathode fall and his derived magnitudes accordingly incorrect. The relative merits of the two theories are otherwise insignificant compared with the main question: *Is there a large and relatively abrupt difference of potential between the cathode and adjacent gas, or is there none?*

In the writer's opinion the experiments described below give incontestable evidence that there is not only a distinct polarization P.D. between the cathode and the gas, but that it is of the magnitude indicated by the wire sound measurements.

Ashton's Experiments.—Skepticism as to the accuracy of wire sound measurements of potential in Crookes dark space is based on reasonable

¹ C. A. Skinner, *Phys. Rev.*, N. S., Vol. V., p. 483, 1915; Vol. VI., p. 158, 1915.

² F. W. Ashton, *Proc. Roy. Soc.*, A 84, p. 526, 1911.

³ H. A. Wilson, *Phys. Rev.*, N. S., Vol. VIII., p. 227, October, 1916.

considerations. But to assume that the error is, for example, eighty volts when an aluminium cathode is used and one hundred fifty volts when a steel cathode is used, seems hardly justified, when the experiments supporting the assumption are carefully considered.

Ashton measured the electric intensity in his Crookes dark space by directing a fine stream of cathode rays—from a branch tube—parallel to the face of a rather large disk cathode and noting the deflection produced. He had to deal with the average value of the field through which his stream passed. From this deflection he calculated the potential, and concluded that there was no appreciable P.D. between the conducting gas and the electrodes.

There are several reasons for questioning the validity of Ashton's conclusions. He worked with his cathode ray tube in open connection with the one under investigation, so that it was necessary to have the gas pressure low enough to give a cathode ray stream. It is a well-known fact, however, that the normal markings of the glow current disappear at pressures giving cathode ray streams satisfactory to work with. With the disappearance of the customary glow the current condenses largely into a bundle of cathode rays proceeding from a small area on the cathode, and little if anything definite can be concluded as to the current density or electric intensity at points off side.

That Ashton was not working under normal glow conditions seems evident from the results of some tests made by the writer under the conditions given by Ashton. With hydrogen at a pressure giving a Crookes dark space 32 millimeters in extent, Ashton had only 265 volts between his electrodes. With a smaller tube (3 cm. in diam.) having as cathode an aluminium disk and as anode a wire extending into the negative glow I have been unable to maintain at such pressures any discharge whatsoever with a P.D. less than about 650 volts. The highest P.D. Ashton records for hydrogen was 610 volts, and this was used at a pressure giving a dark space 28 millimeters in extent.

With an applied P.D. of about 700 volts the current in the present case was started at a higher gas pressure and the change observed as the pressure was gradually reduced. With a dark space about 25 millimeters in extent its boundary began to fade out rapidly, the ordinary current going over into a small bundle of cathode rays proceeding from the center of the cathode. At the pressure which should have given a dark space of 32 millimeters, there was no trace of such a boundary left. At that pressure which should give 38 millimeters dark space the P.D. of 700 volts could not maintain any discharge through the tube.

Ashton's results should not therefore be taken as representing normal glow current conditions.

POLARIZATION AT A CATHODE PLACED IN THE NEGATIVE GLOW OF A SEPARATE CIRCUIT.

Apparatus and Method of Experiment.—Entirely within the negative glow of what will be designated the ionizing current, were placed two relatively small plane electrodes *A* and *B* (Fig. 1) with their faces parallel to the direction of the ionizing current and their supporting stems sheathed in fine glass tubing. These electrodes, together with the gas of the negative glow, formed part of an otherwise carefully insulated

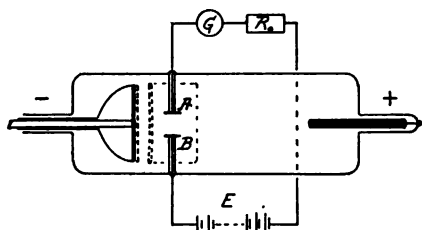


Fig. 1.

circuit, consisting of a storage battery *E*, a d'Arsonval galvanometer *G*, and a protecting resistance *R*₀.¹ The battery *E* was arranged to furnish readily any P.D. ranging from 2 to 300 volts—as measured by a Weston voltmeter kept in connection during all measurements. The current in the ionizing circuit was furnished by a similar but stronger battery. To keep the electrodes *AB* in the negative glow under different gas pressures, the cathode of the ionizing circuit was readily shifted along the axis of the tube by a screw device actuated through a ground joint.

Let *R*_i be the resistance, real or apparent, between *A* and *B* inside the discharge tube, and *R*₀ the resistance of that part of the circuit outside the tube. Let *E* be the P.D. furnished by the battery, and *c* the current it produces—referred to in the following as the transverse current.

Then

$$R_i = \frac{E}{c} - R_0,$$

In case, with values of the transverse current that are small compared with the ionizing current, *R*_i is found to be relatively large, it is to be concluded that it is concentrated between the cathode and the gas. For, the gas in the negative glow is known to be highly conductive; and

¹ H. A. Wilson (Phil. Mag., V., 49, p. 505, 1900) used such a scheme to investigate the relative conductivity of the different parts of the glow current.

no appreciable difference of potential between an anode placed in the negative glow and the gas can be observed.¹

The following experiments reveal surprisingly large values for R_i , and substantiate in various ways the conclusion that it is located between the cathode and the gas.

Measurements with Transverse Electrodes of Aluminium.—The gas used throughout was hydrogen. It was introduced into the discharge tube by diffusion through a palladium tube heated to redness in a stream of commercial hydrogen. The electrodes serving for the ionizing current were of aluminium—the cathode being a disk (diam. about 2 cm.) with its back enclosed in glass to confine the current to its front face.

Table I. gives a representative series of results obtained with A and B

TABLE I.

Aluminium Electrodes (Area: .25 sq. cm.).

Gas Pressure { before 1.65 mm.
after 1.67 " Ionizing Current: 1.50 m.a.

$V_A - V_B$ (Volts).	c (Milli-amp.).	R_i (Ohms).	$V_A - V_B$ (Volts).	c (Milli-amps.).	R_i (Ohms).
1.8	.0099	1.8×10^6	77.3	.109	7.1×10^6
3.85	.0165	2.33	81.5	.112	7.3
5.65	.0231	2.45	89.5	.121	7.4
7.5	.0280	2.85	97.5	.129	7.5
9.2	.0405	3.20	108.	.142	7.6
12.9	.0405	3.20	120.	.155	7.7
18.5	.0495	3.75	132.	.174	7.6
27.8	.062	4.50	143.	.192	7.4
39.0	.074	5.25	51.3	.087	5.9
51.5	.086	6.0			
64.1	.096	6.7			

of thin plates of aluminium, rectangular in form (2.25×5 mm.), and both faces exposed. Their exposed surface, counting the edges, was very close to .25 sq. cm. The resistance R_0 was practically concentrated in a 20,000-ohm box.

The procedure was to measure by the galvanometer G , the current c when a definite E was applied. From the values of c and R_0 the drop in the external part of the transverse circuit was then calculated, and

¹ The potential gradient in the negative glow under the currents used here is distinctly less than 1 volt/cm. With a transverse current of one-tenth the magnitude of the main current the drop of potential in the gas between the transverse electrodes A and B (3 mm. apart) should therefore be less than one tenth volt. The anode fall in the same gas is not much larger than this. (See Skinner, Phil. Mag.) These values are therefore negligible compared with the P.D. applied between A and B throughout this investigation.

this magnitude—always relatively small—subtracted from E , giving the applied difference of potential ($V_A - V_B$) recorded in the tables.

These values together with the calculated value of R_i are given in each case. This calculated resistance is usually designated as the apparent cathode resistance. The observations are recorded in the sequence taken, those at the end being check observations.

Table I. shows that, with an ionizing current of 1.50 m.a., the resistance R_i between the transverse electrodes rises from 180,000 ohms under a transverse current of .01 m.a., to 770,000 ohms under one of .155 m.a. With a further increase in the transverse current R_i then slowly decreases. In the preliminary experiments no protecting resistance R_o was introduced. The transverse current increased slowly with E until about 100 volts was reached when a further increase in E produced a sudden rush of current which completely vaporized the transverse electrodes.

Table II. gives a similar series obtained with an ionizing current of

TABLE II.

Aluminium Electrodes (Area: .25 sq. cm.).

Gas Pressure { before 1.57 mm.
after 1.59 " Ionizing Current: 2.50 m.a.

$V_A - V_B$ (Volts).	c (Milli-amp.).	R_i (Ohms).	$V_A - V_B$ (Volts).	c (Milli-amp.).	R_i (Ohms).
3.1	.025	1.25×10^6	81.	.200	4.05×10^6
9.7	.068	1.4	85.8	.210	4.1
15.2	.088	1.7	95.0	.225	4.2
22.7	.104	2.2	110.8	.260	4.3
29.6	.118	2.5	126.2	.290	4.35
41.3	.140	2.95	140.6	.320	4.4
53.8	.158	3.4	87.7	.215	4.1
66.7	.180	3.7	10.4	.064	1.6
75.1	.194	3.9			

2.50 m.a. For the same transverse current c the resistance R_i between A and B is distinctly lower for the larger ionizing current, but its variation with c is similar.

Measurements with Platinum Electrodes.—Table III. gives the results obtained when the aluminium electrodes A and B were replaced by platinum, having the same dimensions and at the same distance apart as the aluminium. The two metals are readily compared by referring to their respective curves in Fig. 2, in which, for the same ionizing current, the apparent resistance R_i is plotted against the transverse current c .

The two curves are similar in form but that for platinum distinctly

TABLE III.

Platinum Electrodes (Area: .25 sq. cm.).

Gas Pressure { before 1.77 mm.
after 1.91 " Ionizing Current: 2.50 m.a.

$V_A - V_B$ (Volts).	i (Milli-amp.).	R_t (Ohms).	$V_A - V_B$ (Volts).	i (Milli-amp.).	R_t (Ohms).
3.1	.022	1.4×10^6	80.5	.165	4.9×10^6
6.2	.038	1.65	92.5	.180	5.1
9.7	.049	2.0	105.	.193	5.4
14.1	.061	2.3	117.	.206	5.65
19.1	.073	2.6	128.5	.220	5.85
30.6	.095	3.2	142.5	.233	6.1
43.2	.116	3.7	92.5	.176	5.25
52.5	.130	4.05	52.	.127	4.1
65.	.147	4.4	6.7	.041	1.65
74.	.159	4.65			

higher than that for aluminium. This difference gives support to the argument that R_t cannot be located in the gas between the electrodes,

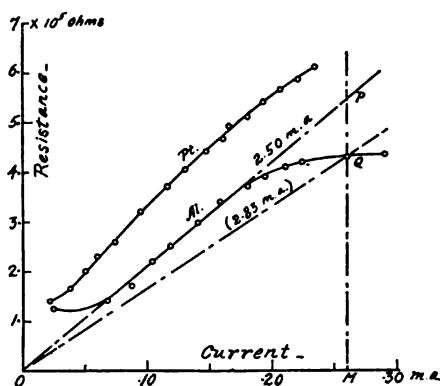


Fig. 2.

for then it should remain unchanged when the materials composing the electrodes are changed.

With the highest values of $(V_A - V_B)$ used in this series, platinum, unlike aluminium, shows no appreciable tendency toward a maximum R_t . Later tests however running to higher potential differences show the maximum to be as clearly marked with platinum as with aluminium.

Measurements with a Platinum-Aluminium Pair.—A more interesting and more accurate comparison of these two metals was made by having one of the transverse electrodes of platinum and the other of aluminium—both having as nearly as possible the same dimensions, and similarly

situated in the negative glow. By simply reversing the applied P.D. first the one metal then the other served as cathode under identical conditions.

The results of this simultaneous comparison, for the ionizing currents 2.50 m.a. and 1.50 m.a., are given in Tables IV. and V. respectively.

TABLE IV.

Simultaneous Comparison of Aluminium and Platinum Cathodes (Area: .25 sq. cm.).

Gas Pressure { before 1.72 mm.
after 1.76 "

Ionizing Current: 2.50 m.a.

$V_A - V_B$ (Volts).		c (Milli-amp.).		R_i (Ohms).	
Al. Cathode.	Pt. Cathode.	Al. Cathode.	Pt. Cathode.	Al. Cathode.	Pt. Cathode.
0.9	2.8	.0043	.0132	2.1×10^6	2.1×10^6
4.6	6.6	.0296	.032	1.55	2.05
10.1	11.9	.063	.058	1.6	2.05
17.7	20.	.089	.077	2.0	2.6
27.3	29.5	.109	.099	2.5	3.0
37.4	39.7	.132	.117	2.8	3.4
49.5	52.	.151	.138	3.3	3.8
63.5	66.	.175	.160	3.6	4.1
77.5	80.5	.196	.181	3.95	4.45
94.5	97.	.225	.206	4.2	4.7
112.	114.	.260	.234	4.3	4.9
131.	134.	.295	.260	4.45	5.15
155.	158.	.346	.295	4.45	5.35
191.	195.	.475	.365	4.0	5.3
17.8	20.	.088	.079	2.05	2.55

The existence of a contact potential between A and B was revealed and its magnitude approximately determined from the applied P.D. necessary to produce zero transverse current. For Table IV. this was about 1 volt, and for V. 1.2 volts. The values given under $(V_A - V_B)$ are therefore the applied P.D. corrected by this amount. Inasmuch as this correction was not large anyway, the possible error, arising from the tacit assumption that it was the same at all values of the transverse current, is of no real moment.

Inspection of these tables and the corresponding curves of Fig. 3, show as expected, a distinct difference in the magnitude of R_i on simply reversing the direction of the transverse current. These results locate the resistance at the cathode; for, with aluminium as cathode and platinum as anode the value of R_i is the same as when both electrodes were aluminium. With platinum as cathode and aluminium as anode they are distinctly higher, though not quite so large as in the previous experiment with both electrodes of platinum.

TABLE V.

*Simultaneous Comparison of Aluminium and Platinum Cathodes (Area: .25 sq. cm.).*Gas Pressure { before 1.68 mm.
after 1.71 "

Ionizing Current: 1.50 m.a.

$V_A - V_B$ (Volts).		c (Milli-amp.).		R_t (Ohms).	
Al. Cathode.	Pt. Cathode.	Al. Cathode.	Pt. Cathode.	Al. Cathode.	Pt. Cathode.
.7	3.0	.002	.007	3.5×10^5	4.3×10^5
4.5	6.9	.0152	.0178	3.0	3.9
10.1	12.5	.0335	.033	3.0	3.8
18.2	20.5	.047	.0485	3.85	4.2
25.7	28.1	.056	.054	4.6	5.2
36.0	38.0	.068	.065	5.3	5.85
48.0	50.0	.079	.075	6.1	6.7
62.0	64.0	.091	.088	6.8	7.3
76.5	78.5	.102	.098	7.5	8.0
94.0	96.0	.119	.111	7.9	8.6
112.0	114.0	.136	.124	8.2	9.2
130.5	133.0	.157	.138	8.3	9.6
150.	153.	.186	.158	8.1	9.7
191.	193.	.266	.200	7.2	9.6
76.5	78.5	.102	.097	7.5	7.9

These results should remove all doubt as to the existence in general of a polarization at the cathode when an electric current flows through an ionized gas. For it is found to exist here under circumstances most favorable to a vanishing magnitude of the electrode fall; namely, in a gas so highly ionized that the potential gradient necessary to maintain

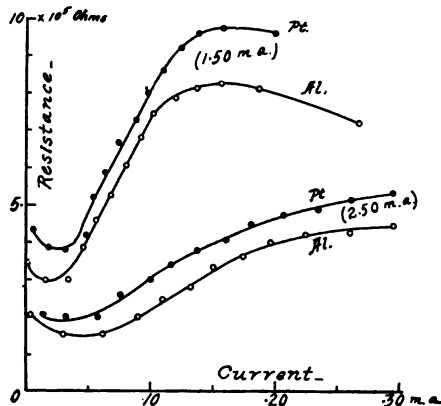


Fig. 3.

a current through it is too small to be measurable. Under many of the working conditions this polarization P.D. may not have an appreciable value, but the probability of its existence in general requires that it be taken into account.

In these last tests the similarity in form of the resistance curves for both metals is clearly evident. Both show a minimum for low values of c and a maximum at higher values of c . The minimum and maximum both become larger and more abrupt with decreased ionizing current. It was suspected at first that the evidence of an appreciable value of R_0 for a zero value of c arose from extraneous conditions, but its persistence in all tests renders such a conclusion at least questionable.

Variation in the Apparent Cathode Resistance with Ionizing Current.—A complete investigation of this problem would consist in obtaining the family of (R_0, c) curves for various ionizing currents and different gas pressures. The curves of Fig. 3 suggest the general change in form with change in ionizing current. Such an investigation requires the metal tested to be in use so long that its condition is likely to undergo important changes before the series of measurements is complete. A simpler plan was devised. This consisted in making a study of R_0 under a definite applied P.D. and different ionizing currents—at various gas pressures.

For this the transverse electrodes were again of aluminium—the same as used in Tables I. and II., their surfaces having been carefully cleaned by scraping. By reversing the applied P.D. first the one electrode, then the other, was used as cathode. In this way the one served to check the results obtained from the other. They were found to differ throughout by about five per cent., probably arising from a slight difference in surface.

Tables VI. and VII. give an uninterrupted series of measurements all with an applied P.D. of about 80 volts between the transverse electrodes. Each group of measurements consisted in observing, with a given gas pressure, the transverse current c with various ionizing currents. Then another group was taken with fresh gas, and so on. The pressure recorded at the top of each group was the initial; that at bottom, the final. The slight increase recorded did not, as shown, appreciably change the results.

The magnitude of the applied P.D. was chosen at 80 volts for various reasons. It was large enough to eliminate the possibility of disturbance arising from contact potential, resistance in the gas, anode fall, and the like. It is distinctly below the magnitude which causes, as explained later, a disturbing ionization by the electrons freed from the transverse cathode. Finally, it is of the magnitude which the wire sound indicates as present between gas and cathode under "normal" cathode current density; so that a comparison could be made between the resistance of the cathode placed in the negative glow and that of the main cathode working under normal conditions.

TABLE VI.

Aluminium Cathodes—(Area: .25 sq. cm.)—Various Ionizing Currents.

Gas Press. (mm.),	Ioniz. Current (m. a.),	$V_A - V_B$ (Volts).	c (m. a.).		R_i (Ohms).		Prod. of R_i by Ionizing Current.	
			Cathode A.	Cathode B.	Cathode A.	Cathode B.	Cathode A.	Cathode B.
1.63	1.50	79.5	.107	.102	7.4×10^5	7.8×10^5	11.1×10^5	11.7×10^5
	2.00	78.5	.161	.152	4.9	5.2	9.8	10.4
	2.50	79.5	.210	.200	3.8	4.0	9.5	10.0
	3.00	78.5	.255	.243	3.1	3.2	9.3	9.6
	3.50	80.0	.305	.290	2.6	2.75	9.1	9.6
	4.00	79.5	.335	.325	2.4	2.45	9.6	9.8
1.67	1.50	79.0	.106	.102	7.5	7.8	11.2	11.7
1.39	1.00	79.5	.070	.066	11.3	12.0	11.3	12.0
	1.50	78.5	.120	.115	6.5	6.8	9.8	10.2
	2.00	77.5	.174	.167	4.45	4.65	8.9	9.3
	2.50	78.0	.225	.213	3.5	3.7	8.8	9.2
	3.00	77.0	.270	.255	2.85	3.0	8.5	9.0
	3.50	78.5	.315	.300	2.50	2.60	8.8	9.1
	4.00	77.5	.350	.335	2.22	2.32	8.9	9.3
1.43	1.00	79.0	.072	.069	11.0	11.5	11.0	11.5
1.15	.75	79.5	.057	.054	13.9	14.7	10.4	11.0
	1.00	79.0	.086	.082	9.2	9.6	9.2	9.6
	1.50	77.5	.143	.137	5.4	5.6	8.1	8.4
	2.00	78.5	.187	.178	4.2	4.4	8.4	8.8
	2.50	78.0	.230	.220	3.4	3.55	8.5	8.9
	3.00	77.5	.270	.255	2.9	3.05	8.7	9.1
	3.50	78.5	.310	.300	2.53	2.62	8.9	9.2
1.20	.75	79.5	.058	.056	13.6	14.2	10.2	10.6

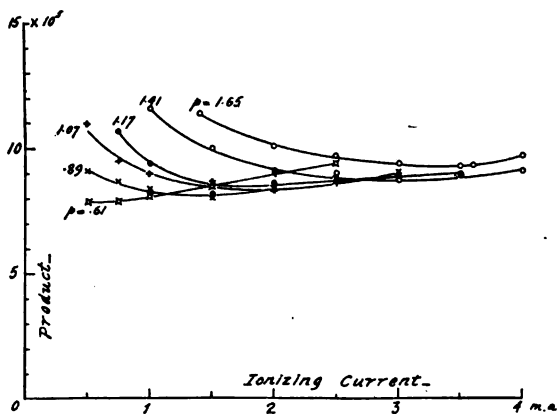


Fig. 4.

An interesting relation is brought out in these tables (VI. and VII.), and shown also in the corresponding curves of Fig. 4, where the products of cathode resistance by the ionizing current are plotted against ionizing currents.

TABLE VII.

Continuation of Table VI.

Gas Pres. (mm.)	Ioniz. Current. (m. a.)	$V_A - V_B$ (Volts.)	c (m. a.).		R_i (Ohms).		Prod. of R_i by Ionizing Current.	
			Cathode A.	Cathode B.	Cathode A.	Cathode B.	Cathode A.	Cathode B.
1.05	.50	80.0	.038	.035	21.0×10^6	23.0×10^6	10.5×10^6	11.5×10^6
	.75	79.0	.061	.061	12.5	12.9	9.4	9.7
	1.00	79.0	.091	.085	8.7	9.3	8.7	9.3
	1.50	78.0	.136	.132	5.7	5.9	8.6	8.8
	2.00	78.0	.188	.185	4.15	4.2	8.3	8.4
	2.60	77.5	.230	.217	3.35	3.55	8.4	8.9
	3.00	77.0	.265	.250	2.9	3.1	8.7	9.3
1.10	.75	79.0	.063	.059	12.5	13.3	9.4	10.0
.87	.50	78.5	.0435	.040	18.0	18.6	9.0	9.3
	.75	78.0	.069	.065	11.3	12.0	8.5	9.0
	1.00	77.5	.094	.090	8.2	8.6	8.2	8.6
	1.50	76.5	.146	.138	5.2	5.5	7.8	8.2
	2.00	78.5	.192	.183	4.1	4.3	8.2	8.6
	2.50	78.0	.225	.215	3.45	3.65	8.6	9.1
.90	3.00	77.5	.260	.250	3.0	3.1	9.0	9.3
.60	.50	79.0	.051	.0485	15.5	16.3	7.8	8.1
	.75	78.5	.076	.073	10.3	10.8	7.7	8.1
	1.00	78.0	.099	.094	7.9	8.3	7.9	8.3
	1.50	77.5	.140	.134	5.5	5.8	8.3	8.7
	2.00	79.0	.180	.173	4.4	4.6	8.8	9.2
	2.50	78.5	.213	.203	3.7	3.85	9.2	9.6
.63	.50	78.5	.051	.049	15.4	16.0	7.7	8.0
.40	.50	79.5	.055	.051	14.5	15.6	7.2	7.8
	.75	79.0	.077	.073	10.2	10.8	7.7	8.1
	1.00	78.5	.095	.091	8.3	8.6	8.3	8.6
	1.50	78.0	.131	.124	6.0	6.3	9.0	9.4
	2.00	79.5	.168	.158	4.75	5.0	9.5	10.0
.43	.50	78.5	.053	.050	14.8	15.7	7.4	7.8

Assuming that the transverse P.D. is concentrated at the cathode we find: for a given polarization P.D. at the transverse cathode, its apparent resistance is nearly inversely proportional to the ionizing current, and nearly independent of the gas pressure.

This observed relation appears to have a simple explanation. Whatever may be the exact nature of the apparent cathode resistance it must be separately encountered by each discharging ion. The current should therefore be proportional to the number of ions simultaneously engaged in discharging, and this number—other conditions remaining the same—should be proportional to the number of positive ions per c.c. in the gas surrounding the cathode. To have this explanation fit the case it would require that the number density of ions in the negative glow be

nearly proportional to the current density of the ionizing current. General considerations indicate this to be true. One of the unique features of the glow current is that the potential gradient changes but very slowly with increasing current density, meaning that the number of ions per c.c. carrying the current must be nearly proportional to the current density.

If this explanation be accepted the above results are easily interpreted. The curves all show a minimum product at some definite ionizing current. Here the ratio of number of positive ions per c.c. to the ionizing current is a maximum, and the number density proportional to the current. As the gas pressure is lowered, this ratio slowly increases.

For the purpose of later reference we will express the relation in mathematical form:

$$R_i \propto \frac{1}{n} \quad \text{for } (V_A - V_B) = \text{const.} \quad (1)$$

where R_i is the apparent cathode resistance, n the number of positive ions per c.c. and $(V_A - V_B)$ the polarization P.D. at the cathode.

Resistance at the Transverse Cathode Compared with that at the Main Cathode.—This comparison is especially valuable because it proves that wire sound measurements in Crookes dark space are reliable; and, it is from such measurements that knowledge of conditions from point to point in the gas is obtained.

The problem to solve is: Given the same polarization P.D. at both cathodes, to calculate the apparent resistance per sq. cm. at each when the current density in the tube is the same.

In a previous article on the cathode fall¹ are given potential measurements taken with a wire sound at different distances from an aluminium cathode in hydrogen. From these the magnitude of the polarization P.D. at the cathode was obtained by theoretical extrapolation—practically linear. Two of the gas pressures given there are the same as two given in Table VI., namely: 1.6 mm. and 1.1 mm. For these, at "normal" cathode current density, the average polarization P.D. at the cathode was 78 volts. The "normal" current density for the first pressure was .192 m.a./cm².; for the second, .091 m.a./cm². The apparent resistance per sq. cm. for the first was therefore

$$\frac{78}{.192 \times 10^{-3}} = 4.1 \times 10^5 \text{ ohms}$$

for the second

$$\frac{78}{.091 \times 10^{-3}} = 8.6 \times 10^5 \text{ ohms.}$$

¹ C. A. Skinner, *Phys. Rev.*, N. S., VI., pp. 160, 161.

In Table VI. the minimum product of R_i by the ionizing current, for a pressure of 1.63 mm., was 9.3×10^5 . This was from a transverse cathode having an area of .25 cm². It is also with the total ionizing current which was spread over a cross-section of about 3 cm².—the area of the main cathode. To obtain the product in terms of cathode resistance *per square centimeter*, and *ionizing current density*, we have

$$\frac{.25}{3} \times 9.3 \times 10^5 = 7.8 \times 10^4.$$

From this, the resistance per sq. cm. of the transverse cathode for "normal" current density at the main cathode at the same pressure, may be calculated if the product be assumed to remain constant. We have then as calculated resistance for the first case:

$$\frac{7.8 \times 10^4}{.192} = 4.1 \times 10^5 \text{ ohms,}$$

similarly, for the second case:

$$\frac{7.0 \times 10^4}{.091} = 7.7 \times 10^5 \text{ ohms,}$$

as compared with the corresponding magnitudes at the main cathode of 4.1 and 8.6×10^5 ohms. The average agreement is unexpectedly good, and *proves that the wire sound measurements in Crookes dark space are reliable.*

The Form of the Resistance Curve.—The curve for aluminium in Fig. 2 may be taken as typical of the (R_i, c) curves. At the lower magnitudes of c the values are rather unexpected but they are regularly present. Beyond this region however the curve assumes almost a straight line direction from the origin, and holds it for some distance. This straight line gives a simple law for the cathode resistance so long as the transverse current remains between two fairly well separated magnitudes. The deviation of the curve from the straight line, at the higher values of c , is probably caused by the increased density of ionization which the electrons from the transverse cathode produce in the surrounding gas.

Assuming this we calculate the resistance at Q (Fig. 2) as that which would be obtained if an ionizing current of 2.83 m.a. were used instead of 2.50 and there were no superposed ionization arising from the electron current from the transverse cathode.

We will now formulate a tentative law governing the polarization P.D. and test its applicability to the general results.

Let V_b be the polarization P.D. at the cathode—the same as ($V_A - V_B$) in the tables— j the cathode current density, R_b its apparent resistance

per sq. cm., and n the number of positive ions per c.c. in the gas surrounding the cathode. From (1)

$$R_b \propto \frac{1}{n} \text{ for } V_b = \text{const.} \quad (2)$$

From the above considerations

$$R_b \propto j \text{ for } n = \text{const.} \quad (3)$$

Equation (2) gives

$$\begin{aligned} V_b &= jR_b \\ &\propto \frac{j}{n} \end{aligned}$$

hence

$$j \propto n \text{ for } V_b = \text{const.} \quad (4)$$

Similarly from (3)

$$j \propto V_b^{1/2} \text{ for } n = \text{const.} \quad (5)$$

Combining (4) and (5)

$$j \propto n V_b^{1/2}.$$

This gives the simple tentative law for the polarization P.D.

$$V_b = g \left(\frac{j}{n} \right)^2 \quad (6)$$

where g is a constant. Inspection of the experimental results indicate that, with an aluminium cathode in hydrogen, it should hold for values of V_b between about 20 and 80 volts.

Something of the nature of this polarization phenomenon is suggested by substituting the customary relation between j and n . For those conditions where the electron current from the cathode is a negligible part of the total current, we have simply

$$j = neu \quad (7)$$

in which e is the ionic charge and u the velocity of the positive ions where their number density is n . Substituting (7) in (6)

$$V_b = ge^2u^2. \quad (8)$$

In those cases where it is simply necessary to maintain a terminal velocity of the ions, the P.D. necessary is proportional to that velocity. Here, we find it proportional to the square of the velocity, as if it produced an acceleration. This is in keeping with the view that the polarization P.D. is necessary to overcome the kinetic energy of rebound of the discharging ions.

The applicability of (8) to the general results obtained from the cathode will now be tested. The following general relation has been

found to hold in the space between the cathode and negative glow: *With the cathode at zero potential, and its current density a definite multiple of the "normal" value, the potential at any definite number of mean free paths from the cathode is the same, over a wide range of gas pressures.*¹ Stated more definitely for the purpose of present application: The polarization P.D. at the cathode remains constant over a wide range of gas pressures so long as the potential gradient per cm. E in the gas just outside the polarization region, is proportional to the gas pressure p . That is, we have the experimental relation

$$V_b = \text{const. for } E \propto p. \quad (9)$$

From the law of mobility

$$u \propto \frac{E}{p}.$$

Substituting this in (8) gives

$$V_b \propto \left(\frac{E}{p}\right)^2$$

which states that, as in (9), V_b is constant for $E \propto p$. Thus we find our tentative law predicts the general cathode relations.

SUMMARY.

1. Considerations and experimental results are given showing that Ashton's experiments, indicating that there is no difference in potential between a cathode and the conducting gas adjacent to it, were not made under normal glow current conditions, as he assumed.

2. A series of experiments are described which show that a cathode placed in the highly ionized negative glow, where a polarization P.D. would be least expected, shows it present in large magnitude.

3. For a constant polarization P.D. at the cathode, the apparent cathode resistance per sq. cm. is *nearly* inversely proportional to the ionizing current and *nearly* independent of the gas pressure. From these relations it is concluded that the apparent cathode resistance is simply inversely proportional to the density of the positive ions in the gas around the cathode.

This relation is applied to calculate the apparent resistance of the transverse cathode with "normal" cathode current density at the main cathode. The calculated magnitude was the same as that obtained from wire sound measurements at the main cathode, under the same conditions. It therefore proves the reliability of wire sound measurements in Crookes dark space.

¹ C. A. Skinner, *l. c.*; W. L. Cheney, *Phys. Rev.*, N. S., Vol. VII., p. 241, 1916; *Wm. News-wanger*, *id.*, p. 253.

4. Between two fairly well separated values of the polarization P.D. the apparent cathode resistance is, for constant ionizing current, practically proportional to the current density. The deviation of the resistance curve from a straight line at the higher potentials probably arises from the extra ionization produced by the electrons escaping from the transverse cathode.

BRACE LABORATORY,
UNIVERSITY OF NEBRASKA,
November, 1916.

THE INFLUENCE OF WATER OF CRYSTALLIZATION UPON
THE FLUORESCENCE AND ABSORPTION SPECTRA
OF URANYL NITRATE.

BY EDWARD L. NICHOLS AND ERNEST MERRITT.

THE most striking characteristic of the fluorescence spectra of the uranyl salts is the presence of series in which the bands, when plotted on the frequency scale, are spaced at constant intervals. The frequency interval is apparently the same for the different series in the spectrum of a given salt and varies only slightly from one salt to another. The spectra observed with different uranyl salts are so similar in their general characteristics that we can scarcely doubt that the nature of these spectra is chiefly determined by the radical UO_2 . Apparently the uranyl radical contains a group of electrons whose arrangement is such as to permit of vibrations that give this type of spectrum; and although UO_2 is not stable in the chemical sense and must be combined with some acid in order to form a stable compound, yet the effect of the acid radical is merely to modify the constants of this vibrating system in the UO_2 radical without changing the type of vibration. It is natural to expect that the addition of water of crystallization would produce a similar effect; and it is our intention to present in this paper the results of a study of the influence of water of crystallization upon the fluorescence and absorption spectrum in the case of uranyl nitrate.

The nitrate is particularly suited for such an investigation because of the fact that several different hydrates are formed. Crystals grown from a water solution contain six molecules of water. In an acid solution crystals are formed with three molecules of water. In both cases crystals may be obtained which are large enough to permit of observations being made with a single crystal. By methods described later small crystals containing only two molecules of water are readily obtained. It is a matter of some difficulty to push the dehydration further, but specimens have been prepared for us by Mr. D. T. Wilber which we have reason to believe are either anhydrous or formed of a mixture of the anhydrous salt and the monohydrate.

In the case of the hexahydrate wave-lengths were in most cases determined photographically. Visual observations were, however, also made,

although these could not be extended throughout the whole spectrum. The agreement between measurements made by the two methods was surprisingly good. In the case of weak bands lying near to bands of great intensity the visual observations were found to be best. The results given for the fluorescence spectra of the other hydrates and for the anhydrous salt are based upon visual observations exclusively.

For observations of the fluorescence spectrum the carbon arc was generally used as an exciting source, suitable absorbing screens being interposed to cut wave-lengths longer than about 4800 A.U. In studying absorption the continuous spectrum of the gas filled tungsten lamp was found more satisfactory, although the carbon arc was also used.

All observations were made at the temperature of liquid air.

THE HEXAHYDRATE, $\text{UO}_2(\text{NO}_3)_2 \cdot 6\text{H}_2\text{O}$.

The hexahydrate crystallizes in the rhombic system with the axial ratio $a : b : c = .6837 : 1 : 6.088$. The crystals were grown in the form of plates by using a water solution whose depth was equal to the thickness of the plate desired. Single crystals as large as 15 mm. in diameter were obtained with relatively little difficulty. All of the results here discussed are based upon observations made with single crystals. The general character of the spectrum is shown in Fig. 1.

In the great majority of cases wave-lengths were measured photographically, both in the fluorescence spectrum and in the absorption



Fig. 1.

Fluorescence and absorption spectra of uranyl nitrate hexahydrate. Fluorescence bands are plotted above the line; absorption bands below. The intensity is roughly indicated by the length. The horizontal scale is one of reciprocal wave-lengths.

spectrum. In the few instances where visual observations were also made, these were found to agree remarkably well with the results obtained from photographs. In selecting the data to be used in taking a final average each negative was carefully studied and measurements that seemed for any reason doubtful were discarded. The elimination of doubtful observations was made without reference to the agreement or lack of agreement between the different measurements, and was in fact completed before the measurements of the different negatives were compared. About forty negatives were used, although the number used for any one line was rarely more than ten.

The errors of calibration of the spectrograph and spectrometer can hardly exceed 1 A.U. except perhaps in the extreme red end of the spectrum. The uncertainties due to the faintness of certain bands, to their finite width, and to photographic broadening are more difficult to estimate and undoubtedly differ greatly with the character of the band and its position in the spectrum. In the case of the sharper bands of

TABLE I.

Series in the Fluorescence Spectrum of Uranyl Nitrate Hexahydrate $[UO_2(NO_3)_2 + 6H_2O]$.

	Intensity. ¹	Reliability. ²	$\frac{1}{\lambda}$.	$\Delta \frac{1}{\lambda}$.		Intensity.	Reliability.	$\frac{1}{\lambda}$.	$\Delta \frac{1}{\lambda}$.
A	v. d.	2	F 1760.1		F	m.	1?	F 1631.3	
	v. d.	1	1846.0	85.9		d.	3	1718.4	87.1
	d.	2	1930.1	84.1		m.	4	1803.6	85.2
	m.	4	F 2018.2	88.1		m.	5	1889.6	86.0
B					F	m.	5	1976.4	86.8
	v. d.	3	F 1689.5			d.	4	F 2061.9	85.5
	d.	3	1775.0	85.5					
	m.	4	1861.1	86.1		m.	4	A 2061.7	
	m.	5	1947.1	86.0		m.	4	A 2148.7	87.0
	str.	5	F 2034.5	87.4		m.	4	A 2234.1	85.4
C	d.	3	A 2207.3	86.4(2)	G	str.	3	A 2321.0	86.9
						v. d.	1	A 2491.9	85.5 × 2
	d.	3	F 1699.0						
	d.	2	1785.1	86.1		d.	3	F 1810.4	
	d.	2	1869.0	83.9		d.	4	1897.3	86.9
	v. d.	1	1956.2	87.2		v. d.	2	1983.1	85.8
D	m.	2	2041.5	85.3	H	d.	1	A 2241.6	86.2 × 3
	v. d.	1	F 1534.9			v. d.	3	F 1649.7	
	d.	3	1621.0	86.1		v. d.	2	1737.4	87.7
	m.	4	1706.8	85.8		d.	1	1822.0	84.6
	str.	5	1792.5	85.7		m.	4	1906.7	84.7
	str.	5	1877.8	85.3		m.	4	1993.4	86.7
	str.	5	1963.6	85.8	I	v. d.	2	1826.0	
E	d.	3	2050.0	86.4		v. d.	2	1911.7	85.7
					J				
	v. d.	1	F 1540.1			d.	3	F 1665.5	
	d.	2	1629.2	89.1		m.	3	1751.8	86.3
	m.	3	1715.0	85.8		d.	3	1837.8	86.0
	str.	5	1800.0	85.0		m.	4	F 1923.1	85.3
	str.	5	1886.1	86.1					
	str.	5	1972.5	86.4		d.	4	A 2268.3	86.3 × 4
	str.	5	F 2058.5	86.0		d.	1	A 2440.2	86.0 × 2
	m.	5	A 2058.6						

¹ Estimated. Str., strong; m., medium; d., dim; v. d., very dim.

² The most reliable results (as indicated by the number and consistency of the individual measurements, the appearance of the negatives, etc.), are marked 5; the least reliable by 1.

³ The unit in which $1/\lambda$ is expressed is such that for $\lambda = 5,000$ A.U. $1/\lambda$ is written 2000.

moderate intensity we feel that the averages that are here tabulated are reliable within 1 A.U. In other words the reciprocal wave-lengths are accurate to within about .02 per cent. For the faint or hazy bands the possible error is undoubtedly much greater.

Of the fifty-five fluorescence bands observed forty-six can be arranged in nine series as tabulated below, the frequency interval being nearly constant in each series. Two of the remaining bands have the same interval, and apparently form part of a series whose other members were too weak to detect. The seven bands that do not fall in any series arrangement are all extremely weak and since in most cases they are recorded only once their existence is subject to considerable doubt. Estimates are given in the table of the intensities of the different bands and of the reliability of the measurements. In some cases the series seem to extend into the region of absorption and in such cases the absorption bands that seem to form part of the series are also given.

The data for series B, D, E and F, which are made up of the stronger bands and those of medium intensity, are undoubtedly the most reliable. The values of the average interval between bands in these series are 86.0, 85.8, 85.9 and 86.1 respectively. In taking these averages the first band in the case of series D and E has been left out of consideration on account of its relative uncertainty. For the other series the interval, although less certain, has nearly the same value. It will be noticed that there is nothing to indicate any change in the interval as we pass from the longer to the shorter waves.

In a spectrum consisting of so many bands the occasional repetition of any given interval between bands is to be expected, even if the bands are distributed at random. It is proper to inquire, therefore, whether this interval of about 86.0 really occurs more frequently than would be expected for a random distribution. Data bearing on this point are plotted in the upper curve of Fig. 2. In this curve horizontal distances indicate the lengths of different possible intervals between bands, while ordinates give the number of times each interval occurred. The range of possible error in the location of each band is arbitrarily assumed to be 2 units. Thus for a frequency interval 32 (abscissa) the ordinate is 10. This means that ten pairs of bands were found for which the interval lay between 31 and 33.

It is evident from the chart that certain intervals occur with much greater frequency than would be expected if the bands were distributed at random, and this is most conspicuously true of the interval 86. It will be noted that the curve also shows lesser maxima for several other frequency intervals: *e. g.*, 8, 16, 70, 78 and 94. These intervals correspond

to the spacing of the bands in the successive groups which make up the spectrum.

On account of the fact that large clear crystals could be obtained the hexahydrate offered an especially favorable case for the study of the

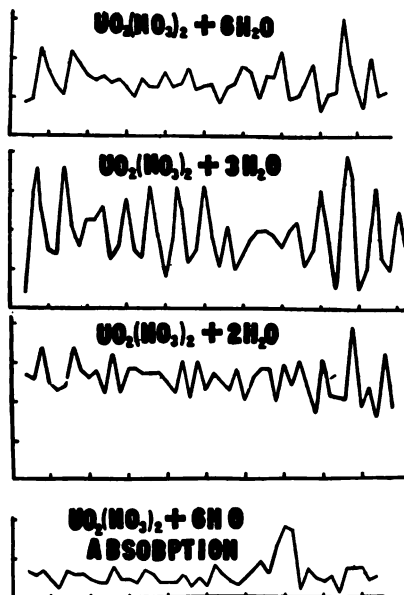


Fig. 2.

Showing the frequency of occurrence of different intervals between bands. Abscissas show the intervals (1 division = 10). Ordinates show the number of times the interval occurs (1 division = 10).

absorption spectrum. Observations were made with a number of different crystals ranging in thickness from a few tenths of a millimeter to three or four mm. The averages given in Table II. are in many cases based upon fifteen or more independent measurements. In the case of the band at 2148.7, for example, 17 measurements of wave-length were made, of which three were discarded because of the unsatisfactory character of the negatives. In the fourteen measurements used in forming the average the reciprocal wave-length ranged from 2147.8 to 2149.6, most of the values lying close to the average. In other cases the wave-length is much more uncertain. The extremely faint band at 2536.4, for example, was observed on only two negatives; while the dim broad band at 2720.3 was observed only once. The reliability of the recorded average has been estimated in each case and is indicated in the table.

A study of the absorption spectrum shows that an interval of about

TABLE II.

Series in the Absorption Spectrum of Uranyl Nitrate Hexahydrate $[UO_2(NO_3)_2 \cdot 6H_2O]$.

Series.	Intensity.	Reliability.	$\frac{1}{\lambda}$.	$\Delta \frac{1}{\lambda}$.	Series.	Intensity.	Reliability.	$\frac{1}{\lambda}$.	$\Delta \frac{1}{\lambda}$.
c	v. d.	2	2127.2		h	m.	5	2148.7	
	str.	4	2200.4	73.2		str.	5	2219.2	70.5
	d.	5	2272.3	71.9		str.	4	2290.2	71.0
d'			[2053.4]			m.	4	2359.4	69.2
	v. d.	4	2125.0	71.6		m.	4	2430.1	70.7
	d.	2	2196.8	71.8		d.	4	2500.0	69.9
	d.	4	2268.3	71.5	α	d.	5	2164.0	
	str.	3	2340.4	72.1		d.	4	2235.4	71.4
	m.	3	2412.0	71.6	β	str.	4	2321.0	
	str.	3	2484.1	72.1		m.	3	2390.0	69.0
	str.	3	2555.3	71.2	γ	str.	2	2464.3	
e	str.	5	[2058.5]			str.	4	2533.2	68.9
	m.	5	2058.6		δ	str.	4	2552.8	
	str.	4	2131.2	72.6		str.	4	2623.3	70.5
	m.	4	2203.8	72.6		d.	2	2695.0	71.7
	d.	4	2277.8	74.0	δ'	d.	3	2559.5	
f	d.	4	[2061.9]			m.	4	2630.5	71.0
	m.	4	2061.7						
	str.	5	2134.1	72.4					
	m.	4	2207.3	73.2					

71 between bands is of relatively frequent occurrence. (See the lower of curve Fig. 2.) In several instances definite series exist with this constant interval. The values of $1/\lambda$ for the bands forming these series are given in Table II.

The two series e and f begin with reversible bands. Thus the first band of series e, at $1/\lambda = 2058.6$, cannot be distinguished in position from the last band, $1/\lambda = 2058.5$, of the fluorescence series E, while the first band $1/\lambda = 2061.9$, of series f is coincident with the band 2061.7 of fluorescence series F. There is some indication that several other absorption series may be looked upon as associated with fluorescence series in the same way. Thus the series d' may perhaps be associated with a very weak fluorescence series falling between D and E. Two bands of such a series were occasionally observed at 1968.0, and 2053.4 (interval 85.4). The interval between the line at 2053.4 and the first line of series d' is 70.6, which is almost exactly the average interval for the absorption series. Again in the case of series h we might expect an absorption band to fall at 2078.7, while series H might have a fluorescence band at nearly the same point, viz., 2079.3. Neither band was observed.

But it must be remembered that the detection of reversible bands is only possible when the conditions of excitation are suitable. Any trace of fluorescence tends to mask an absorption band, and vice versa. The scarcity of bands, either of fluorescence or absorption, in the "reversal region" lying between 2060 and 2120 is perhaps due to this cause. There are other cases which suggest the same relationship between fluorescence and absorption series, although less definitely.

While there are thus strong reasons for believing that certain fluorescence series are to be looked upon as associated, in the manner indicated above, with absorption series, yet there are several series in the fluorescence spectrum for which no related absorption series have been observed; and on the other hand there are several absorption series which do not appear to be related with the observed fluorescence.

The observed intervals between bands are not so nearly constant in the case of the absorption series as in the fluorescence series. It seems to us probable that this is due to the greater uncertainty in the wavelength determinations; for on account of the lack of sharpness of the absorption bands and their greater width, as compared with the fluorescence bands, the accuracy that is attainable in determining their location is considerably less than in the fluorescence spectrum.

It will be noted also that the interval between bands is different for different series. For series e the average interval is 73.1; for series d' it is 71.7; for series h, 70.3; while for the two pairs of lines in the ultra-violet, which have been designated as series β and γ , the interval is in one case 69.0 and in the other 68.9. This change in interval as we pass from one series to another, which is too great to be accounted for by experimental errors, appears of especial significance when it is remembered that in the fluorescence spectrum the interval is the same for all the series.

One of the most puzzling points brought out by the detailed study of the observed spectra is the fact that a considerable number of the absorption bands are spaced with the interval corresponding to the fluorescence series, and in some cases appear to form a continuation of these series. In such cases the reciprocal wave-lengths for these bands are included in Table I but are preceded by the letter A. Thus there are four absorption bands, in addition to the reversible band, which apparently belong to series F. If it is assumed that they do form a part of this series the average interval for the whole series comes out exactly the same as for the fluorescence bands alone. Series J appears to include two absorption bands, and series B, E, and G each show one band. On the whole, however, we are inclined to look upon these cases as the

result of accidental coincidences and to believe that the fluorescence series do not extend into the absorption region beyond the reversible band.

The interval of about 70, which appears to be characteristic of the absorption spectrum, is also found in the fluorescence spectrum. Thus the bands of series C are displaced from those of series E by intervals ranging from 69.0 to 70.2. A similar relation appears to exist between series J and series H, the average displacement being 70.1. Between series B and A the average shift is 70.6, between series F and H, 68.1, between series D and B, 69.1. With the exception of series I the series thus seem to be grouped in pairs, the interval between pairs being in the neighborhood of 70. It seems not unlikely that a companion series exists for I also, since faint bands were occasionally observed at 1826.0 and 1911.7 (interval 85.7) which are displaced by the intervals 71.3 and 71.4 from the corresponding bands of series I.

THE TRIHYDRATE, $\text{UO}_2(\text{NO}_3)_2 + 3\text{H}_2\text{O}$.

The trihydrate crystallizes in the triclinic system with the axial ratios $a : b : c = 1.2542 : 1 : 0.70053$.¹ The crystals were grown by evaporating the nitric acid solution in a desiccator over caustic potash and sulphuric acid or over calcium chloride. To obtain large crystal plates the bottom of a dish 6 cm. in diameter was covered with the solution to a depth of about 2 mm. and the solution was "seeded"

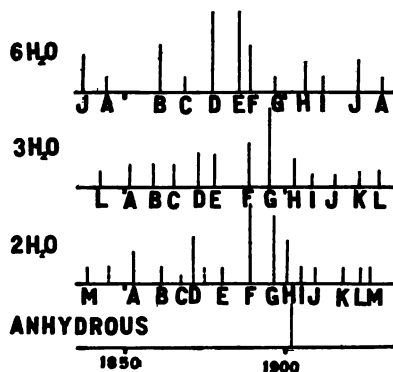


Fig. 3.

Showing one group of bands from the fluorescence spectrum of each of the salts studied. The spectrum of the hexahydrate contains seven such groups; that of the trihydrate, six; the dihydrate, five; and the anhydrous salt, three.

near the center. A cover was then placed over the solution with a small opening at the center, so that evaporation took place directly over the crystal. The trihydrate was also obtained in the form of a fine powder

¹ Wyrouboff, Sur quelques composés de l'uranium, Bull. Soc. Française Mineral, 32, 349-50, 1909.

by efflorescence of the hexahydrate crystals in dry air. Although most of the observations recorded below were made with single crystals, a few measurements were made, with concordant results, on the powder. In the fluorescence spectrum visual observations only were made. In the absorption spectrum the measurements were in most cases photographic.

The fluorescence spectrum of the trihydrate was found to consist of 63 bands, of which 55 fell into 12 constant interval series of from three to six bands each. The intervals for the different series ranged from 86.5 to 87.5, the average being 86.8. The reciprocal wave-lengths are given in Table III. The numbers in brackets refer to absorption bands which seem to fall into the fluorescence series. The relative intensities of the bands are shown roughly in Fig. 3 where a typical group, *i. e.*, one band from each series,—has been plotted for each of the salts studied.

TABLE III.

Series in the Fluorescence Spectrum of Uranyl Nitrate Trihydrate, $UO_2(NO_3)_3 + 3H_2O$.

A.	1677.0, 1765.0, 1851.5, 1938.8, 2025.0, (2112.7, 2200.2)
B.	1686.1, 1772.0, 1858.7, 1945.7, 2033.2, (2120.7)
C.	1778.7, 1865.1, 1952.9, 2041.1, (2041.1)
D.	1699.8, 1785.9, 1873.0, 1959.1, 2046.8, (2134.0, 2220.6, 2307.9)
E.	1704.2, 1791.8, 1877.4, 1965.3, 2051.0
F.	1629.2, 1715.2, 1802.1, 1889.0, 1976.4, 20643.?
G.	1637.2, 1722.8, 1808.7, 1895.3, 1982.3, 2070.7
H.	1643.9, 1729.2, 1816.1, 1903.0, 1989.9, 2076.3, (2076.5, 2251.2)
I.	1821.3, 1908.9, 1995.9, (2083.8)
J.	1741.8, 1828.2, 1915.9, 2002.1, (2089.7)
K.	1748.6, 1835.3, 1923.3, 2009.8
L.	1667.2, 1755.0, 1842.4, 1930.1, 2017.4, (2103.5, 2189.2, 2277.9)

The second curve of Fig. 2, which shows the frequency of occurrence of the different possible intervals between the bands of the fluorescence spectrum, indicates a remarkably regular grouping of the bands. Besides the principal interval 87.0 which is characteristic of the series in this spectrum, a number of other intervals are almost equally prominent, *e. g.*, 7, 14, 30, 36, 43.5, 50, 80, 94. In the case of each of these intervals the frequency of occurrence is far above the average. These intervals, of course, correspond to the spacing of the bands in the groups. Thus the interval 43.5, which is just half the principal interval 87.0, occurs twice in each group, the bands of series K lying half way between the bands of series F and the bands of series L half way between those of series G. In each case the two series might be combined to form a single series with half the interval. Since, however, the bands of the combined series would be alternately strong and weak it does not appear that such a combination is justified.

In the absorption spectrum of the trihydrate 48 bands were observed, several of which, however, were so faint and indistinct as to make their existence doubtful. As in the case of the hexahydrate an interval between bands of a little more than 70 is of frequent occurrence, and 37 of the bands (including all that are strong and well defined) can be arranged in 9 constant interval series. The interval does not appear to be the same for all of these series, however. In one case the interval is as high as 73.8, while in another case its value is 71.0. For most of the series the interval lies near 72.0.

In many instances the absorption series start with reversed fluorescence bands. Reversals are especially sharp and definite in the case of the final bands of series C, G and H and L ($1/\lambda = 2041.1, 2070.7, 2076.3, 2104.9$). In other cases the absorption series begins at a point where a fluorescence band might be expected, but where none was actually observed. Thus we should expect the final bands of series I, J and K to lie at 2082.7, 2090.0 and 2096.6 respectively. These bands were not observed, probably because of the fact that the three series in question are made up of very faint lines. But the first bands of the absorption series i, j and k, fall at 2083.8, 2089.7 and 2095.6 and it would seem, therefore, that they might properly be looked upon as resulting from the reversal of the final bands of the corresponding fluorescence series, even though these bands escaped observation.

The reciprocal wave-lengths for the principal absorption series are given below. Each series is lettered in such a way as to indicate the

TABLE IV.

Series in the Absorption Spectrum of Uranyl Nitrate Trihydrate, $UO_2(NO_3)_2 + 3H_2O$.

b	B = 2033.2	b	= (2033.2), 2107.0, 2180.8
c	C = 2041.1	c	= 2041.1, 2112.7, 2186.7
c'	C' = ?	c'	= 2043.3, 2116.0, 2189.7, 2261.0
f'	F' = 2058.5	f'	= 2057.6, 2129.0, 2200.2, 2271.7
g	G = 2070.7	g	= 2071.7, 2142.7, 2213.2, 2285.7, 2357.9
h	H = 2076.3	h	= 2076.5, 2148.6, 2220.6, 2368.0
i	I = (2082.7)	i	= 2083.8, 2154.5, 2226.9, 2297.8, 2368.0
j	J = (2090.0)	j	= 2089.7, 2162.0, 2235.4, 2307.9
k	K = (2096.6)	k	= 2095.6, 2166.7
l'	L' = ?	l'	= 2103.0, 2173.4, 2245.7, 2317.5

fluorescence series with which it appears to be related: *e. g.*, the first band of series *c* is the reversal of the last band of series *C*. In each case also the reciprocal wave-length is given for the last band of the fluorescence series. The numbers in brackets indicate bands that were not observed but would be expected to occur with the indicated value of $1/\lambda$.

It is to be observed that no absorption series were found which corresponded to the fluorescence series A, D, E and L. On the other hand no fluorescence series were observed to correspond with the absorption series c' and l' .

THE DIHYDRATE, $UO_2(NO_3)_2 + 2H_2O$.

Although the dihydrate has been made by several observers the crystalline form does not appear to have been studied. The crystals used in this investigation were in most cases made by heating a tube containing the crystallized trihydrate to a temperature somewhat above 100° C. and passing through it a current of air which had been run through a mixture of sulphuric acid and nitric acid and over phosphorous pentoxide. The melted trihydrate slowly evaporated and recrystallized as dihydrate without losing nitric acid. The tube was then sealed up to prevent the entrance of moisture. The dihydrate was also prepared synthetically by treating dry H_2UO_4 with the "monohydrate" nitric acid, the two being sealed in a glass tube and allowed to react. The crystals obtained were in each case small so that the observations were made on a mass of crystals having no systematic arrangement of the axes.

Seventy-four bands were observed in the fluorescence spectrum, sixty-one of which fell into twelve constant interval series of from four to six bands each. The interval ranged from 87.6 in series F to 88.3 in series A, but in most cases lay near the average of all, viz., 88.1.¹ The values of $1/\lambda$ are given in Table V., and a typical group, consisting of one band from each series, is shown in Fig. 3. Many of the bands that do not fall into these twelve series nevertheless seem to belong to similar series of which only two or three bands could be detected. Two of these suspected series have been included in Fig. 3 where they are indicated by dotted lines.

TABLE V.

Series in the Fluorescence Spectrum of Uranyl Nitrate Dihydrate, $UO_2(NO_3)_2 + 2H_2O$.

A.	1676.7, 1765.9, 1853.0, 1942.7, 2030.7
B.	1684.9, 1773.7, 1861.6, 1951.0, 2037.9
C.	1780.6, 1867.8
D.	1609.0, 1695.5, 1783.5, 1871.8, 1959.7, 2047.4
E.	1618.9, 1705.9, 1794.4, 1880.3, 1967.5
F.	1625.3, 1714.3, 1801.5, 1889.5, 1977.1, 2063.1
G.	1632.7, 1721.2, 1808.5, 1896.8, 1985.2, 2072.5, (2072.7)
H.	1637.7, 1725.3, 1814.6, 1900.8, 1989.7, 2077.3, (2078.2)
I.	1641.5, 1729.8, 1905.1, 1993.8
J.	1645.3, 1734.6, 1821.2, 1909.0, 1998.4
K.	1830.5, 1918.3
L.	1658.4, 1745.5, 1834.9, 1923.8, 2009.3, (2095.9)
M.	1751.0, 1838.2, 1926.8, 2015.5, (2102.7, 2191.3, 2278.0)

¹ For series C, which contained two bands only, the interval was 87.2.

The absorption bands were located by observing the spectrum of light from a continuous source after diffuse reflection from a mass of small crystals. Although the bands observed in this way are surprisingly sharp the method is not so satisfactory as that in which the light is analyzed after direct transmission through a single crystal. It is doubtful whether the reflection method gives as great accuracy in the location of the bands, and many of the weaker bands, which would have been easily detected if large crystals had been available, were probably not observed at all. For this reason, perhaps, the absorption spectrum of the dihydrate shows only four well defined series. The reciprocal wavelengths for these series are given in Table IV. The interval between bands is 70.0 for series m, 70.4 for series j, and 71.3 for series g and h. In each case the first band in the absorption series occupies nearly the same position as the last band in one of the fluorescence series.

TABLE VI.

Series in the Absorption Spectrum of Uranyl Nitrate Dihydrate, $\text{UO}_2(\text{NO}_3)_2 \cdot 2\text{H}_2\text{O}$.

G = 2072.4	g. = 2072.7, 2144.7, 2215.3
H = 2077.3	h = 2078.2, 2149.4, 2218.9, 2290.4, 2362.9
J = (2085.7)	j = (2086.1), 2156.6, 2227.2, 2297.3
M = (2103.6)	m = 2102.7, 2172.7, 2242.7

THE ANHYDROUS NITRATE.

Specimens of uranyl nitrate that were in all likelihood anhydrous, and which certainly contained less water than the dihydrate, were prepared by allowing nitric anhydride, N_2O_5 , to react with uranic oxide. The nitric anhydride was distilled from a mixture of nitric acid and phosphorous pentoxide, while the uranic oxide was prepared by heating uranic acid, H_2UO_4 . In preparing the oxide the heating was not continued so long as to completely drive off the water from the uranic acid, since when this was done no reaction occurred between the oxide and the nitric anhydride.

At temperatures above 30°C . the N_2O_5 reacts with the mixture of UO_3 and H_2UO_4 to form uranyl nitrate, presumably anhydrous, and a considerable amount of HNO_3 . To free the specimen from acid the tube containing it was placed in a freezing mixture until the N_2O_5 was frozen, when the HNO_3 , which still remained a liquid, was poured off. This process was repeated several times. While the amount of water remaining after this treatment must have been extremely small, we cannot feel certain that all traces were removed, and it is possible therefore that the nitrate formed may have consisted in part of the monohydrate. No fluorescence bands belonging to the other hydrates could be observed.

The method of preparation was varied by changing the temperature at which the reaction was allowed to occur, and by heating the salt, after it had been formed, to different temperatures and for different periods.

The fluorescence spectra of the different preparations differed widely. In one case, the spectrum was found to consist of three narrow bands only. But in all other cases bands were observed which remained broad (about 100 A.U.) even at the temperature of liquid air. These bands were spaced with a constant frequency interval of approximately 88–89. It seems probable that these broad bands were due to the solution of the anhydrous salt in nitric acid. To test this point a specimen was prepared under conditions which made certain the presence of a considerable excess of nitric acid. The fluorescence spectrum contained two series of broad bands, the central band of the stronger series lying at $1/\lambda = 1939.0$ and that of a weaker series at $1/\lambda = 1920.0$. The specimen was then gently heated and the nitric acid driven off was condensed in a connecting tube. After this process had continued for a short time, several series of narrow bands or lines appeared in the fluorescence spectrum (at -186°C.). As this procedure was repeated the line spectrum became more prominent and the broad bands fainter. In one instance the specimen was heated nearly to decomposition—in fact part of the salt was undoubtedly decomposed—and in this case the very faint fluorescence spectrum consisted of three narrow bands only. The same three bands were observed in the case of most of the specimens that were prepared in the attempt to remove the water of crystallization, although in other cases they were accompanied by other lines or broad bands. It seems probable that these three bands constitute the brightest part of the fluorescence spectrum of the anhydrous salt, and that the additional lines and bands that were observed in some specimens are due to traces of the monohydrate or to a solution of the nitrate in HNO_3 . The three bands formed a series with the interval 88.5, the central band lying at $1/\lambda = 1902.0$.

SUMMARY AND CONCLUSIONS.

1. In the case of each of the nitrates the fluorescence spectrum is made up of series in which the intervals between bands are constant and the same for all of the series. The interval increases slightly, but unmistakably, as the amount of water of crystallization decreases. For the hexahydrate the interval is 86.0; for the trihydrate 86.8; for the dihydrate 88.1; and for the anhydrous salt 88.5.

2. Numerous constant interval series occur in the absorption spectrum, the interval being approximately 71. But the interval does not appear

to be the same for different series even when these occur in the spectrum of the same salt. No systematic variation with the amount of water of crystallization could be detected.

3. Nearly all of the series in the absorption spectrum have their origin in the "reversing region," the first member of the absorption series being in coincidence with the last member of a fluorescence series and constituting a "reversible" band.

4. There is some slight resemblance between the different hydrates as regards the grouping of the bands (see Fig. 2). In each case, for example, a certain short interval appears with a frequency considerably above the average. In the case of the hexahydrate and the dihydrate this interval is about 8; in the case of the trihydrate is almost exactly 7. The interval 14, in the case of the trihydrate, and 16, in the case of the other two salts, is also of unusually frequent occurrence. It will be interesting to determine whether this peculiarity in the distribution of intervals is characteristic of all uranyl salts.

5. It is clear from inspection of Fig. 3, in which a characteristic group of bands is shown for each of the hydrates studied, that the different spectra are not in the least similar in their general appearance. It might at first appear that the three hydrates have one series in common, viz., that designated as series F. But while the central band of this series does occupy practically the same position in each of the three spectra, the fact that the interval between bands is different for the different salts causes the bands to fall more and more out of step as we proceed in either direction from the center of the spectrum.

On the whole the spectra of the different hydrates differ from one another fully as much as do the spectra of two different uranyl salts. This result is surprising, since it is customary to think of water of crystallization as rather loosely attached and therefore incapable of exerting a great influence upon properties which depend upon the internal structure of the molecule. In the uranyl salts it appears that we must look upon the connection as more intimate than has generally been supposed. The results also tend to confirm the view that fluorescence spectra are due to vibrations which occur in the outer parts of the molecule and which are therefore readily modified by outside sources of disturbance.

PHYSICAL LABORATORY OF CORNELL UNIVERSITY,
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THE MICA X-RAY SPECTROMETER.
A REPLY TO MR. SIEGBAHN.

BY W. S. GORTON.

IN the September number of the PHYSICAL REVIEW Mr. Manne Siegbahn criticised a paper of the author's which had appeared some months previously.¹ The present article points out some errors made by Mr. Siegbahn in interpreting the author's results.

Mr. Siegbahn, in arriving at his conclusions, evidently made two assumptions, neither of which is in accord with the writer's results. He assumed that all the lines found were alike in appearance, that is, that there was no ocular evidence that some of the lines were due to different constituents in the mica, and, further, that for a certain set of lines the measurements of $\sin \theta$ were consistently in error by about 10 per cent. The accuracy obtained was precisely that stated in my paper, namely, 0.5 per cent. for values of $\sin \theta$ in the neighborhood of 0.350, the accuracy being *inversely proportional to* $\sin \theta$. Mr. Siegbahn considered that the lines, which the author ascribed to four distinct constituents in the mica, were the second, third, fourth and fifth orders due to one single constituent. He assigned the same wave lengths to all of the stronger lines in his second, third, and fifth orders as the author did to those in his fourth, second and first series respectively. To assign wave-lengths to the lines of these series is easy, as the intensity of the lines is comparatively large and the stronger lines can be identified at a glance. In order to bring the lines in his second order into the general scheme Mr. Siegbahn had to assume that the measurements of $\sin \theta$ for these lines were consistently in error, as compared with the other measurements, by 10 per cent. This assumption is entirely inadmissible. The accuracy of the measurements in this region was 1.5 per cent.; and it will be noticed that in Siegbahn's Table I. the values of $2d \times 10^8$ for the lines of his second order do not differ from their mean value by more than 1.25 per cent. This fact alone should dispose of the idea that these particular values are consistently in error by 10 per cent. It is impossible, then, that the lines of this fourth series can be the second order lines due to a structure which might possibly have a third and fifth order in the lines of series 2 and 1 respectively. The lines of series 4, moreover,

¹ PHYSICAL REVIEW, March, 1916.

extend only part of the way across the plate and have a rather sharply defined terminus. This certainly shows that they cannot be due to the structures responsible for the lines of series 2 and 1 as the lines of these series extend entirely across the plate.

The above considerations leave the lines of series 3 still to be discussed. In this series the values of the wave-length assigned to the various lines by Siegbahn and the author are entirely different. If it were only a question of assigning such values of the wave-length to the lines of this series as to establish a reasonably good proportionality between the wave-length and $\sin \theta$, there would be little choice between the two methods, but such is not the case. The lines of series 3 extend only part of the way across the plate, showing that they are due to a structure closely connected with that to which the lines of series 4 are due. Of the lines assigned by Siegbahn to the fourth order, some extend entirely across the plate and some extend only part of the way. Moreover, the author's choice of wave-lengths enables more lines to be accounted for than does Siegbahn's choice.

The above considerations show that the lines of series 3 and 4 are due to structures other than those giving rise to series 1 and 2. No simple relation exists between the interplanar distances calculated for series 3 and 4; consequently two separate and distinct structures must be considered to exist in order to account for these two series. When the interplanar distances for series 1 and 2 are calculated, it is found that they are very nearly in the ratio 3 : 5. This raises the question whether these two series are not, as Siegbahn asserts, the third and fifth order spectra due to one structure. In order for this to be so, the structure would have to be such as to suppress all orders of spectra except the third and fifth. Such a structure would almost certainly be a very complicated one and the writer considered it more probable, in view of the known complex nature of mica, that there were two structures responsible for series 1 and 2. On this view it is easy to understand why only a few faint lines appear beyond $\sin \theta = 0.377$, the limit of the first order lines. If strong fifth order lines are present one would expect that numerous lines of the sixth, seventh, and higher orders would be visible.

Siegbahn refers to a paper by de Broglie in which a plate shows plainly six orders of spectra due to a mica analyzer. In this connection it should be remembered that mica is a substance of variable composition, and it should cause no surprise if different specimens show differences in composition and structure.

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THE TUNGSTEN ARC UNDER PRESSURE.

BY GEORGE P. LUCKEY.

INTRODUCTION.

THE present investigation was undertaken to determine, if possible, whether a change in the melting point of tungsten due to a change in pressure could be noted in the case of the tungsten arc. From previous work done on the effect of pressure on the melting point of metals having low melting points¹ it seemed doubtful if any changes could be noticed with the low pressures at which the tungsten arc could be burned in the apparatus at hand, pressures up to 35 atmospheres. On the other hand, work done by Lummer² seemed to indicate that in carbon, at least, there was a large pressure effect, even though it could not be attributed directly to a change in the melting point. Due to a lack of data on the latent heat of fusion, and the volume change on melting, the temperature change in the melting point of tungsten could not be calculated.

APPARATUS.

The arc was burned in a specially constructed brass tank, designed to hold pressures up to forty atmospheres. A cross-section of the tank is shown in Fig. 1. The central portion is a flanged tube three inches in diameter and four inches long. Brass plates were fastened onto each end by means of twelve bolts which passed from one end of the tank to the other. Lead washers were placed between the end plates and the central portion of the tank to secure air tight joints. Three openings were made in the tank, one in an end plate into which a plate glass window *w* for observing the arc was fitted, and two in the central portion. One of these was used in evacuating and filling the tank, and through the other an insulated wire was led in. The outside of the tank was wrapped with a coil of copper tubing, through which a stream of water was passed, and which served as a very effective cooling device.

A small holder that could readily be removed from the tank was constructed for holding the electrodes. The electrodes were held in

¹ Johnston and Adams, *Am. Jour. of Sci.*, Vol. 31, p. 501, 1911; Tammann, *Zs. Anorg. Chem.*, Vol. xi, p. 54, 1904; Bridgman, *Proc. Am. Ac. of Arts and Sci.*, 47, pp. 347, 391, 415, 1911.

² Lummer, *Verflüssigung der Kohle und Herstellung der Sonnentemperatur.*

the center of the tank and the arc was burned in an axis parallel to the walls of the tube. The positive electrode, and the one on which the temperature measurements were made, was fastened to the center of the frame, and was insulated from it by several layers of mica. This electrode was connected to the leading-in wire, and the remainder of

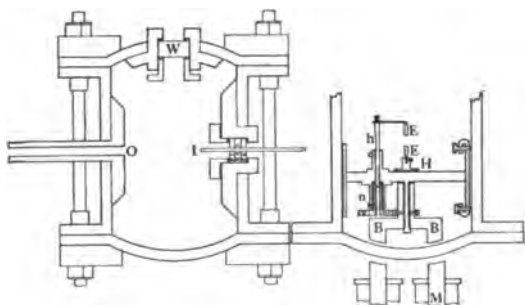


Fig. 1.

Diagram of pressure tank and electrode holder.

Pressure Tank.—*W*, window; *O*, gas inlet; *I*, insulated leading-in wire, held between two layers of fiber, *f*₁ and *f*₂ with a soft rubber plug *r*, in the center.

Arc Holder.—*E*, *E*, Tungsten electrodes; *h*, movable electrode holder; *H*, stationary electrode holder, insulated from frame by layers of mica; *B*, soft iron bar, the rotation of which turns nut, *n*, on the end of the movable electrode holder, *h*, through gears, *g*₁ and *g*₂, and causes electrode holder, *h*, to move in or out; *M*, magnet used to rotate soft iron bar.

the circuit was made through the tank and the frame of the holder. The negative electrode was fastened to one end of a shaft which could be moved back and forth. On the other end was a nut which engaged a thread on the shaft. As the nut was turned the shaft was drawn in or thrust outwards. The nut was geared to an iron bar fitted centrally to the back of the electrode holder. The iron bar could be rotated by a magnet outside of the end plate of the tank. Thus by rotating the magnet the two electrodes could be brought together, and on reversing the direction of rotation they could be moved apart.

When measuring the temperatures of the tungsten the negative electrode was set slightly out of the center of the tank, so that an unobstructed view of the whole positive electrode could be obtained through the window, and so that there were no disturbing reflections of the image of the negative electrode from the smooth mirror-like surface of the electrode under observation.

The arc was burned in nitrogen, which was obtained from a tank under a pressure of 1,800 pounds per square inch. The gas had been freed from all impurities, with the exception of water vapor, before it was pumped into the tank. In order to remove the water vapor, before

filling the small tank in which the arc was burned, the gas was passed through a steel tube, one end of which was filled with soda lime, and the other with phosphorous pentoxide. The pressures in the small tank were measured by a twelve-inch Ashcroft test gauge graduated to read from 0-500 pounds per square inch.

METHOD OF DETERMINING TEMPERATURE.

The temperatures were measured by means of a Holborn-Kurlbaum optical pyrometer of the type in general use in this laboratory.¹

The pyrometer lamp was calibrated by the use of a tungsten lamp which had been standardized with a black body at the melting point of palladium, taken as 1822° K. The effective wave-lengths of the red glass screen (Jena Rotfilter No. F-4512) between different temperatures were obtained from data calculated by Hyde, Cady and Forsythe.² The data for the absorption coefficient of tungsten at the melting point was extrapolated from readings made by Worthing³ at temperatures up to 3200° K. Below this point the absorption coefficient of tungsten is practically a linear function of the temperature. It was assumed that this function continued up to, and through the melting point without suffering any abrupt changes. The assumption that the absorption as a function of the temperature suffers no abrupt changes between the solid and the liquid states of tungsten seemed justified from the fact that in looking at the surface of the melting tungsten there is no sharp defining line between the solid and the liquid (in case the solid tungsten has a smooth surface) which there would be if the absorption suffered an abrupt change. This result is at variance with the observation of Langmuir⁴ (who states) that in the arc the solid tungsten always appeared brighter than the molten tungsten in contact with it. This latter effect was at times noticed in the present experiments in case the solid surface of the tungsten was not smooth, but apparently this was due to the diffuse reflection of the light from the negative electrode.

The value of the absorption as determined from the data of Worthing in the above manner and as used, was 0.393 for $\lambda = 0.661\mu$. This value is much lower than that used by Langmuir,⁵ 0.425 at $\lambda = 0.667\mu$. It might be stated in this connection that the method used by Langmuir in determining the emissivity of molten tungsten appears open to doubt, as in the present series of experiments it was at no time possible to

¹ Worthing and Forsythe, *Phys. Rev.*, Vol. IV., p. 163, 1914.

² Hyde, Cady and Forsythe, *Astrophys. Jour.*, Vol. XLII., p. 294, 1915.

³ Worthing, *Jour. Frank. Inst.*, Vol. 181, p. 417, 1916.

⁴ Langmuir, *Phys. Rev.*, Vol. 6, p. 138, 1915.

⁵ *Loc. cit.*

obtain a pool of melted tungsten in which the temperature throughout was that of the melting point. The temperatures obtained varied from that of melting tungsten at the edges of the pool to a temperature three or four hundred degrees higher near the center. The only time when it was possible to obtain nearly uniform temperatures throughout the visible surface of the molten tungsten was at low pressures when the whole end of the electrode was melted, and the temperatures found under these circumstances were several hundred degrees higher than the melting point.

THE MELTING POINT OF TUNGSTEN UNDER PRESSURE.

A series of readings of the melting point of tungsten were taken under various pressures from one to thirty atmospheres. Electrodes 0.1 cm. in diameter were used, the length of arc varied from 0.1 to 0.2 cm. The results of these determinations are given in Table I. The value found at atmospheric pressure agrees closely with that found by Worthing¹ in the tungsten arc, 3630° K., while it is about eighty degrees higher than the average value of the melting point as determined by Langmuir, 3540° K. The discrepancy between the values determined by Langmuir

TABLE I.

Apparent Melting Point of Tungsten under Pressure.

Pressure.		Red Black Body Temperature. $\lambda = .661\mu$.	True Temperature.
Atmospheres.	Kilo/Cm ² .		
1	1.03	3138° K.	3623° K.
4	4.13	3129° K.	3611° K.
8	8.26	3116° K.	3594° K.
11	11.36	3113° K.	3590° K.
14	14.46	3107° K.	3582° K.
18	18.6	3099° K.	3572° K.
21	21.7	3092° K.	3562° K.
28	28.9	3093° K.	3564° K.

and those determined in the present investigation is due to the different value of the emissivity of tungsten taken in the two cases. The values of the red black body temperatures obtained in all cases give good agreement. The value as determined by Langmuir for $\lambda = 0.667\mu$ was 3124° K., by Worthing for $\lambda = .666\mu$, 3140° K., while that determined in the present investigation was 3138° K. for $\lambda = 0.661\mu$.²

¹ *Loc. cit.*

² Various other determinations of the melting point of tungsten are well summarized by Pirani and Meyer (Berichte d. Deut. Phys. Gesell., 14, p. 426, 1912) who compare the values found by other observers with those they obtained. In addition to these might be mentioned the determinations made by Forsythe (Astrophys. Jour., Vol. 34, p. 353, 1911). All of these values are lower than those obtained here.

The usual appearance of the positive electrode during temperature measurements may be described as follows. There is a bright spot in the center where the main arc discharge falls, around this is a pool of melted tungsten, which in turn is surrounded by solid tungsten. If the tungsten on the end of the electrode is melted and allowed to cool slowly by gradually decreasing the current through the arc the surface remains smooth after the tungsten has become solid, if, however, the tungsten is cooled very rapidly by suddenly decreasing the current the surface is roughened. This roughened condition of the solid surface was of great advantage in determining the boundaries of the pool of melted tungsten. By increasing the current the melted pool of tungsten is made larger, and the pyrometer lamp filament may be matched with the tungsten in the moment at which it changes from the solid to the liquid state. Or the current may be left constant, and it may be assumed that the temperature of the tungsten at the boundary line between the solid and the melted portions has a value very near to that of the melting point. Readings made by these two different methods gave the same values or the melting point. It is necessary that the regions of solid and melted tungsten should be sharply differentiated in order to get accurate results. The roughness of the solid portion and the mirror-like surface of the melted tungsten gave a very sharp differentiation at low pressures. Above twelve atmospheres, however, the refraction of the light by the heated gas near the electrodes created such disturbances in the image that the position of this dividing line could not be determined with any great accuracy, so that a rather large uncertainty is introduced into the measurements.

Assuming no change in the emissive power of tungsten with pressure the melting point of tungsten as found apparently decreases slightly as the pressure increases. The change observed, however, is large in comparison with the changes found by previous experimenters (*loc. cit.*) in the case of metals having a low melting point, on the other hand it is very small in comparison with the changes found by Lummer in carbon, and has the opposite sign. It is also open to doubt whether the observed change is due to an actual change in the melting point, or whether the apparent diminution in brightness might not be due to a layer of cooler tungsten vapor above the electrode, the absorption of which would increase as the density of the vapor was increased at higher pressures. Unfortunately the apparatus used would not allow this point to be investigated further, but the order of such an effect might well be the same as that observed.

THE HIGHEST ATTAINABLE TEMPERATURES.

As has been previously noted, that part of the electrode on which the main arc discharge falls is much more brilliant than the surrounding melted tungsten. A series of readings were made on the red black body temperature of this portion of the arc, and temperatures were found which varied largely with the pressure, the current through the arc, and the area over which the discharge took place. The highest attainable red black body temperatures found at each pressure are given in Table II. Owing to the melting of the whole tungsten electrodes by the high currents that were necessary to obtain these temperatures only a few readings were possible at each pressure. If it be assumed that the absorption coefficient, as determined by Worthing, maintains the same relationship with respect to the temperature up to the temperatures here obtained, then the true temperature of the tungsten would be given by the values in the last column of Table II.

TABLE II.
Highest Attainable Temperatures.

Pressure.		Red Black Body Temperature. $\lambda = .661\mu.$	Assumed Absorp- tion Coefficient.	Computed True Temperature.
Atmospheres.	Kilo/Cm. ²			
1	1.03	3559° K.	0.374	4235° K.
8	8.3	3794° K.	0.363	4602° K.
15	15.5	3907° K.	0.358	4782° K.
22	22.8	3982° K.	0.354	4912° K.
29	30	4022° K.	0.352	4978° K.
33	34.1	4086° K.	0.349	5084° K.

There is a question as to what this temperature represents. It is doubtful whether it can be assumed to be the boiling point of tungsten, as at a given pressure the temperature varies with the current strength through the arc. It is more probable that it represents some temperature lying between the melting and the boiling points. The observed increase in the temperature with the pressure is possibly due to either an increase in the boiling point of tungsten, or to an increased density of the arc discharge, which would tend to concentrate the heating effect, or to both of the above causes. For a given current as the pressure was increased the arc discharge became more concentrated, till at high pressures the whole discharge fell upon a very small portion of the electrode surface.

ELECTRICAL CHARACTERISTICS.

In connection with the temperature determinations of the tungsten arc, measurements were also made of the electrical characteristics in nitrogen under pressures from one to thirty-three atmospheres.

Rapid fluctuations of the current and especially of the voltage, made it necessary to use an instrument with which both readings could be taken simultaneously. For this purpose a Siemens and Halske Oscillograph was used, and exposures of about two minutes were made, during which time the current was varied throughout the range of possible currents. The variation, however, was made slowly enough to enable the arc to reach stable conditions after each change.

Direct currents of from one to ten amperes, with an outside potential on the circuit of 225 volts were used. The length of arc was varied from 0.06 to 0.24 cm. At the highest pressure the longest arc that could be maintained stably with the voltage used was about 0.06 cm. With the exception of the characteristics taken at atmospheric pressure, electrodes having a diameter of 0.14 cm., and about 1 cm. long were used. At atmospheric pressure the positive electrode melted down when currents larger than three amperes were sent through the arc. In order that characteristics for larger current values could be obtained a positive electrode of 0.38 cm. diameter and a negative electrode 0.14 cm. in diameter were used. This change in the size of the positive electrode apparently made no change in the form of the characteristic.

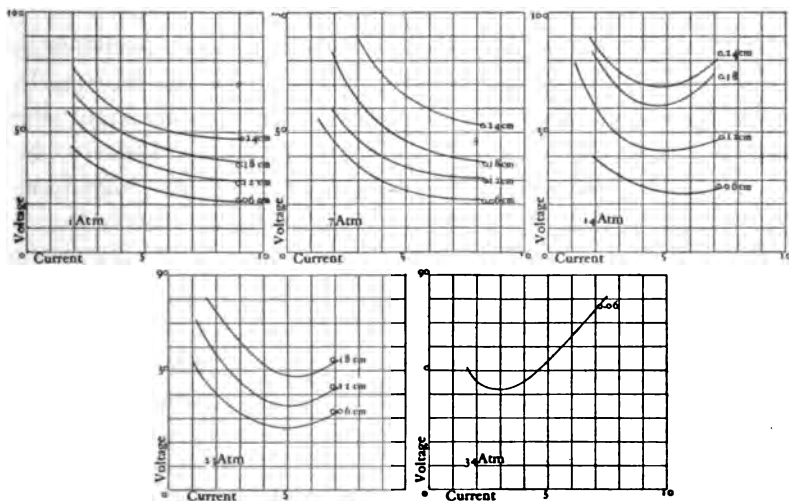


Fig. 2.

The characteristics of the arc are shown in Fig. 2. Owing to the large fluctuations of the voltage and current averages were made of several determinations, which are plotted above. The characteristics of the arcs of the same length tend to have higher values of voltage throughout the whole range of current as the pressure increases. This effect of pressure is similar to that noticed in arcs burned under pressures

lower than atmospheric, an effect especially predominate in the mercury vapor arc, and to that found by Duncan, Rowland and Todd¹ in the carbon arc burned at pressures up to ten atmospheres.

It will be noticed also that at high pressures the characteristics, instead of falling continuously, as is the case in the usual metal arc, have a tendency to rise. That is, as the current increases the voltage does not drop continuously, but after a certain current has been reached tends to rise if the current through the arc is made larger. A rising characteristic similar to this was noted by C. D. Child² in the case of the flaming carbon arc. He attributed this increase in voltage with larger currents to a tendency of the arc to flare out to one side. This would have the same effect as increasing the length of the arc. The rising characteristic found in the tungsten arc is probably partly the result of this same effect. The convection currents in the gas tending to blow the arc out of its normal path would become more marked as the pressure was increased, thus causing a higher potential drop across the arc for greater pressures, other conditions being the same. The effect of these convection currents in the gas at high pressures was seen in the fact that when all the external resistance was removed from the arc circuit, the current through the arc would not rise indefinitely; instead, after the current had reached a value of ten or twelve amperes, the arc would flare up and go out. The effect of the increase in the pressure in the tank due to the heating of the gas by the larger currents would have a similar effect on the characteristics. However, as this change in pressure was never very large, the increase in voltage from this cause would not wholly account for the large change observed.

SUMMARY.

Measurements were made of the temperature of the melting point of tungsten in the electric arc under pressures from one to thirty atmospheres. The melting point found at one atmosphere was 3623° K., and the apparent melting point decreased to 3564° K. when the pressure was raised to twenty-eight atmospheres. It is doubtful whether this decrease is due to a change in the melting point or merely to an increased absorption of the vapor surrounding the arc.

The highest temperatures attainable in the arc were measured, and a large increase in this temperature as the pressures increased was noted. At atmospheric pressure a probable temperature of 4235° K. was found, while at thirty-three atmospheres the highest temperature in the arc

¹ Duncan, Rowland and Todd, *Electrot. Z. S.*, 14, 603, 1893.

² C. D. Child, *Electric Arcs*, p. 60, D. Van Nostrand Co., N. Y.

was increased to 5084° K. It is probable that these temperatures represent a value somewhere between the melting and the boiling points of tungsten in each case.

The electrical characteristics of the tungsten arc were investigated. The drop in potential across the arc, other conditions being the same, was found to increase with the pressure. At high pressures the voltage dropped as the current increased until values of current of about five amperes were reached, after which the voltage rose with increasing current. This latter effect might be ascribed to the tendency of the arc to flare out to one side with larger currents, as well as to the increase of pressure in the tank caused by the heating of the gas.

In conclusion I should like to take this opportunity to express my appreciation to the director and the members of the staff of the Nela Research Laboratory for the suggestion of this problem and for their valuable assistance.

NELA RESEARCH LABORATORY,

NATIONAL LAMP WORKS OF GENERAL ELECTRIC COMPANY,

NELA PARK, CLEVELAND, OHIO,

August 24, 1916.

NOTE ON THE ELASTIC CONSTANTS OF ANTIMONY AND
TELLURIUM WIRES.

BY P. W. BRIDGMAN.

IN the course of an investigation on the effect of pressure on the electrical resistance of a large number of metals, I have had occasion to prepare wires of antimony and tellurium. The preparation of these metals in the form of wire appears to be somewhat of a novelty; I have not found any previous mention of it. Once the metals are obtained in the form of wire they may be utilized in the measurement of other physical properties besides electrical resistance. In particular, the elastic constants of neither of these metals seem ever to have been measured, and since measurements of this kind are particularly easy, I was unwilling to let the opportunity go, even though there is no especial connection with the original purpose for which the metals were prepared. The data given in this paper make no pretense to great accuracy, but are intended to give merely the approximate magnitude of the constants; in fact, for a reason that will appear, probably a serious attempt for much greater accuracy would be fruitless.

The wires were prepared by extrusion in a hydraulic press through a steel die. It is necessary to warm the metals considerably, and even then it is possible to obtain results with antimony only by a trick of manipulation. If the pressure is run up slowly until the pressure of extrusion is reached, the antimony is forced through the die in the form of wire, but continually breaks up (probably because of the release of internal strains) into very small pieces. The die used in this work had a diameter of 0.013 inch; with this it was difficult to get by this procedure pieces longer than one sixteenth of an inch. If the pressure is raised slowly above the minimum pressure of extrusion, the rapidity of outflow increases, but the spontaneous breakage becomes worse. By working excessively slowly at the minimum pressure of extrusion, I have been able to get a few pieces several inches long and 0.013 inch in diameter. But if the pressure is *rapidly* raised high enough above the minimum, a point is suddenly reached at which the antimony is expelled with explosive velocity in long coherent pieces of apparently limitless length. These may be caught and preserved in a vessel filled with water.

This point of explosive expulsion is doubtless the melting point of antimony under a non-hydrostatic pressure. The melting point is always depressed by this sort of a stress,¹ and the amount of depression is proportional to the square of the stress. The antimony so obtained is excessively brittle and must be handled with extreme care.

Tellurium wire is not so difficult to prepare, and I obtained a number of pieces one foot long by slow extrusion at the lowest pressure. There is every reason to think that rapid extrusion at the depressed melting point would be successful here also, but the design of the apparatus was not such as to allow this, since under the higher pressures tellurium comes out in curved pieces, whereas antimony comes out nearly straight. Tellurium made into wire in this way is, like antimony, very brittle.

No attempt was made to measure the temperature of extrusion. For antimony it was high enough for the temper color of the steel cylinder to reach a green. The pressure necessary was 10,000 kg./cm². or over. The tellurium was not heated so hot, to only about 330°, and the extrusion pressure was about 4,000 kg. As the cylinder cools during extrusion, a critical temperature is sharply reached below which a pressure of over 15,000 kg. is not sufficient. Of course, in either case, the extrusion pressure is lower the higher the temperature.

The antimony was from Eimer and Amend, and showed by analysis traces of As and Fe, other foreign metals none, and Sb 99.45 per cent. The tellurium I owe to the kindness of Professor G. W. Pierce, who has been using it for thermo-couples. He obtained it from Mr. Samuel Wien, of New York City, by whom it had been especially refined, but no analysis was available.

Two different sorts of test were made, bending and twisting. The first gives Young's modulus and the second the modulus of rigidity; these two constants completely determine the elastic behavior, provided the metal acts like an isotropic body.

The bending tests were made by fixing the wire with deKhotinski cement into the chuck of the milling attachment of a jeweler's lathe, loading the end with a rider of known weight, and observing the deflection of the free end with a cathetometer. By rotating the chuck, the bending in different orientations may be obtained, thus eliminating the effect of inequalities of cross section. Three readings were made at each setting; settings were at angular intervals of 45°. The length of the wires was from 7.5 to 9.0 cm., the load was 0.014 gm., and the deflection varied from 0.3 to 0.7 mm. Two different specimens of antimony were measured, agreeing within 2 per cent. Young's modulus was calculated from

¹ P. W. Bridgman, *Phys. Rev.*, 7, 215-223, 1916.

these measurements by the formula of elasticity $E = Wl^3/3I\xi$ where W is the load in dynes, l the length, I the moment of inertia of the cross section about a horizontal diameter of the section, and ξ is the deflection of the free end under the weight.

The rigidity was measured by attaching coaxially to the wire with cement a glass fiber, twisting wire and glass together, and observing simultaneously the amount of twist of glass and wire. Then by using the glass fiber separately as a torsion pendulum with a suspended disc, the moment of the force for a given amount of twist of the glass was found, and so the twisting force on the wire. It was not feasible to use the wire directly as a torsion pendulum because of its extreme fragility. A large number of readings were made for each metal with increasing angles of twist up to beyond the point at which permanent set appeared, but not to the rupture point. For both antimony and tellurium, the relation between twist and moment of force is linear up to the appearance of set. The antimony (diameter 0.0357 cm.) showed a very slight set after a twist of 1° per cm. of length; the maximum twist given was 3.6° per cm. after which the set was 0.4° per cm. Tellurium (diameter 0.0349 cm.) showed very slight set after 1.3° per cm., and after the maximum twist of 6.7° per cm. a set of 0.2° per cm. The rigidity was calculated from the twist by the formula

$$\mu = \frac{2 \text{ Moment of Force}}{\pi r^4 \tau}$$

where τ is the twist in radians per unit length.

	C. G. S. $\frac{E}{\text{Units.}}$	C. G. S. $\frac{\mu}{\text{Units.}}$
Antimony.....	7.8×10^{11}	2.0×10^{11}
Tellurium.....	4.1×10^{11}	1.54×10^{11}

The results are shown in the table. The order of magnitude of these constants is the same as that for ordinary grades of glass, which have an E around 6×10^{11} and a μ around 2.5×10^{11} .

The reason is now apparent for the statement in the introduction that the results could give only a rough average value, and that any attempt to secure greater accuracy would be illusory. The relation between the constants for antimony is not one that is possible for an isotropic material, because if such a material is stable 3μ must be greater than E , and this is not true for antimony. If such a substance existed, it would expand under pressure. It has of course been known for a long time that many metals in the form of wire do not behave like isotropic materials. This example of antimony is perhaps an extreme instance of departure from isotropy, but is no more than one would expect for an exceptionally crystalline substance like this. The only use to which

the constant E as determined above can be legitimately put in calculating the bending of a wire under conditions similar to those of the test. It would be most desirable to determine E by a direct measurement of the extension under tensile load, but this would be a matter of great difficulty, because the extension of a piece 10 cm. long under the breaking load would be in the neighborhood of only 0.01 mm. This load would not be more than sufficient to remove any initial slight curvature of the wire.

The departure of tellurium from isotropy is not so extreme as that of antimony, for at least the constants satisfy the necessary conditions that the constants of an isotropic body must satisfy. If we assume complete isotropy, we will find the cubic compressibility to be 0.0000024 kg. per cm². This computation is probably not worth a great deal, but is perhaps worth recording because there seem to be no measurements whatever of the compressibility of tellurium. It is interesting to notice that this value fits easily into Richards's curve showing the compressibility of the elements as a periodic property.¹ Poisson's ratio, also computed on the assumption of complete isotropy, is 0.33, which is an entirely normal value.

In addition to the elastic constants, the behavior beyond the elastic limit was just touched. The breaking load in pure tension was 108 kg./cm². for antimony, and 115 kg./cm². for tellurium. Although exceptionally brittle, both antimony and tellurium will receive a slight set before breaking. A piece of antimony wire 8 cm. long and 0.0356 cm. diameter, fastened rigidly at one end and bent by a weight at the other, showed a maximum permanent set of 0.28 cm. and broke when deflected through 1.5 cm. The weight to produce rupture was 0.36 gm. A similar test on a piece of tellurium wire of 8 cm. length and 0.0348 cm. diameter showed a maximum set of 0.11 cm. and broke under a deflection of 1.82 cm. The brittleness is extreme for a metal, but is probably not as high as that of glass. The strength, however, is only about one fifth that of ordinary glass.

Summary.—The chief results are contained in the figures of the table on page 140. These were computed on the assumption of perfect isotropy. Antimony wire affords an extreme example, however, of non-isotropy, and possibly tellurium also is not isotropic. The constants must, therefore, be used with reserve in computing strains of other types than those involved in these tests.

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¹ T. W. Richards, Jour. Amer. Chem. Soc., 37, 1649, 1915.

THE ACTINIUM-URANIUM RATIO IN COLORADO
CARNOTITE.

BY KARL H. FUSSLER.

IT is generally accepted by investigators in radio-activity that actinium is a branch product from the uranium transformation series. This is based on the following facts: (1) Actinium is always found in uranium minerals; (2) Boltwood¹ separated the actinium from North Carolina uraninite and determined the ratio of its activity, with its products in radio-active equilibrium, to the activity of the uranium with which it was associated. The values of this ratio, from the four determinations which he made, were 0.14, 0.15, 0.24 and 0.36. He gives the value 0.28 as the weighted mean of these results. (3) He¹ also measured the total activity of various uranium minerals and determined what fraction of this total activity is due to each of the separate radio-elements contained in the minerals. His results indicate that the activity of a uranium mineral, containing equilibrium amounts of the various radio-elements, is about 4.7 times the activity of the uranium in the mineral. The sum of the activities of the separate radio-elements, uranium to polonium, plus $0.28 \times$ uranium for the actinium series, gives $4.64 \times$ uranium as the total activity of the mineral. If actinium is a branch product of the uranium series there should be a constant ratio between the activities of the actinium and of the uranium in all uranium minerals.

Since this ratio has been directly measured for only one uranium mineral and for only two specimens of that mineral² it was thought desirable to determine it using another uranium mineral. This paper describes the experiments carried out using Colorado carnotite.

Through the courtesy of the Cummings Chemical Company, manufacturers of radium, of Lansdowne, Pa., I was enabled to select my specimens from ton lots of mineral. Only large, firm lumps of the higher grade ores were chosen. The final selection was made at the laboratory from the bulk of the mineral brought from the factory, and, as a rule, only the centers of the larger lumps were chosen. The two

¹ Boltwood, Amer. Jour. Sci., XXV., p. 269, 1908.

² The specimens used are described by Boltwood as no. 3 and no. 4 uraninite from Spruce Pine, N. C. Two of the results were obtained from one sample of specimen no. 3, one result from another sample of the same specimen, and one from a sample of specimen no. 4.

specimens finally selected represented two different shipments of ore. Specimen A after pulverizing and passing through a 40-mesh screen consisted of about 300 grams of carnotite. Specimen B was a lower grade material. There were about 75 grams in this lot.

The radio-active measurements were made in an alpha-ray electroscope which will not be described as it differed in no essential detail from one described by Boltwood.¹ The gold-leaf was charged by connecting, through a water resistance, to the negative terminal of a 400-volt battery of "Spindler and Hoyer" cadmium cells. The guard ring was permanently connected to the negative terminal of the battery. The case of the electroscope and the positive terminal of the battery were connected to earth.

A microscope, with a graduated scale in the eye-piece, was rigidly clamped in front of the electroscope. The readings consisted in timing the passage of the tip of the gold-leaf over a certain definite portion of the graduated scale. The sensitiveness of the electroscope was determined for every series of measurements by taking a reading of the leak produced by a standard film of uranium oxide. This standard was made several years ago by Professor D. H. Kabakjian, from a very pure specimen of uranium oxide. It was deposited in the form of a thin film on a light brass disc. It has been carefully preserved and from time to time its activity has been compared with the activity of a primary uranium standard, which was similarly prepared, and which has been used only as a comparison specimen for the secondary standards.

The natural leak of the electroscope was determined for every series of measurements and the corrections made for it. It was fairly low and has not varied materially during the past one and one half years.

The general plan of the experiments was to separate chemically the ionium, radio-actinium, and actinium, as a group, with the rare earths as oxalates, from the other radio-elements in a specimen of carnotite, to determine the weight of the mixture of these elements and the non-radio-active elements separated with them and to make a very thin film, of known weight, on a metal plate, from this active material. The growth of activity of this film was measured at intervals over a period of about 100 days, at which time the equilibrium value had been attained. The activity due to ionium and actinium plus its products in equilibrium in one gram of carnotite was calculated from the equilibrium value of the activity of this film. A film, of known weight, was also made from the uranium separated from the same specimen of mineral. The activity due to the uranium alone in one gram of carnotite was calculated from

¹ *Loc. cit.*, p. 272.

the activity of this film. Boltwood¹ has shown that the activity of ionium in a uranium mineral is 34 per cent. of the activity of the uranium with which it is associated. This result was used in calculating the activity of the ionium per gram of mineral. Deducting the activity due to ionium, from the combined activity of ionium and the actinium series, gives the value of the activity due to actinium and its products in equilibrium in one gram of the mineral. This latter value divided by the activity due to the uranium in one gram of the mineral gives the value of the actinium-uranium activity ratio.

Because of the minute quantities of these radio-elements in a mineral the usual method used in separating them is to introduce into the solutions other elements, or carriers, which are either isotopic, or chemically similar to them. The best method of separating ionium from a mineral is to introduce thorium, if not already present, and then separate the thorium by the well-known chemical processes. Wishing to avoid the introduction of another radio-element, cerium was used as a carrier, the chemical properties of which are not very different from those of ionium. In the same way lanthanum was used as a carrier for the actinium.² As radio-actinium is isotopic with ionium,³ evidently it will be precipitated with the ionium. The details of the experiments are as follows:

Experiment 1.—Twenty-five grams of specimen A carnotite were decomposed by heating with dilute nitric acid and the solution evaporated to dryness. The residue was treated with hot dilute nitric acid. The insoluble matter was separated by decantation and again treated with the hot dilute acid. The solutions were combined and the insoluble matter, consisting largely of silica, after washing, was found to have negligible activity. The solution was saturated with hydrogen sulphide to separate the insoluble sulphides after the addition of a small quantity of bismuth nitrate to ensure complete precipitation of the polonium. The filtrate was boiled to remove the excess of hydrogen sulphide. Small amounts of cerium nitrate, and lanthanum nitrate, were added, and while the solution was still hot, an excess of oxalic acid was added and allowed to stand for about twenty hours. The precipitated oxalates were removed, a minute quantity of the cerium and lanthanum salts was again added, and the precipitation by oxalic acid repeated.⁴ The oxalates were combined, converted into nitrates, and again precipitated

¹ *Loc. cit.*, p. 289.

² Auer von Welsbach, *Sitzungber. K. Akad. Wiss. Wien*, 1910, 119, ii., a, 1.

³ Fleck, *Journ. Chem. Soc.*, 103', 381, 1913.

⁴ In work preliminary to these experiments it was found that a third oxalate precipitate had a negligible activity.

by oxalic acid. The oxalates were converted into oxides by intense ignition over the blast-lamp. The rare earth oxides obtained in this manner weighed 1.0230 grams. A film weighing 0.0082 gram was made. The activity of this film when it had reached equilibrium, was 5.202 divisions per minute. This gives $\frac{1.0230}{25 \times 0.0082} \times 5.202 = 26$ divisions per minute for the activity due to the ionium and the actinium, plus its products in equilibrium, in one gram of carnotite.

The filtrates, from which the oxalates were separated, were evaporated to dryness and gently heated to destroy the oxalic acid. The residue was taken up in dilute hydrochloric acid, a small amount of barium chloride was added, and the radium precipitated as sulphate with the barium. The addition and precipitation of the barium was twice repeated to ensure complete removal of the radium. The filtrate was boiled with an excess of sodium carbonate, containing some ammonium sulphide, and the carbonates filtered off. The uranium was precipitated and weighed as sodium uranate. It weighed 6.6042 grams and the film made from it weighed 0.0096 gram. The activity of the film was 1.990 divisions per minute. This gives an activity of 55 divisions per minute for the uranium in one gram of the mineral.

The activity of the ionium per one gram of the mineral, using Boltwood's value, should be (0.34×55) 18.6 divisions per minute. Deducting this value from the activity calculated from the ionium-actinium film (26 divisions per minute per one gram of mineral) gives 7.4 divisions per minute as the activity due to actinium and its products in equilibrium. This is 0.135 times the activity of the uranium.

Experiment 2.—Fifty grams of specimen B carnotite were taken for this experiment. The material was treated in the same manner as that of experiment 1. The activity of the ionium-actinium film 100 days after separation was 2.116 divisions per minute which gave 18.2 divisions per minute as the total activity of the ionium and actinium per one gram of mineral. The sodium uranate separated weighed 8.5234 grams, and the film weighed 0.0041 gram, which showed an activity of 0.849 division per minute. This gave an activity of 35.2 divisions per minute for the uranium in one gram of carnotite. The activity of the ionium, calculated as before, is (0.34×35.3) 12.0 divisions per minute. Subtracting this from the combined ionium-actinium activity gives 6.2 divisions per minute as the activity of the actinium series. This divided by the activity of the uranium (35.3 divisions per minute) gives 0.176 as the value found for the ratio of the activity of the actinium and its products to that of the uranium with which it is associated.

Experiment 3.—Twenty-five grams of specimen A carnotite were treated in the same manner as in experiment 1. The activity of the uranium per gram of carnotite was 51.5 divisions per minute. The combined activity of the ionium and the actinium series per gram of carnotite was 23 divisions per minute. The calculated activity due to the ionium (0.34×51.5) is 17.5 divisions per minute which leaves 5.5 divisions per minute as the activity due to actinium and its products in equilibrium. This divided by the activity of the uranium gives 0.107 as the actinium-uranium ratio.

The values for the relative activity of the actinium series obtained in these experiments were, therefore, 0.135, 0.176, and $0.107 \times \text{Ur}$.

A theoretical curve was plotted on the assumption that an equilibrium amount of radio-actinium was precipitated with the actinium. This curve was obtained by adding the ordinates of curves I and II, Fig. 1.

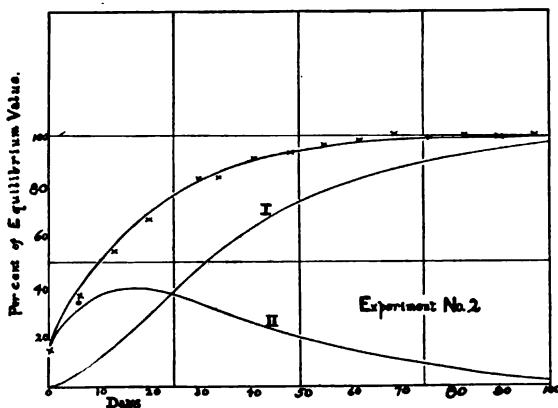


Fig. 1.

Curve I is the recovery curve of actinium freed from all its active products.¹ The ordinates represent the activity and the abscissae the time in days from separation. The equilibrium value of the activity is taken as 100. McCoy and Leman² have shown that the equilibrium activity of radio-actinium is 17.6 per cent. of the total activity of the actinium series. This value is used as the initial value of the radio-actinium curve (Curve II). The crosses give the experimental values of the growth with time of the activity of the film of experiment no. 2. The curves for experiments no. 1 and no. 3 were similar in shape, but with initial activities somewhat lower. (13.9 per cent. for no. 1 and 11.7 per cent. for no. 3.) The agreement is well within the limits of experimental error. This, I believe, indicates that the film did not contain

¹ Hahn, Phil. Mag., XIII., 165, 1907.

² PHYS. REV., 4 Ser. 2, 1914, p. 409.

other radio-active elements in quantities sufficient to appreciably alter the shape of the actinium plus radio-actinium curve. The disagreement between the experimental and the theoretical curves may be accounted for by an incomplete precipitation of the ionium in the specimens. A computation shows that if 1.5 per cent. (for film of experiment no. 2) of the ionium remained in the solution the starting point of the two curves would agree. This would make the actinium ratio larger by about 2.8 per cent.

CONCLUSIONS.

1. The ratio of the activity of the actinium series to the activity of the uranium associated with it in Colorado carnotite has been determined.

2. The results indicate that the quantity of actinium in carnotite is proportional to the quantity of uranium.

3. The value of the ratio is of the same order of magnitude, but somewhat lower than that obtained by Boltwood using North Carolina uraninite.

4. The results are in agreement with the theory that actinium is a branch product of the uranium series.

In conclusion I wish to acknowledge my indebtedness to Professor Arthur W. Goodspeed for kindly placing at my disposal all the facilities of the Randal Morgan Laboratory, and to Professor D. H. Kabakjian for suggesting the subject of this research and for his continued co-operation and valuable criticisms during the course of the investigation.

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THE OPTICAL PROPERTIES OF AN ISOLATED CRYSTAL OF SELENIUM.

BY CHARLES H. SKINNER.

THE optical properties of various forms of selenium have been investigated by Meier,¹ Foersterling and Fréedericksz,² Gripenberg,³ Pfund,⁴ Wood⁵ and others, but no results have yet been published on the optical properties of an isolated crystal of selenium. The present paper is to extend the investigations to this particular form.

The polarimetric, or Babinet compensator, method was used. The investigation was confined to a prism face of a crystal of the hexagonal prism type.⁶ The reflecting surface was about 2 mm. by 5 mm., and required no polishing. A frequent cleansing in alcohol helped prevent the formation of surface films. Owing to the purity of such crystals, the surface may be taken as very probably representing the state of matter within the crystal.

The theory of metallic reflection for crystals of this type has been fully developed by Drude.⁷

Let $\bar{\phi}_1, \bar{\psi}_1$, be the angles of principal incidence and principal azimuth, respectively, when the crystal is in the first principal position (*i. e.*, when the principal axis of the crystal is parallel to the plane of incidence);

$\bar{\phi}_2, \bar{\psi}_2$, be the angles of principal incidence and principal azimuth, respectively, when the crystal is in the second principal position (*i. e.*, when the principal axis of the crystal is perpendicular to the plane of incidence);

$n_1, n_2, k_1, k_2, R_1, R_2$, be the indices of refraction, absorption coefficients, and coefficients of reflection, respectively, for the two principal positions:

¹ Meier, Ann. d. Phys., 31, p. 1029, 1910.

² Foersterling and Fréedericksz, Ann. d. Phys., 43, p. 1227, 1914.

³ Gripenberg, Phys. Zeitschr., 14, p. 123, 1913.

⁴ Pfund, Phys. Zeitschr., 10, p. 340, 1909.

⁵ Wood, Phil. Mag., S. 6, Vol. 3, pp. 612-613, 1912.

⁶ These crystals were obtained through the courtesy of Dr. F. C. Brown, of the physical laboratory of the State University of Iowa.

⁷ P. Drude, Ann. d. Phys., 32, p. 613, 1887; 34, p. 515, 1888; 36, p. 542, 1889.

then,

$$\frac{\sqrt{\alpha}}{\cos \phi_1} - \sqrt{\beta} \cos \bar{\phi}_1 = \cos 2\bar{\psi}_1 + i \sin 2\bar{\psi}_1;$$

$$\frac{\sqrt{\beta}}{\cos \bar{\phi}_2} - \sqrt{\alpha} \cos \bar{\phi}_2 = \cos 2\bar{\psi}_2 + i \sin 2\bar{\psi}_2;$$

$$\alpha = a_{11} + ia_{12}; \quad \beta = a_{21} + ia_{22};$$

$$\frac{a_{12}}{a_{11}} = \tan x; \quad \frac{a_{22}}{a_{21}} = \tan \epsilon;$$

$$k_1 = \tan \frac{x}{2}; \quad k_2 = \tan \frac{\epsilon}{2};$$

$$n_1^2 = \frac{2 \sin \frac{x}{2} \cos^3 \frac{x}{2}}{a_{12}}; \quad n_2^2 = \frac{2 \sin \frac{\epsilon}{2} \cos^3 \frac{\epsilon}{2}}{a_{22}};$$

$$R_1 = \frac{n_1^2(1 + k_1^2) + 1 - 2n_1}{n_1^2(1 + k_1^2) + 1 + 2n_1};$$

$$R_2 = \frac{n_2^2(1 + k_2^2) + 1 - 2n_2}{n_2^2(1 + k_2^2) + 1 + 2n_2}.$$

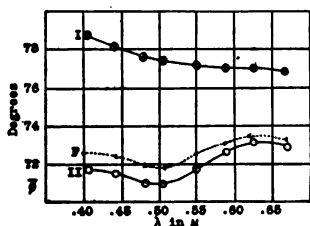


Fig. 1.

Principal Incidence.

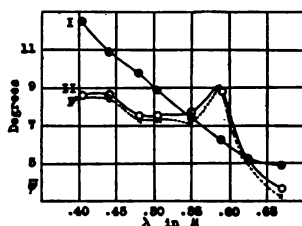


Fig. 2.

Principal Azimuth.

In the following table, the values of $\bar{\phi}$ and $\bar{\psi}$ are the means of several observations. The results are shown graphically in Figs. 1-5, where

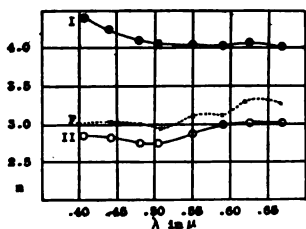


Fig. 3.

Index of Refraction.

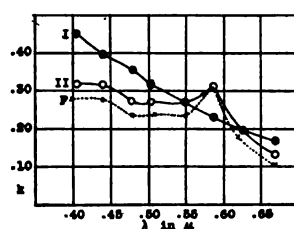


Fig. 4.

Absorption Coefficient.

$\bar{\phi}$, $\bar{\psi}$, and the calculated constants, are plotted with the wave-length. Curves marked I and II refer to the crystal in the first and second principal position, respectively.

It was found that when the crystal was placed in the second principal position, the results obtained agreed well with the best published results for crystalline selenium in mass. The curves marked *F* are from the results of Foersterling and Fréedericksz, who used mirrors obtained by spreading melted selenium over a well cleaned glass plate, and heating in a sand-bath at 180° C. Pfund determined *R* by a direct method, using mirrors obtained in the above manner. His results are shown in curve *P*, Fig. 5.

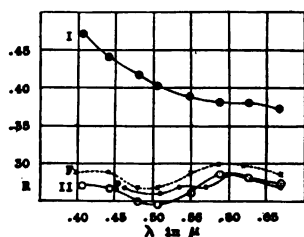


Fig. 5.

Reflecting Power.

With the crystal in the first principal position, quite different results were obtained, there being no similarity whatever to the results for the other position. It is interesting to note that the index of refraction as determined for this position is more than 15 per cent. greater than that found by any observer for other forms of selenium. There is no other known element with so high a refractive index.

The results show that isolated hexagonal crystals of selenium are doubly refracting. This points to the possibility that all such crystalline substances that reflect metallicly have in general three sets of optical constants (two in any one plane); and that when one determines the

λ in μ .	First Principal Position.					Second Principal Position.				
	$\bar{\phi}_1$.	$\bar{\psi}_1$.	n_1 .	k_1 .	R_1 .	$\bar{\phi}_2$.	$\bar{\psi}_2$.	n_2 .	k_2 .	R_2 .
.406	78° 47'	12° 32'	4.40	.458	.472	71° 45'	8° 40'	2.86	.320	.273
.441	78 13	10 54	4.26	.395	.442	71 37	8 42	2.83	.319	.269
.480	77 34	9 51	4.12	.357	.419	71 07	7 35	2.77	.277	.252
.505	77 23	8 53	4.05	.317	.404	71 01	7 38	2.75	.276	.249
.550	77 11	7 31	4.05	.269	.393	71 46	7 42	2.87	.275	.264
.589	76 59	6 18	4.04	.229	.384	72 40	8 54	3.00	.315	.288
.626	77 01	5 20	4.09	.188	.383	73 13	5 16	3.17	.184	.285
.668	76 47	4 58	4.02	.172	.374	72 57	3 42	3.15	.132	.276

optical constants for any crystalline substance, used in mass, only a certain mean set of values is obtained.

In conclusion, I wish to acknowledge indebtedness to the staff of the physical laboratory of the State University of Iowa for the use of apparatus, and especially to Professor L. P. Sieg for suggesting the problem.

PHYSICAL LABORATORY,
STATE UNIVERSITY OF IOWA.

MEASUREMENTS IN FRICTIONAL ELECTRICITY.

BY N. R. FRENCH.

HISTORICAL.

SINCE all the recent articles bearing upon this subject have appeared in other magazines, a brief outline of the results of the recent researches is given here.

Morris Owen¹ took measurements of the charges produced upon various bodies by known amounts of energy expended in friction and found that with definite amounts of frictional work the charges reached certain fixed maxima, depending upon the kind and size of specimen, but that these same maxima could be reached with a less expenditure of energy by increasing the pressure between the rubbing surfaces. The final maximum charges, however, were independent of the pressure. Owen concluded that Helmholtz's theory that frictional electricity is in the nature of contact electricity, the frictional work being done to bring the surfaces into closer contact, is borne out by his observations, since after several consecutive rubs the maximum charge could be produced by a much smaller amount of work than that required the first time. After several hours' rest the material would return to its original condition in which a much larger amount of energy was required to produce the maximum charge. His theory of this phenomenon was that the surface of the material underwent a slow elastic recovery of its uneven form or that it resulted from the tarnishing of the surface by the atmosphere, thus causing a poor contact during the first rubs. No evidence, however, of the slightest charge was produced by mere contact of the surfaces.

In a recent paper, W. Morris Jones² published the results of his work in this field. He found that by increasing the softness of the rubbers the rest between the rubs could be made shorter and still have the specimen return to its original condition. His results give the interesting facts that transparent insulators are positively charged when rubbed with flannel, silk and chamois leather, while negative charges are produced on opaque insulators by rubbing with these same three materials.

¹ Phil. Mag., April, 1909, p. 457.

² Phil. Mag., Feb., 1915, p. 261.

He found that in the case of metals, using silk as the rubber, the sign and amount of the charge is not as definite a characteristic of the material as in the case of non-conductors. First, reversal of sign often took place, especially in the case of zinc and iron. The following extract is taken from his paper: "When the metals were all cleaned with very fine emery paper, all the specimens except thallium, lead, and bismuth (which have the highest atomic weights) gave a negative charge at the first rub. Continued rubbing, however, produced a polish on the metals, and then it was found that the negative charges on aluminum, iron, copper, zinc, and antimony became less and less and changed to positive, though the other metals, on continued rubbing, did not show this effect." Later in this paper this statement will receive further treatment. Secondly, low readings were obtained when the surfaces were tarnished with oxide, positive charges being obtained for small amounts of work and negative for large amounts.

Plotting atomic weights against the maximum charges, all the metals being rubbed with the same substance, he obtained a curve which exhibits some evidence of periodicity. No effect upon the production of the charge was obtained by placing the specimen in magnetic and electric fields or by changing the temperature.

APPARATUS.

The apparatus for the production of the charge, a sketch of which is shown in Fig. 1, was practically the same as that employed by Owen.

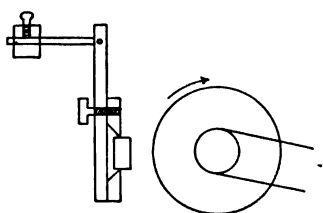


Fig. 1.

A slate wheel, 61 cm. in diameter and 5.1 cm. in thickness, around the rim of which bands of the rubbing material were fastened, was driven by an electric motor through a system of pulleys which made possible several speeds.

The vertical part of the specimen-holder, which possessed an adjustable ebonite vise-like arrangement at the lower end for holding the specimens, consisted of a strip of ebonite 30 cm. long, 5 cm. wide and approximately 0.7 cm. thick; the horizontal part was a steel rod passing through a movable weight. This holder was free to rotate about a horizontal axle which could be moved vertically or in a horizontal plane. The pressure between the specimen and the band on the wheel rim was regulated by means of the movable weight.

After a specimen had been rubbed, the holder was rotated in a horizontal plane till the specimen was above a deep metal can into which

it was lowered, a set-screw having previously been tightened to prevent rotation about the axle. This can was contained within a larger one from which it was insulated by shellaced ebonite posts. This outside can was grounded, the inside can being connected to one pair of quadrants of a quadrant electrometer, the other pair of which was also grounded. On account of the small capacity of this system, the inside can was also connected to one side of a condenser of known capacity, the other side being connected to the ground. It was found that the capacity of the condenser could be taken as the capacity of the system with an error of approximately one part in five hundred. The measuring apparatus was kept within a grounded tinfoil covered box which protected it from outside disturbing influences.

The electrometer needle, the deflections of which were read with a telescope and scale, was kept at a constant potential by connection with one side of a set of storage batteries the other side of which was earthed. The electrometer was frequently calibrated by using known potential differences and noting the deflections, which in most cases were proportional to the applied potential differences. After each set of readings throughout all the work tests were made to make certain that the calibrations still applied. By noting the deflection caused by an unknown charge and then referring to the calibration curves, the value of the charge could be easily computed since the capacity of the system was known.

To improve insulation the surface of the slate wheel was coated with shellac and a band of silk placed next to the rim beneath the bands of wool which were used as rubbers. The insulation of this and the specimen-holder was frequently tested.

The specimens consisted of several metals, all 2.5 cm. square and 0.9 cm. in thickness. Great care is needed in preparing the specimen for rubbing because the condition of the surface determines to a large extent the size and in some cases the sign of the charge. It was found, after trying several methods of preparing the surface, that going over it with a fine file just before each set of observations gave the best results and insured all the metals being in the same condition. It is important to always get the metal in the same position with reference to the rubber; if it is not in good even contact a low charge usually results. It was found that a speed of seventeen r.p.m. of the wheel gave the best results, all the work being carried on at this speed. Since the work done was proportional to the time of rubbing, only the time of rubbing was recorded.

THE RELATIONSHIP BETWEEN THE CHARGE AND WORK AND THE DETERMINATION OF THE MAXIMUM CHARGES FOR SEVERAL METALS AND THEIR ALLOYS.

The following metals were used: aluminum, zinc, an alloy of half aluminum and half zinc, lead, tin, and an alloy of half lead and half tin. In determining the relationship between the charge and the work done, the rubs were taken immediately following each other, *i. e.*, as soon as

TABLE I.

Specimen.	Maximum Charge in E. S. U.
Lead.....	65.4
Tin.....	23.8
Lead and tin alloy.....	23.0
Aluminum.....	46.4
Zinc.....	57.1
Aluminum and zinc alloy.....	45.6

the measurement of the charge obtained from the first rub was completed, the specimen was discharged as well as the rubber, and at once rubbed again, this time for a longer period. This process of rubbing, measuring the charge, discharging, and then rubbing again for a longer period, was continued till the charge showed no increase. After the maximum charge was obtained, a second set of data, commencing again with a short rub, was immediately taken on the same specimen. In constructing the curves showing this relationship, charges were plotted in terms of the electrometer deflections. The temperature and humidity of the air were kept nearly constant for this part of the work.

DISCUSSION OF RESULTS.

These plots showing the relationship of the charge to the work bring out a point mentioned in the paper of Morris Jones and show that his explanation of it (quoted on page 151) is apparently incorrect. The following is taken directly from his paper: "When the metals were all cleaned with fine emery paper, all the specimens except thallium, lead, and bismuth (which have the highest atomic weights) gave a negative charge on the first rub. Continued rubbing, however, produced a polish on the metals, and then it was found that the negative charges on aluminum, iron, copper, zinc and antimony became less and less and changed to positive, though the other metals on continued rubbing did not show this effect."

The following plots show this effect: Curves 1 and 2, Fig. 2, Curves 1A, 1B, 2A, and 3A, Fig. 3. The metals showing this effect are an alloy of lead and tin (although neither lead nor tin alone showed it), aluminum, zinc and an alloy of aluminum and zinc.

On examination of the above extract from the paper of Mr. Jones, one would get the impression that Jones believed the production of the polish to be the cause of the change in sign of the charge after the metal had been rubbed for some time. My results seem to disprove this theory. Curve 2, Fig. 2, is constructed from data taken immediately after the taking of data for Curve 1. Curve 1 shows the change from negative to positive after continued rubbing, the last point in 1 being obtained for a rub of one hour. According to Jones the surface of this metal was polished after being rubbed only 400 seconds, this being the point where reversal takes place. At all times after this, rubbing ought to result in the production of positive charges, which, however, does not

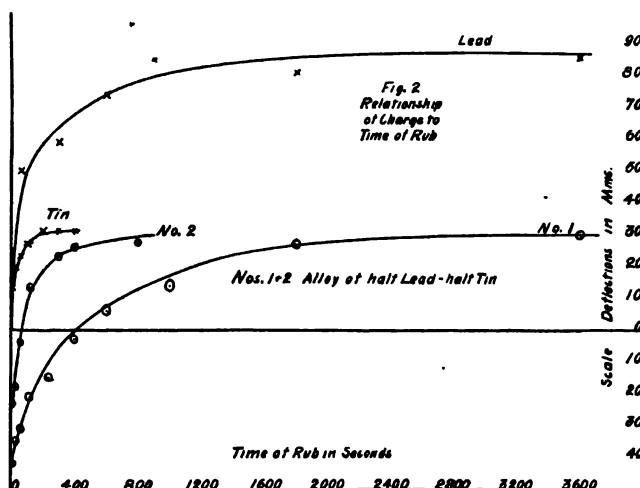


Fig. 2.

take place. Starting once more with short rubs, negative charges are again produced, although as long a time is not now needed to cause the reversal of sign. Nothing, whatsoever, has been done to the specimen between the two series of rubs, the second of which followed immediately after the first. Curve 2B, Fig. 3, also exhibits this same phenomenon, the "B" Curves being plotted from the data taken immediately after that for the "A" Curves.

It will be noticed from the above plots that the negative charges obtained from the second set of readings are in each case less than those obtained from the first set by the same amount of rubbing.

Rapid oxidation of the specimen between rubs would not account for this reversal as it is found, because the specimens, when oxidized, give positive charges at first and then reverse to negative after enough rubbing to wear away this oxide.¹ This condition is not shown on the plots

¹ Jones, *loc. cit.*

because the surface of each specimen was cleaned with a fine file before each set of observations for the purpose of removing this oxide. Jones stated in his paper that temperature changes had no effect upon the production of the charge so it is probable that this condition was not brought about by an increase in the temperature due to friction.

It may be of interest to note in Table I. that of the alloys used, neither one gave a maximum charge larger than the smaller maximum charge of its two constituents.

An attempt to explain the above results from an entirely new point of view follows:

It is known that, under ordinary circumstances, the surfaces of bodies

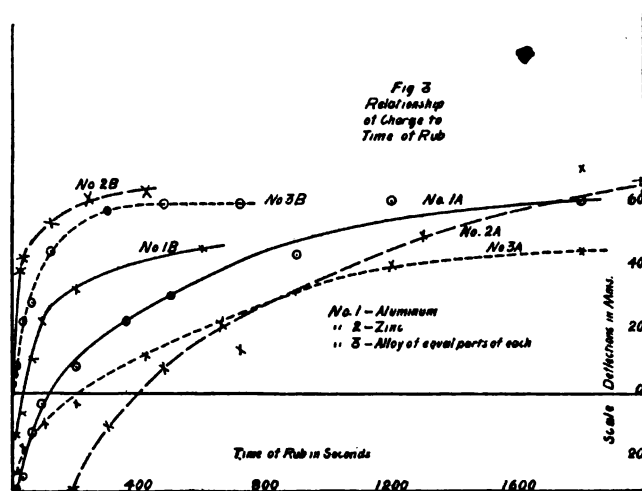


Fig. 3.

are covered with layers of occluded air. This can be shown by what are called "breath figures." Let a glass plate remain in the air for several hours after being cleaned, and then, without scratching the surface, mark upon it with a hard point. If the surface is now breathed upon, the mark is rendered visible. The explanation given for this is as follows: the glass, after being cleaned, has adsorbed air to its surface; the marking removes some of this occluded air, and the breathing distributes vapor unequally over the surface, more being absorbed and condensed where there is still occluded air than in those places where it has been partially or wholly removed. This air is not removed except by hard rubbing or scratching, as can be shown by rubbing this surface after the mark has been made upon it. Breathing upon it again, after this rubbing, will still render this mark visible, showing that the air on the other parts of the surface has not been removed, since it is the con-

trast between the condensation of vapor on the surface where there is and where there is not condensed air, that renders the mark visible.

Ordinary air contains moisture which would be deposited along with the air upon the surface of the solid, so that all substances probably have layers of air upon their surfaces, this air condensing with it the water vapor which it contains.

Supposing, then, that the surfaces of the metals have these layers of condensed air and water vapor, let the rubber be brought in contact with the surface of one of the specimens and the rubbing commenced. Charges then begin to be developed, not only between the metal and the flannel, but also between this surface layer and the flannel. To explain our results we must now make two assumptions concerning the production of the charge from this layer: (1) The charge which the film yields is negative in sign; (2) the maximum charge given by the film alone is smaller than that given by any of the metals used, meaning by "the charge of the metal," the charge that would be obtained if the film gave no charge.

The following equation, developed by Jones, gives the charge in terms of the work and two constants which have values depending upon the substances used:

$$Q = \frac{a}{b} (I - e^{-bw}), \quad (1)$$

where Q equals charge, w the energy expended in frictional work, a and b being constants which come into the expressions for the rate of liberation of electrons from one surface, and the leakage back to the surface, respectively.

Let Q_1 and Q_2 be the charges produced on the surface layer and metal, respectively; a and b constants for the action between the surface layer and woolen band; c and d constants for the action between the metal and wool; w_1 and w_2 the respective amounts of work done in producing Q_1 and Q_2 , where the sum of w_1 and w_2 equals the total amount of energy used in the production of any charge. Then

$$Q_1 = \frac{a}{b} (I - e^{-bw_1})$$

and

$$Q_2 = \frac{c}{d} (I - e^{-dw_2}).$$

The resultant charge is then the difference between Q_1 and Q_2 since, according to Assumption 1, these charges are opposite in sign those

metals which show reversal yielding a positive charge finally, or

$$Q = Q_2 - Q_1 = \frac{c}{d}(I - e^{-dw_2}) - \frac{a}{b}(I - e^{-bw_1}). \quad (2)$$

Although, for any rub, w_2 is probably greater than w_1 , it is possible to assign values sufficiently large to b and small to d , if c/d is greater than a/b (Assumption 2), such that $Q_2 - Q_1$ will be negative for short rubs but positive for those of longer duration.

Or d and b may have such values that for a given ratio of w_1 and w_2 , $Q_2 - Q_1$ can only have positive values.

The above mathematical reasoning shows how it is possible for reversal of sign to take place on the basis of the adsorption theory. It is evident that if reversal took place for the first set of readings it would also for the second, other conditions being the same. But what will actually happen is complicated by the fact that probably some of the surface film is worn away by the rubbing which, it is evident, would result in a smaller negative charge or even a positive being obtained for the first short rub of the second series of observations.

THE RELATIONSHIP OF THE CHARGE TO THE HUMIDITY.

It was soon found that a much smaller charge was obtained during damp weather than during dry. It was thought, at first, that this was due to a leak from the apparatus, but several tests were tried, when the absolute humidity was high, which seem to disprove the leak explanation. A charged electroscope was slowly moved over the rim of the wheel and the rapidity of the collapse of the gold leaves noted. No appreciable amount was noticed, however, and the electroscope could be left in stationary contact with the rim of the wheel for an hour with no greater leak than would take place from the electroscope alone. The condenser and electrometer would remain charged for several hours in the dampest of weather with only a small loss. After a reading had been taken, removing the holder from the jar and again taking the measurement after an interval was tried. The loss produced in this manner varied but very little in damp and dry weather.

The metals used in this part of the work were aluminum, lead, and tin. The length of the rub was different for each metal, 300 seconds, yielding a maximum charge, being the period for tin, 30 seconds for lead, and 10 for aluminum. To do away with numerical calculation, the values of the charges were not worked out. Instead, the potential of the charge was measured and plotted against the absolute humidity, the shape of the curve being unchanged by this substitution. The plots obtained,

Fig. 4, seem to show that there is some relationship between the charge obtained from specimens of metals and the absolute humidity of the atmosphere.

DISCUSSION OF RESULTS.

Using equation 2 we get for the resultant charge in any case,

$$Q = \frac{c}{d}(I - e^{-d w_2}) - \frac{a}{b}(I - e^{-b w_1}),$$

a/b and c/d being the maximum values of the charges obtained from the surface film and metal, respectively. Since, as found by Owen, the charge is directly proportional to the surface area of the specimen, an increase in the absolute humidity will result in an increase in surface area of the surface film, more moisture being deposited, and a corresponding increase in a/b and decrease in c/d . Any positive value of Q will then be decreased by an increase in absolute humidity which is in agreement

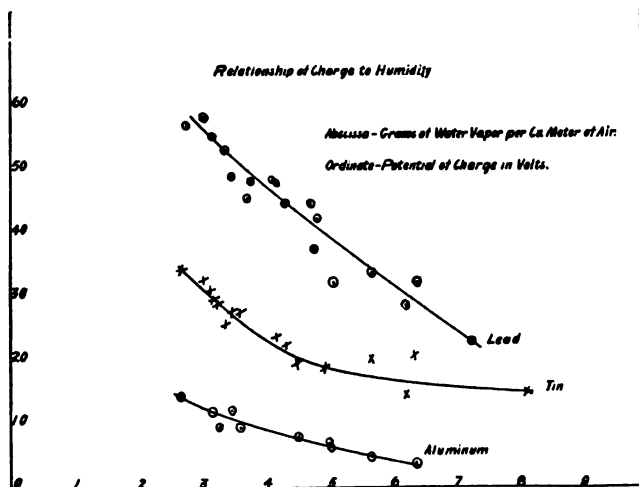


Fig. 4.

with plots for lead and tin in Fig. 4. The data for aluminum was taken before reversal of sign took place—the charges all being negative. According, then, to equation 4 it seems as if, for this particular case, the charges should be increased by an increase in absolute humidity, which is not in accordance with the actuality as shown by the curve for aluminum on Fig. 4. The results are probably complicated, however, by the existence of a changing surface condition upon the flannel rubber, increased moisture being deposited there by the increased humidity, but probably being worn away much more quickly than on the metal. It is to be noticed, however, that the rate of change of charge with respect to

absolute humidity for the aluminum is not as great as for the tin and lead, the average rate between the absolute humidities of three and six grams per cu. meter of air being 8.3 volts fall per increase of 1 gram of water vapor per 1 m³. of air. for lead, 5.0 for tin, and 2.8 for aluminum.

It is barely possible, of course, that this change in the charge was caused by a leak which the writer was not able to detect, although very careful tests were repeatedly made. It seems probable, however, that there should be some change in the charge with a change in the absolute humidity.

TEST OF OWEN'S THEORIES AS TO WHAT TAKES PLACE ON THE SURFACE OF A SPECIMEN BETWEEN RUBS SEVERAL HOURS APART.

After a specimen had been rubbed for a considerable length of time and then discharged, any definite charge can then be obtained with a smaller expenditure of energy than would be required at the first rub. If the specimen be given a rest of several hours, it will then be found that it has returned to its original condition in which a much larger amount of energy is required to produce the same charge. Owen attributed this to either an elastic recovery of the surface of the specimen or to its being tarnished by the atmosphere. The following experiments were tried to test his theories and to determine, if possible, the cause of this phenomenon.

First, the effect of keeping the specimen in a vacuum after being rubbed, was tried. The specimen was first rubbed for quite a long period, the measurement of the time and charge then being taken for several short rubs, after which it was immediately discharged and placed under the bell jar of an air pump and the air removed. After several hours the charge was again obtained.

The effects of keeping the specimen in an atmosphere of water vapor and then in dry air were also tried. One difficulty which prevented more accurate study of these effects lay in the fact that it was necessary to bring the specimen back into the air while the development and measurement of the charge were being taken. Since the results were only qualitative they are left as deflections of the electrometer rather than as charges.

TABLE II.

Specimen Used: Lead.

Deflections Millimeters.	Remarks.
	All rubs thirty seconds in length.
142	Specimen previously cleaned with fine file.
169	After four hours in a vacuum.
170	After three hours in water vapor.
138	After three hours in moist air.

DISCUSSION OF RESULTS.

The data seem to show that the decrease in the charge produced by a definite amount of work on a specimen, which has been allowed to remain in contact with the air after previous rubbing, is caused by an action of the atmosphere upon the surface of the metal and not by an elastic recovery of the surface. In every case tried, and several sets of obser-

TABLE III.

Specimen Used: Aluminum.

Surface cleaned with fine file before first rub of this set.				
No. of observation.....	1	2	3	4
Length of rub—secs.....	30	60	300	600
Deflections—mm.....	3	— 27	— 34	— 52
Specimen put in vacuum after last rub above—removed 12 hrs. later.				
No. of observation.....		1		2
Length of rub—secs.....		30		300
Deflections—mm.....		— 54		— 56
Specimen left in moist air after last rub above—out 8 hrs. later.				
No. of observation.....	1	2	3	4
Length of rub—secs.....	30	60	300	600
Deflections—mm.....	35	19	— 2	— 10

vations were taken which are not recorded here, the charges obtained by equal amounts of work after the specimen had been given a rest of from three to twelve hours in a vacuum, were as large as those obtained just before the specimen was put away. This seems to show definitely

TABLE IV.

Specimen Used: Aluminum.

Surface cleaned with fine file before first rub of this set.				
No. of observation.....	1	2	3	4
Length of rub—secs.....	300	600	7:0	
Deflections—mm.....	— 22	— 25	— 33	
Specimen put in water vapor after last rub above—out 7 hrs. later.				
No. of observation.....	1	2	3	4
Length of rub—secs.....	30	60	30	30
Deflections—mm.....	— 42	— 40	— 30	— 32
Kept in dry air for ten hours after last rub of above set.				
No. of observation.....			1	2
Length of rub—secs.....			30	60
Deflections—mm.....			9	6

that the phenomenon is not caused by an elastic recovery of the surface but by contact with something in the atmosphere.

As shown by Tables II. and III., the charges obtained were larger after the specimen had been given a rest in a vacuum. This seems to be due to the fact that occluded gases and water vapor are given off by the metal when the atmospheric pressure upon it is decreased.

Keeping the specimen in an atmosphere of water vapor only, does not cause a decrease in the charge; in one case, as shown by Table IV., it was increased by this procedure; in another, Table II., it was not changed appreciably.

As can be seen from the last part of Table IV., the charge was decreased and reversed in sign when the specimen was kept in dry air, the change in deflection being 40 mm. Sulphuric acid was used as a drying agent so the air was probably not totally free from moisture. The moist air, however, had over double this effect, the change in deflection being 87 mm., and the time in moist air being eight hours as compared with the ten hours which it remained in the dry air (see Table III.).

These facts seem to indicate that it is a mixture of air and water vapor deposited upon the surface of the specimen which causes this decrease in the charge, for a definite amount of work, after the material has been allowed to rest for some time between rubs. Possibly the formation of oxide upon the surface plays an important part. The dry air alone seemed to produce nowhere near the whole effect and the water vapor practically none at all.

SUMMARY OF RESULTS AND CONCLUSIONS.

1. Jones's theory as to the cause of the reversal in sign of the charge produced on a metal specimen has been proven incorrect.

2. For each of the two alloys examined the maximum charge was not the average of the maximum charges of the constituent metals, being, in fact, no larger than the maximum charge from the constituent giving the smaller charge.

3. A relationship between the charge and the absolute humidity, for a definite amount of frictional work, has apparently been found to exist.

4. The decrease in the charge produced by a definite amount of work on a specimen which has been allowed to remain in contact with the atmosphere after previous rubbing, has been shown to be caused by contact with a mixture of air and water vapor and not by an elastic recovery.

5. To explain the facts, a theory, based upon the adsorption of air to the surface of the metal, has been proposed.

Jones stated that magnetic fields caused no effect upon the production of the charge, but did not mention the kind of metals used for the tests. The writer made a few preliminary observations with paramagnetic substances in a magnetic field and also tried the effect upon the production of the charge of the magnetization of the specimen. Both these tests seemed to influence the results, but nothing definite enough to

warrant the drawing of any conclusions, was obtained. The intention is to investigate this question further at some future date using stronger fields.

In conclusion, the writer wishes to express his thanks to Professors J. S. Stevens, L. E. Woodman and R. M. Holmes, of the University of Maine, for valuable advice during the course of the work, and also to Professor K. T. Compton, of Princeton University, for aid in the arrangement of the material for publication.

THE UNIVERSITY OF MAINE PHYSICAL LABORATORY,
ORONO, MAINE,
June, 1916.

THE MASS OF THE ELECTRIC CARRIER IN COPPER,
SILVER AND ALUMINIUM.

BY RICHARD C. TOLMAN AND T. DALE STEWART.

IN a previous article¹ we have described some experiments in which a coil of copper wire was rotated about its axis at a high speed and then suddenly brought to rest, the ends of the coil being connected with a sensitive ballistic galvanometer which permitted a measurement of the pulse of current which was produced at the instant of stopping by the tendency of the electrons to continue in motion.

We have continued these experiments making use of three new windings of copper wire, and using two different windings each of silver and aluminium wire. These further experiments were made not only because it seems desirable to subject so new a phenomenon to a more rigid test, but because it is also desirable to see if the mass of the carrier of electricity is the same in all different metals and how much it differs, if at all, from the mass of the electron in free space.

The only change that we have made in our apparatus as described in the earlier article was to provide a small bowl-shaped metallic cover to protect the binding posts on the rotating wheel from air friction. This was particularly important when our coil was of aluminium or silver in helping to reduce accidental thermoelectric forces which arose at the junction of the coil with the copper wires which led to the galvanometer.

A summary of our experimental results is given in Tables I., II. and III. The individual runs were made in the way described in our previous article and the deviations of individual results from the mean were of the same order of magnitude as those given in that place.

The first column in Tables I., II. and III. states the number of the coil and this corresponds to the description of the coils given in Table IV. The following columns in the table give the number of individual runs,—the *total* resistance in the circuit, R ,—the length of the rotating coil, l ,—the rim speed of the coil, v ,—the pulse of electricity, Q ,—which passed through the galvanometer at the instant of stopping, and the mass of the carrier, M , as calculated with the help of the following equation:

$$Q = \frac{Mvl}{FR}. \quad (1)$$

¹ PHYS. REV., 8, 97, 1916.

the *effective* mass associated with one equivalent (*i. e.*, $F = 96,540$ coulombs) of electricity.

It will be seen that we have here a record of the results of 624 individual runs made on a number of different coils, using three kinds of

TABLE III.

Silver Wire.

Coil No.	No. of Runs.	Resistance in Ohms.	Length of Wire in Cm.	Velocity in Cm. Per Sec.	Q Coulombs $\times 10^9$.	$M \times 10^4$ ($O=16$.)	$1/M$ ($O=16$.)
I.	11	42	51,200	- 3300	2.76	6.62	1510
	13	42	51,200	+ 3420	2.99	6.94	1440
	15	42	51,200	- 4550	3.86	6.71	1490
	20	42	51,200	+ 4550	3.49	6.06	1650
	11	42	51,200	+ 5460	4.70	6.80	1470
						Av. 6.62	Av. 1510
	14	63	51,200	+ 4670	2.47	6.29	1590
	11	63	51,200	+ 5410	3.00	6.58	1520
						Av. 6.43	Av. 1550
II.	34	31	29,930	+ 4390	3.37	7.69	1300
	33	31	29,930	- 4335	2.08	4.78	2090
						Av. 6.23	Av. 1600
Average value for silver						6.50	1540

wire, two different sizes, and two different kinds of insulating binder to hold the coils in place. The runs were made with various total resistances in the circuit, with various lengths of wire, and at various velocities,

TABLE IV.

Description of Coils.

All coils wound on wooden wheel described in previous article (*loc. cit.*).

Copper Coil No. I.—46,550 cm. no. 20 B. & S., double silk-covered magnet wire. Not wound as carefully as later coils, alternate layers bound with hot paraffin.

Copper Coil No. II.—30,370 cm. no. 20 B. & S., double silk-covered magnet wire. Made from previous coil by removing outer third of wire.

Copper Coil No. III.—35,400 cm. no. 18 B. & S., double silk-covered magnet wire. Each layer bound with an alcoholic solution of shellac and allowed to dry.

Copper Coil No. IV.—28,500 cm. no. 18 B. & S., double silk-covered magnet wire. Same wire as used in previous coil, unwound and soaked in alcohol to remove shellac, then rewound with great care, each layer being bound with paraffin run in well with a hot iron and excess paraffin carefully removed.

Copper Coil No. V.—52,900 cm. no. 20 B. & S., double silk-covered magnet wire. New wire, careful winding, and paraffin binding.

Aluminium Coil No. I.—38,700 cm. no. 20 B. & S., double silk-covered wire. Shellac binding.

Aluminium Coil No. II.—50,600 cm. no. 20 B. & S., double silk-covered wire. Paraffin binding, new wire.

Silver Coil No. I.—51,200 cm. no. 20 B. & S., double silk-covered wire. Shellac binding.

Silver Coil No. II.—29,930 cm. no. 20 B. & S., double silk-covered wire. Made by removing part of wire from No. I.

rotating sometimes in one direction and sometimes in the other, as indicated by the plus and minus signs in the tables. Not only was the pulse of electricity every time in the direction which would be predicted on the basis of a mobile *negative* electron as the carrier of electricity, but the experiments have led to concordant results for the mass of this carrier.

We believe that this constitutes a reasonably rigid test of the phenomenon in question but hope in the future to construct an apparatus which will eliminate two sources of accidental electromotive force which still seem to be present in our apparatus. The first source of trouble lies in the binding posts where connection is made with the external circuit, and the second more serious source of trouble appears to lie in the mechanical buckling of the wire or its slipping relative to the insulation when the wheel is stopped. We inferred that this latter was an important cause of accidental effects, since we found that the more carefully we wound and bound the coil the more concordant were the results which it gave. We are inclined to believe that this latter source of trouble accounts for the somewhat wide deviation of the results obtained with copper coil no. I, since this was our first and least carefully wound coil.

In conclusion we may point out that for all three metals copper, aluminium and silver the effective mass of the carrier comes out somewhat larger than the accepted value for the mass of a slow moving electron in free space. In free space the mass of the electron may be taken as $1/1,845$ times that of the hydrogen atom, while we have found for the carrier in copper $1/1,660$, in aluminium $1/1,590$, and in silver $1/1,540$. In our previous article we have mentioned some of the factors which might lead to a difference between the effective mass of an electron when in free space and when in the body of a metal. It is doubtful, however, if our present results are accurate enough to warrant much speculation of this kind. We hope in the future to construct an apparatus which will provide a more accurate determination of the mass of the carrier in metals and believe that such results will throw further light not only on the constitution of metals but on the nature of the electron as well.

The experimental work recorded in this article was carried out in the Chemical Laboratory of the University of California.

UNIVERSITY OF ILLINOIS AND UNIVERSITY OF CHICAGO,
October 30, 1916.

PROCEEDINGS
OF THE
AMERICAN PHYSICAL SOCIETY.

MINUTES OF THE CHICAGO MEETING.

THE eighty-fifth meeting of the American Physical Society was held in the Ryerson Laboratory of the University of Chicago on Saturday, December 2, 1916. There were two sessions, 9:00 A.M. and 2:00 P.M. The following program of papers was presented:

On the Velocity of Sound in Metal Tubes. KARL K. DARROW.

Collapse of Thin Tubes Shorter than the Critical Length. A. P. CARMAN.

An Acoustical Thermometer. F. R. WATSON.

The Intensity-factor in Binaural Localization and an Extension of Weber's Law. G. W. STEWART AND O. HOVDA.

Report of Progress on the Measurement of Earth Rigidity. A. A. MICHELSON AND HENRY G. GALE.

The Accuracy with which Gravity may be Predicted at any point in the United States. JOHN F. HAYFORD.

A Proposed New Method for the Determination of the Acceleration due to Gravity. HERBERT BELL.

On Some Very Large Variations in the Adsorption of Certain Specimens of Charcoal. HARVEY B. LEMON.

The Principle of Similitude. C. S. FAZEL.

Preliminary Notes on the Torsional Elasticity of Drawn Tungsten Wires. L. P. SIEG.

A Precision Calorimeter for Measuring Heats of Dilution. D. A. MACINNES AND J. M. BRAHAM.

Note on the Amount of Error in Applying to Non-Parallel Plates the Formula for Electrical Capacity of Parallel Plates. L. E. DODD.

The Kinetic Theory of Non-Spherical Rigid Molecules. YOSHIO ISHIDA.

The Photo-electric Emission from Crystals of Selenium. F. C. BROWN.

The Production of Light by Cathode Rays in Air. GORDON S. FULCHER.

The Structure of the Bismuth Line at Wave-Length 4722. HENRY G. GALE AND LESTER ARONBERG.

The Single-lined and the Many-lined Spectrum of Mercury. T. C. HEBB.

Note on the Single-lined and the Many-lined Spectrum of Mercury. R. A. MILLIKAN.

A Single Bar and Yoke Method for the Magnetic Testing of Iron Bars.
ARTHUR WHITMORE SMITH.

Measurement of Wave-lengths with the X-ray Spectrometer. ELMER
DERSHEM.

An Extension of the Mayer Experiment. R. R. RAMSEY.

The Optical Constants of Liquid Alloys. CARLETON V. KENT.

Visual Diffusivity. HERBERT E. IVES.

The Derivation of the Retarded Potentials. MAX MASON.

Some Effects of Cross-Magnetizing Fields on Hysteresis. N. H. WILLIAMS.

A. C. and D. C. Corona in Hydrogen. JOHN W. DAVIS.

The Magnetic Properties of Fe, Ni and Co above the Curie Point, and
Keesom's Theory of Magnetization. EARLE M. TERRY.

A Simple Method for Determining the Audibility Current of a Telephone
Receiver. (By title.) EDWARD W. WASHBURN.

The Mass of the Electric Carrier in Copper, Silver and Aluminium. RICHARD
C. TOLMAN AND T. DALE STEWART.

An Experimental and Theoretical Investigation of Binaural Beats. G. W.
STEWART.

Contact Electro-motive Forces and the Energy of Emission of Electrons
under the Influence of Monochromatic Light. R. A. MILLIKAN.

The Permanence of the Wave-length Sensibility Characteristics of Photo-
electric Cells. HERBERT E. IVES.

An Effect of Light on the Contact Potential of Selenium and Cuprous Oxide.
E. H. KENNARD AND E. O. DIETERICH.

A Peculiar Gas-Crystal Resistance Change in Selenium. W. E. TISDALE.

The Variation in the Blackening of a Photographic Plate with Time of
Exposure, Total Energy Remaining Constant. P. S. HELMICK.

Note on the Ionizing Potential of Metallic Vapors. (By title.) H. J. VAN
DER BIJL.

A General Method of Producing the Stroboscopic Effect, and its Application
in the Tonodeik. L. E. DODD.

An Apparatus for the Demonstration to an Audience of Simple Harmonic
Motion. (By title.) PAUL E. KLOPSTEG.

A considerable number of members of the Physical Society attended a
session of the Physics Section of the Central Association of Science and Mathe-
matics Teachers on Friday afternoon, which was devoted to questions con-
nected with the teaching of physics. One of the speakers was Professor G. W.
Stewart, a member of the Council of the Physical Society.

A. D. COLE,
Secretary.

MINUTES OF THE NEW YORK MEETING.

THE eighty-sixth meeting of the American Physical Society was held at
Columbia University, December 26-29, 1916. Sessions on Tuesday
afternoon, Thursday forenoon and afternoon, and Friday forenoon and after-
noon were joint sessions with Section B, A. A. A. S., and were held at the

School of Journalism. The two sessions on Wednesday were joint sessions with Sections B and C, and were held in Havemeyer Hall. The following program of papers was presented:

A Proposed New Form of Seismograph. HERBERT BELL.

The Velocity of Sound in Gases in Metal Tubes, as a Function of Density. KARL K. DARROW.

Measurements in Frictional Electricity. L. E. WOODMAN AND N. R. FRENCH.

The Preparation of Metallic Mirrors, Transparent Metallic Prisms and Films by Distillation. OTTO STUHLMAN, JR.

Our Part in the Advancement of World Physical Sciences. L. A. BAUER.

Some Experiments Concerning Magnet-Photography. L. A. BAUER AND W. F. G. SWANN.

On Growth of Crystal Structure in Selenium. F. C. BROWN.

Experimental Evidence for the Parsons Magneton. L. O. GRONDAHL.

The Effect of Pressure on the Resistance of Metals and a Possible Theoretical Explanation. P. W. BRIDGMAN.

The Infra-red Absorption Bands of Gases and the Application of the Quantum Theory to Molecular Rotations. EDWIN C. KEMBLE.

A Criticism of the Rutherford-Bohr Atomic Hypothesis, based upon a Theorem of Phase-equilibrium of two Electrons. ALBERT C. CREHORE.

A Physical Conception of the Reason for the Existence of Planck's Constant "h" based upon the Classical Electrodynamics. ALBERT C. CREHORE.

The Magnetization of Iron, Nickel and Cobalt by Rotation and the Nature of the Magnetic Molecule. S. J. BARNETT.

The Internal Structure of Atoms. A. W. HULL.

A New Count Method of Determining the Elementary Electrical Charge. HARVEY FLETCHER.

A Lecture Demonstration of the Capture of Ions by Falling Drops. E. P. LEWIS AND W. A. SHEWHART.

Some Undescribed Disintegration Products of Radioactive Elements. FANNY R. M. HITCHCOCK.

Recent Progress in Spectroscopy. (Vice-Presidential Address before Section B.) E. P. LEWIS.

The Photo-Electric Effect of Radiations in the Extreme Ultra-violet. JAMES BARNES.

Aluminum and Mercury Atoms under an Electric Field. REINHARD A. WETZEL.

Photography of Spectra in Red and Infra-red Regions. WILLIAM F. MEGGERS.

A Relationship between Fluorescence and Planck's Radiation Law. E. H. KENNARD.

The Infra-red Arc Spectra of the Metals of the Fe Group. H. M. RANDALL AND E. F. BARKER.

Some Spectra in the Photographic Infra-red. CHARLES F. MEYER.

The Effect of Longitudinal Alternating Magnetic Fields Upon the Hysteresis Curves Produced by Slowly Varying Currents in a Series of Iron-Carbon Alloys. C. W. WAGGONER AND H. M. FREEMAN.

Experiments with the Electric Furnace on the Anomalous Dispersion of Metallic Vapors. (By title.) ARTHUR S. KING.

The Effect of Oxygen on the Production of Band and Line Spectra in the Electric Furnace. (By title.) ARTHUR S. KING.

A Polarization Flicker Photometer. HERBERT E. IVES.

Test of Absorption Screen for Optical Pyrometry. E. P. HYDE, F. E. CADY AND W. E. FORSYTHE.

A New Direct-Reading Precision Refractometer with Uniformly Divided Scale. G. W. MOFFITT.

The Minimum Potential Required to Excite the Balmer Series of Hydrogen. JAMES BARNES.

Impact of Electrons on Mercury Atoms. C. D. CHILD.

The Stark Effect. REINHARD A. WETZEL.

A Proposed Method for Measuring Disturbances in the Earth's Magnetic Field. (By title.) HERBERT BELL.

The Kathodo-Luminescence Produced by Certain Tribo-Luminescent Salts of Zinc. (By title.) C. W. WAGGONER.

Variations in Glow Discharge Produced by a Longitudinal Magnetic Field. R. F. EARHART AND C. B. JOLLIFFE.

A Time-Current Equation for Making Iron Passive. C. MCCHEYNE GORDON.

A Calorimetric Resistance Thermometer. S. LÉROY BROWN.

A New Design of Mercury-Break Buzzer for Generating Electrical Oscillations, and a Study of the Use of Other Buzzers in Radio Measurements. CHARLES MOON.

The Reflectivity of Tungsten. W. WENIGER AND A. H. PFUND.

A Determination of C_2 of Planck's Radiation Law. (By title.) C. E. MENDENHALL.

The Range of Recoil Atoms from Actinium Emanation. L. W. MCKEEHAN.

The Intensity of X-ray Spectra. (By title.) ARTHUR H. COMPTON.

The Distribution of the Electrons in Atoms. ARTHUR H. COMPTON.

The Effect of Transverse Joints on the Magnetic Induction in Iron and Nickel. S. R. WILLIAMS.

A Resonance Method for Measuring the Phase Difference of Condensers of Fixed Capacity and a Comparison of Resonance and Bridge Methods. J. S. WARD.

The Thermophone as a Precision Source of Sound. H. D. ARNOLD AND I. B. CRANDALL.

A Uniformly Sensitive Instrument for the Absolute Measurement of Sound Intensity. E. C. WENTE.

Note on the Ionization Manometer. O. E. BUCKLEY.

An Accurate Method for the Determination of Surface Tension. W. D. HARKINS AND F. E. BROWN.

Surface Tension, Total Surface Energy, Solubility, Emulsification and Polar Setting in Surfaces. W. D. HARKINS.

The Variation of the Mobility of the Negative Ion with Temperature in Air of Constant Density. HENRY A. ERIKSON.

Intensity of Emission of X-rays from Metals. C. S. BRAININ.

Extension of Recently Published Work on Ionization Potentials. J. C. McLENNAN.

The Significance of Certain New Phenomena Recently Observed in Preliminary Experiments on the Temperature Coefficient of Contact Potential. (By title.) A. E. HENNINGS.

The Energy of Emission of Photo-electrons from Film-coated and Non homogeneous Surfaces: A Theoretical Study. (By title.) A. E. HENNINGS.

The Possibility of a Science of Experimental Meteorology. B. P. WEINBURG.

A Proposed Method for the Photometry of Lights of Different Colors. (By title.) IRWIN G. PRIEST.

At the joint sessions on Wednesday with Sections B and C, A. A. A. S., the following papers were presented by invitation.

? Radiation and Atomic Structure. (Presidential address before the American Physical Society.) R. A. MILLIKAN.

The Atom and Chemical Valence. G. N. LEWIS.

Molecular Resonance and Atomic Structure. ROBERT W. WOOD.

The Evolution of the Elements as Related to the Structure of the Nuclei of Atoms. WM. D. HARKINS.

The Relation of Magnetism to the Structure of the Atom. WM. J. HUMPHREYS.

The Relations of Magnetism to Molecular Structure. ALBERT P. WILLS.

The Structure of Solids and Liquids, and the Nature of Inter-atomic Forces. IRVING LANGMUIR.

Electromerism: A Case of Chemical Isomerism Resulting from a Difference in Distribution of Valence Atoms. LAUDER W. JONES.

The following responded to invitations to discuss the above papers: WM. DUANE, A. C. CREHORE and K. G. FALK. Mr. Falk read the discussion of J. M. Nelson. The discussion was then thrown open and participated in by W. F. G. Swann, A. G. Webster, M. I. Pupin and others.

Many physicists attended the address Tuesday evening of the retiring president of the A. A. A. S., Director W. W. Campbell of the Lick Observatory, on "The Nebulæ," and the special program of Section D, Friday evening, on "The Inter-relationship of Engineering and Pure Science." This session was held at the Engineering Societies Building and was followed by a reception to visiting members of the A. A. A. S.

At a short business session the result of the mail ballot for the election of officers was announced. R. A. Millikan, H. A. Bumstead, A. D. Cole and J. S.

Ames are reelected president, vice-president, secretary and treasurer respectively; H. A. Wilson and G. O. Squier are the new members of the Council; F. Bedell is reelected managing editor, and O. M. Stewart, N. E. Dorsey and Wm. Duane are elected on the editorial board of the *PHYSICAL REVIEW*. The reports of the treasurer and the managing editor were presented and on motion, accepted. (These will be printed and mailed to all members.) It was announced that the next meeting of the Society will probably be in connection with the Midwinter Convention of the Amer. Institute of Elec. Engineers at New York, February 14-16.

The subscription dinner on Thursday evening was attended by about eighty, and was much enjoyed. The exhibit of new apparatus and results in the Commons Building was open from 4:00 to 6:00 P.M. daily, and on Friday afternoon the instruction and research laboratories for physics in Fayerweather Hall were on exhibition, with members of the teaching staff in attendance. For these courtesies and many others the Society is indebted to Director Geo. B. Pegram, who also had charge of the physics portion of the apparatus exhibit.

The attendance at this meeting was record-making, about 325 at the joint sessions on Wednesday and about 200 at most of the ordinary sessions. The number of new members elected at the meeting was forty, which also probably establishes a new record.

A. D. COLE,
Secretary.

NOTE ON THE IONIZING POTENTIAL OF METALLIC VAPORS.

BY H. J. VAN DER BIJL.

FRANCK and Hertz found that when the molecules of mercury vapor are struck by electrons with energy corresponding to 4.9 volts, a single line spectrum was emitted, the resulting wave-length being 2536 A.U. which, according to the quantum relation, $Ve = h\nu$, corresponds to an energy equivalent to 4.9 volts. They furthermore concluded that 4.9 volts was just sufficient to ionize the mercury vapor.

It is clear, as has already been pointed out, that this experiment of Franck and Hertz does not necessarily show that ionization takes place under the influence of 4.9-volt cathode rays, but that the collisions are elastic until the colliding electrons have acquired an energy corresponding to 4.9 volts. This merely means that when the electron strikes a molecule with 4.9 volts energy, it gives up its energy to the molecule, but whether this energy is transformed into ionization as well as radiation, as Franck and Hertz assumed, or into radiation alone, cannot be decided from their experiment.

They state, however, that they also find the ionizing potential to be 4.9 volts when they use Lenard's method, in which the positive charges resulting

¹ Abstract of a paper presented at the Chicago meeting of the Physical Society, December 2, 1916.

from ionization were supposed to be actually picked out and measured with an electrometer. This method is as follows:

The hot wire cathode P was maintained at a positive potential of 10 volts and a platinum gauze D of fine mesh at a potential of $10 + V$ volts. Electrons emitted from P are accelerated by a potential difference of V volts and then enter into a retarding field of $10 + V$ volts. Since under such conditions F is always negative with respect to P , no electrons can reach F . If, on the other hand, the electrons acquire sufficient energy to ionize the gas between D and F , the resulting positive ions will be driven onto F and the electrometer will

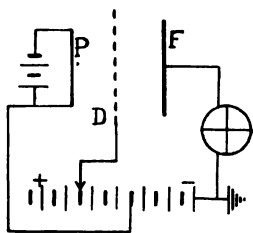


FIG. 1.

acquire a positive charge. We see, therefore, that this method lends a decision regarding the transformation of energy by an inelastic impact.

Later experiments by McLennan, Couchner and others show that the electrometer actually acquires a positive charge when the colliding electrons reach 4.9 volts, but that the positive charge increases very rapidly when about 10 volts is reached, and then all the lines of the series are emitted. McLennan was led to sup-

pose that there might be two distinct types of ionization—one requiring the energy equivalent of the first line and the other the energy equivalent of the limiting line of the series, corresponding for mercury vapor to 4.9 and 10.27 volts respectively.

An accurate determination of ionizing potentials would have an important bearing on the problem of atomic structure. That an amount of energy corresponding to the first member of the principal series can represent what we might call the ionizing potential does not at present seem capable of satisfactory theoretical explanation. In other words, 4.9 volts could not be looked upon as the ionizing potential of mercury vapor. The question therefore arises whether the positive charge acquired by the electrometer in the experiments recorded was actually due to ionization. It seems to me that the answer might be found by considering a factor which has thus far been left completely out of account, and that is: When the electron acquires energy equivalent to 4.9 volts in the case of mercury, for example, the corresponding line 2536 is emitted. There is thus created a source of ultra-violet light in the tube. This light falls on the plate which is connected to the electrometer and so can emit electrons from the plate photoelectrically, thus causing the electrometer to charge up positively. It is just possible that this is what was measured. Such a possible effect was not excluded from any of the experiments published. This seems the more likely, in view of the fact that the positive current increases very slowly from about 5 volts to about 10 volts and then shows a very rapid increase, indicating that the positive current obtained for voltages between 5 and 10 might have been due to the comparatively small photo-electric effect, while for voltages above 10 a current due to intense ionization was superimposed upon the photo-current. If therefore it should

be found that such a photo-electric effect actually is present to an extent great enough to make itself felt, there would be strong reason for believing that what we might call the ionizing potential of mercury vapor is more nearly represented by 10.27 than by 4.9 volts. This would correspond to the limiting line of the series. This would also apply to the determination of the ionizing potentials of other elements.

It may be remarked that such a result, should it receive experimental corroboration, would possibly be capable of explanation on Bohr's theory, but there is a question as to whether such a thing as a definite ionizing potential of an element exists at all. A discussion of this question I hope to present in the near future.

NEW YORK CITY.

THE KATHODO-LUMINESCENCE PRODUCED BY CERTAIN TRIBO-LUMINESCENT SALTS OF ZINC.¹

BY C. W. WAGGONER.

THIS paper is a continuation of a paper² presented at the Columbus meeting of the Physical Society, December 28-30, 1915, in which it was shown that these compounds give the same character of spectra when excited to give Tribo-Luminescence as they give by X-Ray Luminescence. The work was extended to include the spectra of these salts when excited by kathode rays, and it was found that in those compounds where the spectra were not obscured by the kathodo-luminescence of Willemite, the bands are the same as those excited by Tribo- and X-Rays. The intensity of the kathodo-luminescence was sufficient to make it easily possible to photograph the bands by the use of spectrum plates. It appears then that the luminescent spectrum of all these salts is independent of the mode of excitation.

WEST VIRGINIA UNIVERSITY.

ON PHOTOELECTRIC PHOTOMETRY.³

BY JAKOB KUNZ.

IN spite of the very numerous investigations carried out in the field of photoelectricity there are still very few questions which have been settled definitely. Among the open problems appears that of the relation between the intensity of light and the photoelectric current. Elester and Geitel and Richtmeyer found a linear relation between the two quantities; Lenard concluded from his measurements also that the photoelectric current is proportional to the intensity of light. But upon plotting his figures, one finds a

¹ Abstract of a paper presented at the New York meeting of the Physical Society, December 26-30, 1916.

² PHYSICAL REVIEW, Vol. VII., Ser. II., p. 402, 1916.

considerable deviation from a straight line. On the other hand, Griffith and especially Ives found a large variety of curves. This question has been studied in the Nela Research Laboratory at Cleveland, Ohio, during the last summer, with various forms of cells. The results point very strongly toward a direct proportionality, in fact in most cases straight lines have been obtained. But under exceptional conditions marked deviations have been found. The light intensity varied in the interval from 1 to 1300; 3 different methods for the variation of the light intensity and 4 different methods for the determination of the current have been used. The result was in most cases a straight line.

PHYSICAL LABORATORY,
UNIVERSITY OF ILLINOIS.

THE PHYSICAL REVIEW.

ON THE FREE VIBRATIONS OF A LECHER SYSTEM. IV.¹

BY F. C. BLAKE AND CHARLES SHEARD.

SINCE our last paper we have investigated the effect of the presence of the wooden supports for the capacity plates upon the phase relations and the relative wave-lengths of the various tones. The results of this investigation will be published shortly in *The Ohio Journal of Science*. We have also investigated the effect of the kind of induction coil and the current-strength upon the wave system. These results are reported here. The influence of the edge-on distance between plates upon the wave system is also here reported.

APPARATUS.

Since one of the main objects of our research was to determine the factors that controlled the intensity of the various harmonics and since we had found that the wooden supports introduced into our work phase changes that were appreciable, we found it desirable to eliminate these supports entirely. This was done by supporting the plates by means of Barbour's machine thread No. 18, well waxed to prevent changes of tension due to absorption of moisture. Each of the eight plates was bored with four small holes equally spaced around the circumference, and through each hole two strings were passed and knotted on the back side of each plate. The hole was bored diagonally from the back of the plate to the edge. This left the front face wholly intact. The other ends of the strings were fastened to small windlasses attached to a wooden framework of dimensions $3 \times 1 \times 1$ meters. These windlasses were so distributed on the framework as roughly to cause each of the eight strings from each plate to form an angle of 90 degrees with any adjacent string. Roughly the solid angle round each plate was thus divided by

¹ Read before the American Physical Society, December, 1915.

the string into six right-angled pyramids. In this way it was possible to get the plates accurately lined up and to keep them so. Their alignment was always tested daily. The nearest distance of any part of the system to wood or material other than air was thus 40 cm. except at the oscillator spark-gap and at the receiver. At the spark-gap the hard rubber piece that held the glass tubing necessary for the flow of oil, together with the steel rods that formed the gap, formed an intimate part of the oscillator. At the receiver, the thermocouple T , Fig. 1, was

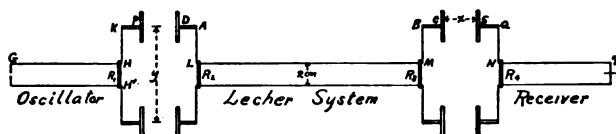


Fig. 1.

enclosed by a hard rubber box $1.9 \times 1.6 \times 1.3$ cm., the walls being about 3 mm. thick. Except at the spark-gap and at the thermocouple the whole vibrating system was surrounded with the one dielectric, air. The small hard rubber pieces, R_1 , R_2 , R_3 , R_4 were assumed to be negligible in their effect. The plates were accurately 5 cm. in diameter and 4.5 mm. thick, to prevent warping under the tension put upon them by the strings. Tension applied by means of other strings attached to $R_1 R_2 R_3 R_4$ gave the necessary tension to the parallel wires. The hard rubber box enclosing the thermocouple T was surrounded with cotton batting to prevent changes of temperature due to air currents. The batting was kept in place throughout the work although it was scarcely necessary except on very windy days. Tension was also applied to T by still other strings leading to a permanent wooden support at a distance of 30 cms. The hard rubber support for the spark-gap was attached to the end of a 2×4 inch plank 26 cm. long which was itself attached to an upright 4×4 inch plank, 1.1 meter long, on top of which the bottle holding the oil was placed. These wooden supports for oscillator and receiver were movable along a 2×6 inch plank by means of iron clamps to which they were attached. In this way the length of oscillator, Lecher system or receiver could be varied at will. As heretofore the distance between the parallel wires was kept constant at 2 cm.

EFFECT OF THE KIND OF INDUCTION COIL.

In the course of the two or three papers that we have now completed upon this research but not yet published we hope to make clear the experimental conditions that will enable anyone to duplicate our results as a laboratory experiment. In this respect a matter of prime importance is the choice of induction coil.

As previously reported¹ the induction coil that we have constantly used was a 10-inch coil without interrupter or condenser on a 110-volt A.-C. circuit. This gave a fat spark over a length of about 3 cm. at the secondary terminals with a primary current of 10 amperes. Experience had shown that a spark that is almost an arc is the best.

There has lately appeared on the market a new type of induction coil for X-ray work manufactured by the Vulcan Coil Co. of Los Angeles. It is a sort of auto-transformer. It is worthy of special study. For instance, by merely changing the length of the spark-gap the nature of the secondary discharge, *i. e.*, the frequency of alternation in the secondary as well as the current strength in the primary can be changed. Since it operates on a 110-volt A.-C. circuit without an interrupter it was similar to the coil we had been using, so that it could be compared with it. With a given spark-gap it is capable of seven variations in current

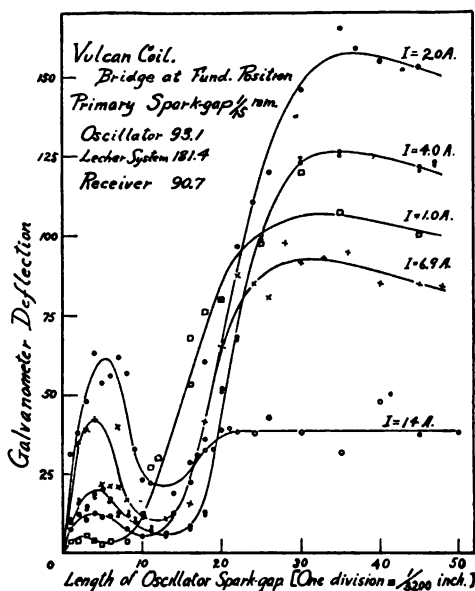


Fig. 2.

strength from 1 to 14 amperes roughly. When it was substituted for the regular coil it gave better results when the water resistance, previously used, was removed. This was accordingly done. A hard rubber micrometer screw-head was attached to the screw that regulated the oscillator oil-gap to enable the length of this gap to be known. The two air gaps were kept constant at 5 mm.

With the Vulcan coil inserted and the water-resistance removed the curves of Fig. 2 were taken. Abscissæ represent the length of the oscil-

¹ PHYSICAL REVIEW, XXXII., p. 534, 1911.

lator spark-gap in oil, the ordinates the intensity of the fundamental tone, represented by the galvanometer deflection for the bridge in the middle of the Lecher wires. Throughout this paper the length of the oscillator will always be expressed as the straight-away length GH , the Lecher wires as the length LM , the receiver as the length NT (see Fig. 1). When the distance "y" is known, that is, the edge-on distance between plates center to center, the total length of oscillator, Lecher system or receiver up to the back of the plates can easily be found as needed. The length of the wire KH at right angles to the main length is always $1/2 (y - 2)$ cm. At K the wire was inserted into a small brass rod, KP , 3.5 cm. long and 4.7 mm. diameter, threaded at the end to receive the plate. For all three systems the bends at H , L , M and N were rectangular.

For the curves of Fig. 2 the spark-gap of the Vulcan coil was estimated

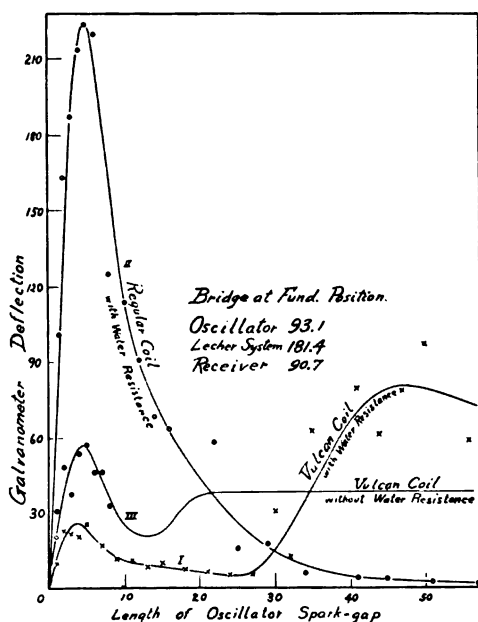


Fig. 3.

at 1/15 mm. In this case the current in the primary coil read 14.0, 6.9, 4.0, 2.0, 1.0 amperes when the coil was adjusted to stops 7, 5, 4, 3, 1 respectively. It is seen at once that there is a rather sharp maximum for oscillator spark-gap varying from 3 to 6 divisions, *i. e.*, from 0.00094 to 0.00188 inch. There is, however, a second broad maximum for much larger lengths of the oscillator spark-gap. The first maximum increases with the current while the second one increases at first and then decreases.

A similar set of curves was obtained when the bridge was placed at the position to give the "third" near the oscillator. Fig. 3 gives a direct comparison between the Vulcan coil and the regular coil. Curve I was obtained with the Vulcan coil, primary current 14 amperes, Curve II. with the regular coil, primary current 13.5 amperes, in both cases the water-resistance being inserted. Curve III. is a Vulcan coil curve without the water-resistance, the portion of the curve for larger oscillator spark-gaps being copied from Fig. 2. In the curves of Figs. 2 and 3, for the oscillator spark-gap in the neighborhood of 50 divisions ($1/64$ inch) the sparking would occur at HH' instead of at G , Fig. 1, and the galvanometer would generally read nearly zero. The curves of Fig. 3 at once bring up the question—To what is this second broad maximum due? Since it is not present in Curve II., taken with the regular coil, we are inclined to think it is spurious. It is plain from Fig. 2, moreover, that the ratio

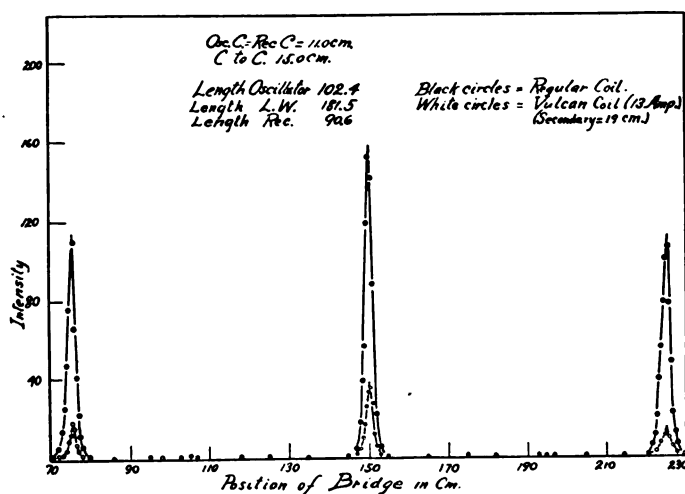


Fig. 4.

of intensities of the fundamental and third is not constant for this second maximum as the current strength is changed, while it is for the first maximum (see also Fig. 5). We found, moreover, that this second maximum is very much more dependent upon the oscillator length and current strength than the first maximum. This is to be expected if it is due to a "slopping-over" of energy from the induction coil upon the vibrating system. We believe that it is brought about in the case of the Vulcan coil because of the relative small inductance of the secondary of the Vulcan coil. This probably enables the secondary to be a part of a vibrating system including the coil and the lead wires to the oscillator and the oscillator itself perhaps. Such a vibrating system would have

a slow period which would tend to dominate effects at the receiver. For the regular coil on the other hand, the inductance of the secondary is, compared with that for the Vulcan coil, exceedingly great. This tends to make the lead wires to the oscillator vibrate as a system independent of the secondary coil and thus enables the free vibrations of the Lecher system (in tune with the oscillator and receiver) to appear to a marked degree as Curve II., Fig. 3, shows.

EFFECT OF PRIMARY CURRENT STRENGTH.

Fig. 4 shows the whole wave-system for the Vulcan and regular coils with the same primary current for a length of oscillator for which only the thirds and the fundamental appear. The advantage of the regular coil is obvious. Fig. 5 was taken with the regular coil for an oscillator length at which the fifths also appear. It is seen that the relation between current-strength in the primary and the tone intensity is strictly

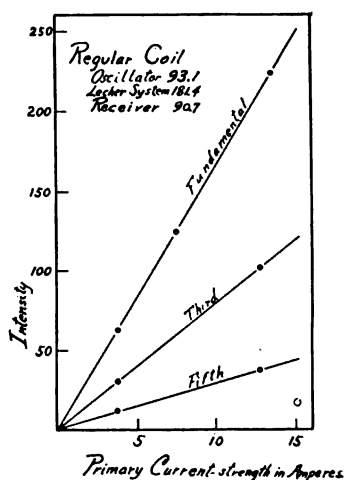


Fig. 5.

a linear one. This accounts for the advantages of a fat spark. We proved the same thing for the Vulcan coil in the case of the third, using the first maximum of course. We have made no attempt to increase the primary current strength beyond 14 amperes for the following reasons. First, the greater the current strength the greater the oscillator deterioration because the greater the amount of carbon soot that tended to deposit on the spark-gap rods. Second, the air-gap rods became red hot much more quickly with increasing current. Third, since we used a Pohl's commutator as a key in the primary circuit the sparking at the mercury cup was rather vigorous

for 14 amperes on breaking the circuit.

EFFECT OF CHANGING THE EDGE-ON DISTANCE BETWEEN PLATES.

Having eliminated the effect of dielectrics in the neighborhood of the capacity plates and having shown the optimum conditions in the choice of an induction coil we reverted to our task¹ of determining the factors that control the intensities of the various harmonics.

At the start, in order to make reasonably sure that the effect of mutual capacity due to the plates edge-on was negligibly small, we chose the

¹ See PHYSICAL REVIEW, Vol. XXXII., p. 534.

distance " y " (Fig. 1) as 35 cm. We had previously¹ shown that the optimum oscillator length for a certain tone varied with the frequency of that tone, being shorter for the higher tones and this was again verified as shown in the curves of Fig. 6. But rather to our surprise the higher

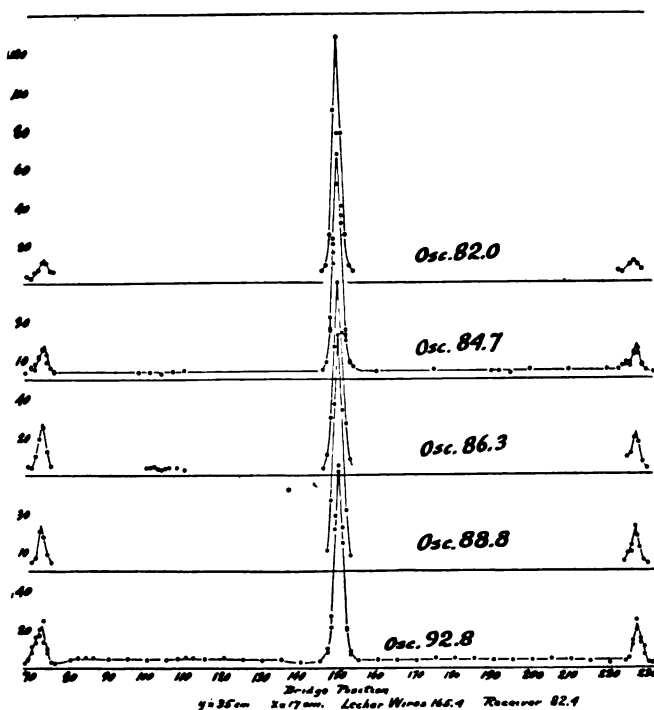


Fig. 6.

tones were very weak. Fig. 6 shows only the thirds and the fundamental, with the maximum value of the thirds only about 15 per cent. of that of the fundamental. Thinking that possibly the very loose coupling (17 cm.) was responsible for the low intensity of the higher harmonics, the coupling was reduced to 10 cm. at each set of plates. This did not help matters. It became increasingly evident that the tone intensity, " I " say, was very much dependent upon the distance " y ," the edge-on distance between the plates center to center. Accordingly we had to settle down to a long investigation of the relation between " I " and " y ." Since the plates were all suspended by strings this investigation was necessarily tedious. Sets of curves were taken for the following values of y , 35, 20, 15, 12, 8, 7 cm. Some of the results are shown plotted in Figures 6, 7, and 8. For all these curves the receiver was kept very

¹ PHYSICAL REVIEW, N. S., Vol. III., p. 217, 1914.

approximately equal to one half the Lecher system. For a given distance "y" the oscillator length was shortened step by step. Then the plates were readjusted to a new distance, new oscillator wires were inserted and again shortened by steps. For any given curve the oscillator spark-gap was adjusted to a maximum for the fundamental; then a curve was taken starting in general with the bridge near the oscillator end of the

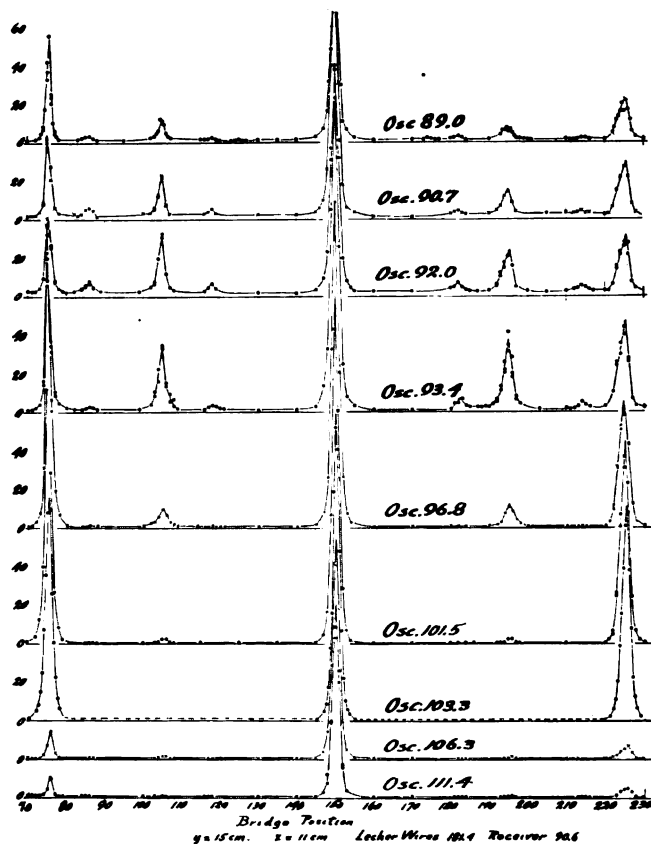


Fig. 7.

Lecher wires. No account is taken in any of these curves of oscillator deterioration. This makes it impossible to compare the various curves directly though one can see clearly that the higher harmonics show only for the lesser distances. Assuming for the moment oscillator deterioration to be zero, many of the curves could be compared directly, for they had the same coupling, and it was plain that for the smallest distances the intensity of the fundamental and the thirds, at least, had decreased considerably. It became plain that there were two factors operating in

opposite directions and that for a certain tone there would be a certain distance " y " at which the tone intensity would be a maximum. Only very roughly though could this distance be determined by the present investigation. A special and still more tedious investigation to determine this has just been completed and will be reported in another paper.

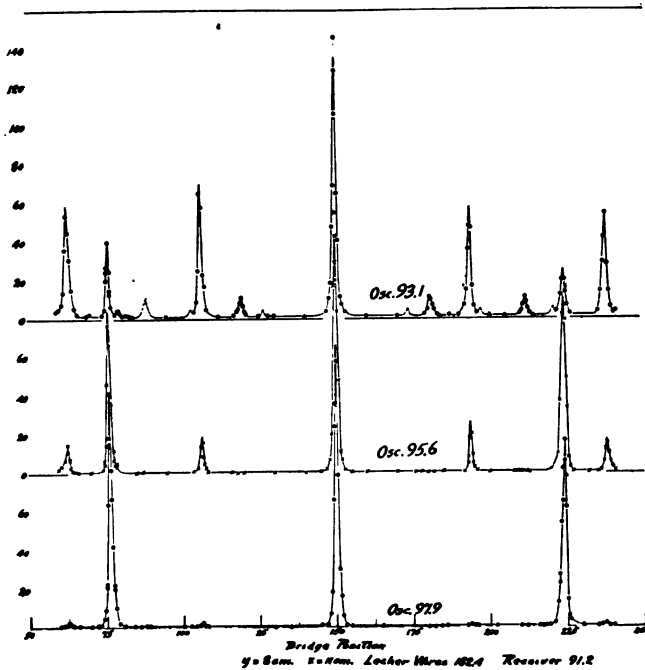


Fig. 8.

Several things of importance fall out of the present study, however, and need to be noted. Of the two factors above mentioned, one is plainly of such a sort that for smaller values of " y " more and more of the electrostatic lines run between any two plates edge-on of the same system for the lower tones, hence less energy gets to the receiver. In a certain rough sense, for comparative results, the ratio y/λ or $(y - 2)/\lambda$ should be a constant for the various tones. This first factor or function we shall speak of as $\theta(y)$. The opposing factor, which we shall call $\phi(y)$, is plainly of such a sort as to introduce a greater phase change for the larger values of " y " in the case of the higher harmonics than for the lower harmonics. This would tend to make the intensity, I , too small for the higher harmonics for a given value of " y ."

If we summarize the results of all the curves for a given figure we should get results typified by the curves of Figs. 9 and 10. In Fig. 9,

for $y = 20$, we see that the length of oscillator at which the fundamental and the two thirds are a maximum is practically 100 cm., while for the two fifths it is about 93.5 cm. In Fig. 10, for $y = 12$, the optimum oscil-

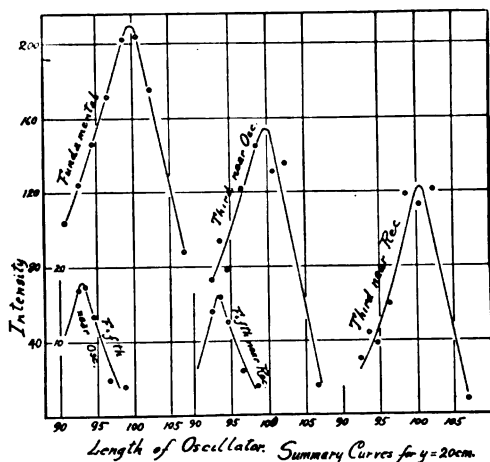


Fig. 9.

lator length for the fundamental and thirds is 96 cm., while for the fifths it is 93.4 cm. and for the sevenths 92.3 cm. There is one essential difference, however, between these two figures in the manner of making the summary. For Fig. 9, the oscillator spark-gap was always adjusted to a maximum

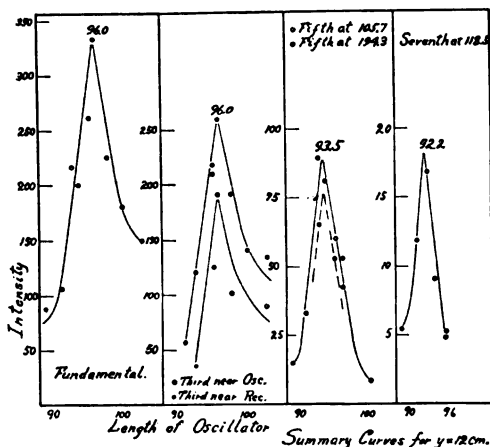


Fig. 10.

for the fundamental and a few readings were taken for the other tones. For Fig. 10, on the other hand, complete spark-gap curves were taken for each tone and the maxima of such curves were plotted. Had spark-gap curves for Fig. 9 been taken the optimum oscillator length for the

fifths would undoubtedly have been over 94 cm. The spark-gap curves for the distance $y = 12$ for the fundamental and the two thirds are shown in Fig. 11. If we summarize the curves of Fig. 11 and other curves not

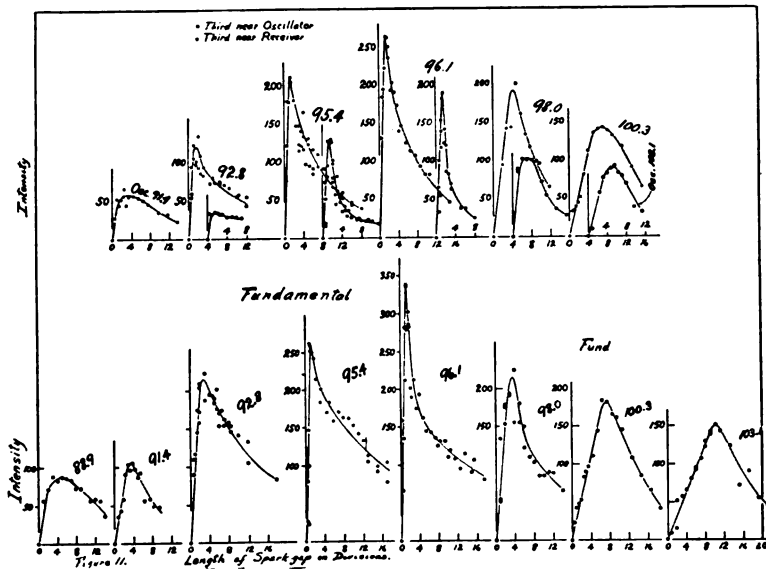


Fig. 11.

shown we get Fig. 12. It proves several points. First, that for each tone maximum intensity occurs at or near the minimum spark length;

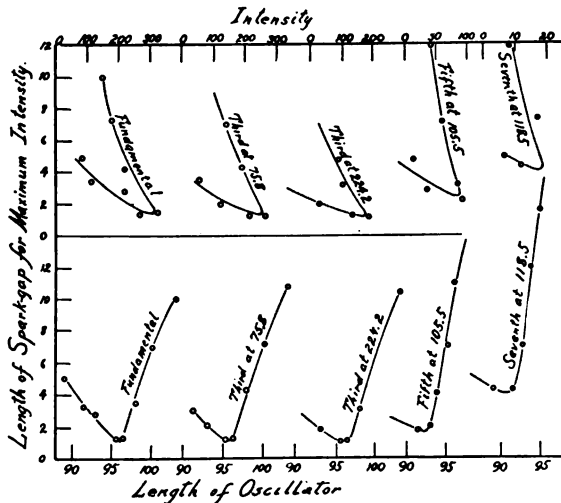


Fig. 12.

second, that this minimum length is practically the same for the fundamental and thirds, about 1.2 divisions, but increases for the higher tones,

being 2.0 divisions for the fifths and 4.0 divisions for the sevenths. Apparently the minimum spark length occurs for oscillator lengths slightly shorter than those that give maximum tone-intensity, at least in the case of the higher tones.

Fig. 13 shows the amounts the peaks for the various tones are displaced

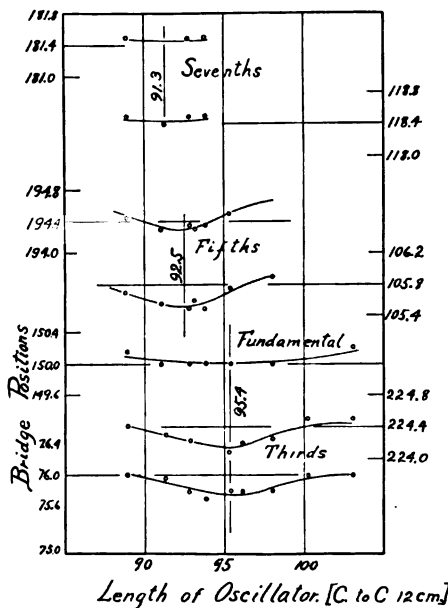


Fig. 13.

as the oscillator length is changed for the case $y = 12$ cm. The amounts the peaks can be varied are very much less than in our previous work owing to the absence of dielectric around the plates. For $y = 12$ the variation is less than for $y = 15$ and for $y = 7$ and 8 it is still less. Moreover, in any case where there is variation the peaks belonging to the same tone vary always in the same direction so that the wave-length is quite independent of the oscillator length. When the plates were mounted on wooden supports this was not the case.¹ Undoubtedly the wave length is equally independent of the receiver length.²

The results shown in Figs. 6 to 8 show clearly another advantage of having air the only dielectric surrounding the plates. The anomalous effects portrayed in Fig. 11 of our paper III. on this subject do not now occur. For a given tone the peak nearest the oscillator is the highest, decreasing as the distance from the oscillator increases, due to damping of course. The damping factor can, of course, be determined.

HOW DOES THE CAPACITY OF THE PLATES VARY WITH THE EDGE-ON DISTANCE BETWEEN THEM?

Plotting all the peaks of the curves taken for the same coupling, viz., $x = 11$ cm., to a much larger scale than shown here and averaging the internodal spaces the wave-lengths shown in Table I. were obtained. For values of " y " equal to 12 cm. and less each peak was determined by moving the bridge across it by intervals of 1 cm. and returning on the half-centimeter divisions. In general this eliminated oscillator deteriora-

¹ See PHYSICAL REVIEW, N. S., Vol. III., p. 229.

² See Figure 5 of our paper III., l. c. The small variation in wave-length due to receiver-length could easily be explained by the presence of dielectrics near the plates.

tion, though when for any reason an observation appeared inconsistent with its neighbors several readings were taken. Such bridge curves would, in general, be equally as good as those shown in our paper I., Fig. 10,¹ and entirely similar to them, though the center of gravity line might not always be quite so vertical as there shown.

The first column of Table I. gives the position, p , of the back side of the Lecher plates near the receiver. The points L and M , Fig. 1, were read directly with a plumb-bob and the 3-meter stick was always adjusted until the midway point between L and M came at 150 cm. Within a fraction of a millimeter the distance $DALMBC$ was always just twice $SQNT$. K , A , B and Q are the points where the copper wire was inserted into the brass rods to which the plates were attached.

In attempting to calculate the value of the end-capacity in terms of the capacity per unit length of the Lecher wires it seemed safest to treat the lengths MB and BC as if they were straight-away prolongations of LM . For it is known² that the capacity and inductance per unit length of the wires fulfill the equation

$$L = \frac{\epsilon\mu}{K} = 4\mu \log_e \left(\frac{d + \sqrt{d^2 - b^2}}{b} \right)$$

where " $2d$ " is the distance between the axes of the parallel wires and " b " is the wire radius. For air both ϵ and μ equal unity and this equation becomes

$$LK = 1 = 4K \log_e \left(\frac{d + \sqrt{d^2 - b^2}}{b} \right)$$

The equation $LK = 1$ holds for the part LM of the Lecher wires without question. It probably holds for the rods DA and BC also. Hence it seems wise to say that it holds for the portions AL and BM . Although there is a change in diameter at the points A and B (and K and Q) we shall assume that there is no appreciable reflection at these points on that account.

Accordingly the position " p ," Table I., can be found by adding $MB + BC$ to the reading of the point M . Of course $MB = 1/2 (y - 2)$. The lengths, l_s , of Table I. are found from the relation

$$l_s = p - \left\{ 150 + (s - a) \frac{\lambda_s}{2} \right\},$$

where " s " equals the frequency numbers 1, 3, 5, 7, etc., and " a " the corresponding natural numbers 1, 2, 3, 4, etc. Thus " l_s " is the distance between the nearest bridge position and the end-capacity. The last

¹ PHYSICAL REVIEW, XXXI., p. 547.

² See, for instance, Abraham, "Théorie der Elektrizität," 2 Aufl., Bd. 1, p. 349.

five columns of Table I. give the ratio K_0/K , calculated for all observed harmonics by means of the formula

$$\frac{K_0}{K} = \frac{\lambda_s/2\pi}{\tan\left(\frac{l_s}{\lambda_s/2\pi}\right)}$$

where K_0 is the value of the end-capacity. Since the frequency changes at most only in the ratio 9 to 1, it is to be expected that the value of K_0/K should be constant, that is, independent of "s." The table shows, indeed, that this is practically the case for small values of "y." The fundamental wave-length λ_1 must, of course, be obtained by extrapolation and this is done by plotting $s\lambda_s$ against s, as shown in Fig. 14. For y

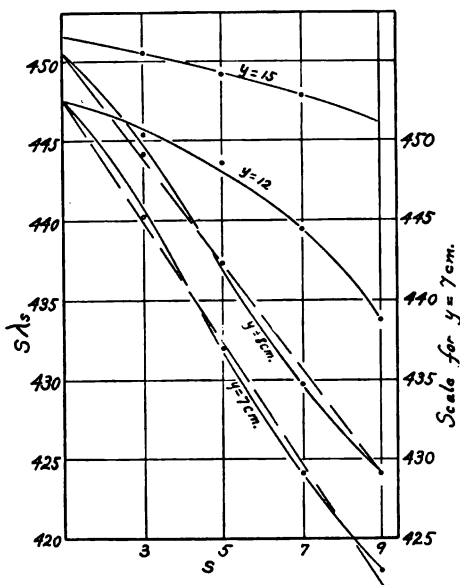


Fig. 14.

equals 7 and 8 cm. the curves shown are theoretical curves obtained for values of K_0/K equalling 16.12 cm. and 15.15 cm. respectively. The points shown are obtained from the observed mean wave-lengths. To emphasize the sinusoidal nature of the theoretical curves straight lines are also plotted. For y equals 12 and 15 cm. the curves are not theoretical curves. Table I. shows clearly that for the larger values of "y" the value of K_0/K is far from constant, apparently increasing rapidly with increasing "s." This goes merely to show that $\phi(y)$ is playing a prominent part in preventing the constancy of K_0/K for these larger values of "y." Naturally the inconstancy of K_0/K will serve to enable one to

judge the form of $\phi(y)$. To show the effect of a slight variation of the wave-length λ , upon the value of K_0/K the comparison between the theoretical and observed values is given in Table I. for the distance $y = 8$

TABLE I.

(x = 11.0 cm.).

μ .	y .	Wave-length.					l_s .					K_0/K .				
		λ_1 .	λ_2 .	λ_3 .	λ_4 .	λ_5 .	l_1 .	l_2 .	l_3 .	l_4 .	l_5 .	$s = 1$.	$s = 3$.	$s = 5$.	$s = 7$.	$s = 9$.
50.6	15	451.64	150.19	89.84	63.98	(49.57)	100.60	25.50	10.76	4.63	(1.51)	12.43	13.18	15.27	25.05	(40.70)
49.35	12	446.80	148.44	88.74	62.80	48.2	99.35	25.10	10.65	5.15	2.95	12.65	13.13	15.12	19.65	18.96
47.70	8		148.23	87.37		47.12		23.60	10.33		3.46		15.15	15.15		15.09
47.25	7	450.50	148.05	87.48	61.40	47.14	97.70	23.67	10.22	5.60	3.42	15.15	15.04	15.42	15.13	15.30
		452.50	148.40	87.40	61.30	47.00	97.25	23.10	9.85	5.30	3.25	16.12	15.85	16.24	16.16	16.12

cm. The lower row of figures are the observed values, the upper row the theoretical values taking $K_0/K = 15.15$ cm. Changing λ_2 0.12 per cent. affects K_0/K 0.73 per cent. Changing λ_3 0.13 per cent. affects K_0/K 1.8 per cent. Changing λ_4 0.04 per cent. affects K_0/K 1.4 per cent. The importance of close measurement of the peak positions can thus be realized.

It will be seen from Fig. 14 that for y equals 7, 8 and 12 cm. the observed points for $s = 3$ all lie below the curves while for $s = 5$ the observed points lie above the curves. One reason for this is that in every case where four fifths were observed the internodal spaces for the fifths either side of the fundamental were slightly less than those for the two outer fifths, *i. e.*, those furthest removed from the fundamental. For $y = 15$ only the two inner fifths could be obtained, hence the point fits on the curve sufficiently well. There must be some second-order effect, possibly the presence of the small ebonite pieces, $R_1 \dots R_4$, Fig. 1, so near the outer fifths, to increase slightly the outer internodal spaces. Apparently some other second order effect serves to decrease λ_3 slightly below its true value.

Plotting the values of K_0/K against " y " we get the curve of Fig. 15. For $y = 7$ and $y = 8$ the value for $s = 1$ was used. Now the equation

$$\frac{1}{K} = \log_e \frac{d + \sqrt{d^2 - b^2}}{b}$$

holds for the capacity per unit length of the Lecher wires. In our case $2d = 2.0$ cm. and $b = 0.038$ cm. Assuming that the plates are simply portions of wires of bigger diameter we can say that this relation holds for the capacity of the plates. If for y equals 7 and 8 cm. we take

$K_0/K = 16.12$ and 15.15 cm. respectively and extrapolate according to the equation

$$\frac{1}{K_0} = \log_e \frac{d_0 + \sqrt{d_0^2 - b_0^2}}{b_0},$$

where $2d_0 = y$ and $b_0 = 2.5$ cm. we get K_0/K to be 13.69 and 13.24 cm.

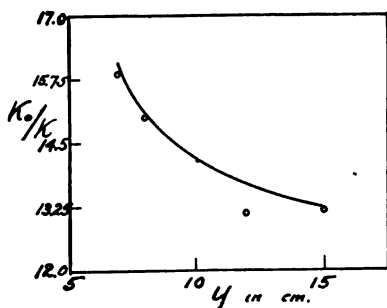


Fig. 15.

for $y = 12$ and 15 cm. respectively. The figures do not differ much from the figures for K_0/K in Table I. for $s = 3$ which presumably are more accurate than the extrapolated figures for $s = 1$. The curve of Fig. 15 is the theoretical curve; the points shown are the experimental values of K_0/K for $s = 3$ taken from Table I.

A research is now in progress to determine the change in the capacity of a condenser with distance between the plates face on (*i. e.*, changing x).

SUMMARY.

We have shown:

1. That the sharpness of the wave-peaks of a Lecher system depends very much upon the induction coil used as a source of power, the best effects being obtained with a coil giving a fat spark.
2. That the peak intensity is directly proportional to the current strength.
3. That the tone intensity is dependent upon the edge-on distance between the capacity plates, there being for any given tone an optimum edge-on distance.
4. That the logarithmic law for the capacity per unit length of two parallel wires also holds for the change in capacity of a condenser as the distance between it and an equal condenser is varied edge-on.

We are indebted to Mr. Bennett H. Jackson for assistance in taking the observations and to the University for financial assistance.

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THE COMPARISON OF A CERTAIN CASE OF THE ELASTIC CURVE WITH ITS APPROXIMATION.

BY R. W. BURGESS.

IN the usual discussion of the elastic curve in elementary textbooks of physics and engineering, a certain term in the differential equation of the curve is neglected and the curve resulting from this approximation is said to be a good substitute for the real elastic curve. It is my purpose in this note to show by a comparison of one case of this approximate solution with the accurate solution, that the one is not always a satisfactory substitute for the other.

If a straight thin rod in which I is the moment of inertia of a cross-section about a line perpendicular to the plane of bending, and E the constant of elasticity, is bent into a bow by two opposing forces each of magnitude H , acting at the ends of the rod, it is assumed, or deduced from more elementary assumptions, that the resistance to bending at any point is proportional to the curvature of the central axis at that point. The bending moment at any point is therefore proportional to the curvature; taking the central axis when unbent as x -axis, and a perpendicular at its mid point as y -axis, we have $Hy = EI/\rho$, that is, $\rho y = EI/H = a^2$, say, as the equation of the central axis. We can without difficulty integrate this equation, subject to the conditions $y = h$, $dy/dx = 0$ when $x = 0$. Substituting the known value of the radius of curvature in the equation $\rho y = a^2$, we obtain

$$a^2 \frac{d^2 y}{dx^2} = -y \left[1 + \left(\frac{dy}{dx} \right)^2 \right]^{\frac{3}{2}}.$$

As usual, put $dy/dx = p$; then

$$\frac{d^2 y}{dx^2} = \frac{dp}{dx} = p \frac{dp}{dy},$$

and therefore

$$\frac{2a^2 p \frac{dp}{dy}}{(1 + p^2)^{\frac{3}{2}}} = -2y.$$

Integrating, remembering that $p = 0$ when $y = h$, we have

$$\frac{2a^2}{(1 + p^2)^{\frac{1}{2}}} = y^2 - h^2 + 2a^2,$$

and hence

$$p^2 = \frac{4a^4}{(y^2 - h^2 + 2a^2)^2} - 1.$$

The length from the top point $y = h$ to the end $y = 0$ is therefore

$$\begin{aligned} \int_h^0 \sqrt{1 + \left(\frac{dx}{dy}\right)^2} dy &= \int_h^0 \frac{\sqrt{1 + p^2}}{p} dy \\ &= \int_h^0 \frac{2a^2}{y^2 - h^2 + 2a^2} \frac{y^2 - h^2 + 2a^2}{\sqrt{4a^4 - (y^2 - h^2 + 2a^2)^2}} dy \\ &= 2a^2 \int_0^h \frac{dy}{\sqrt{2a^2 - (y^2 - h^2 + 2a^2)} \sqrt{2a^2 + (y^2 - h^2 + 2a^2)}}. \end{aligned}$$

Now put $y = h \cos \phi$, and the length from end to end is seen to be

$$\begin{aligned} &= 4a^2 \int_0^{\pi/2} \frac{h \sin \phi d\phi}{h \sin \phi \sqrt{4a^2 - h^2 \sin^2 \phi}} \\ &= 2a \int_0^{\pi/2} \frac{d\phi}{\sqrt{1 - \frac{h^2}{4a^2} \sin^2 \phi}} = 2aK, \end{aligned}$$

where K is the complete elliptic integral of the first kind, with $k = h/2a$.

Using the expansion for this integral, as given for instance in Pierce's Shorter Tables, formula 265, or expanding by the binomial theorem and integrating, we get at once

$$l = \pi a \left[1 + \frac{h^2}{16a^2} + \dots \right].$$

The usual approximation is to omit from the original equation the term $(dy/dx)^2$, which is small if the bow is only slightly bent. The equation is then

$$a^2 \frac{d^2 y}{dx^2} = -y,$$

and the solution is $y = h \cos x/a$, which is said to be a good approximation to the solution.¹ If this equation is considered correct, the length of the bow will be

$$l' = 2 \int_0^{(\pi a)/2} \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx = 2 \int_0^{(\pi a)/2} \sqrt{1 + \frac{h^2}{a^2} \sin^2 x/a} dx.$$

Put $x/a = \pi/2 - \theta$,

$$l' = 2a \int_0^{\pi/2} \sqrt{\frac{a^2 + h^2}{a^2} - \frac{h^2}{a^2} \sin^2 \theta} d\theta$$

¹ See Love's Elasticity, p. 388, or the Encyclo. Math. Weiss., IV., 4, p. 211.

$$\begin{aligned}
 &= 2 \sqrt{a^2 + h^2} \int_0^{\pi/2} \sqrt{1 - \frac{h^2}{a^2 + h^2} \sin^2 \theta} d\theta \\
 &= 2 \sqrt{a^2 + h^2} E.
 \end{aligned}$$

Using Pierce, formula 266, or expanding by the binomial theorem and integrating, we have

$$\begin{aligned}
 l' &= \pi \sqrt{a^2 + h^2} \left[1 - \frac{1}{4} \frac{h^2}{a^2 + h^2} \dots \right] \\
 &= \pi a \left(1 + \frac{1}{2} \frac{h^2}{a^2} \right) \left(1 - \frac{1}{4} \frac{h^2}{a^2} \dots \right) \\
 &= \pi a \left(1 + \frac{1}{4} \frac{h^2}{a^2} \dots \right).
 \end{aligned}$$

In the problem of a bent bow, or of a column supporting a weight, the force (*i. e.*, a) and the length l are really the given constants. Our solution should tell us what the deflection is under these conditions. Using the value found for the length of the elastic curve, we find that the deflection h is approximately $4a \sqrt{(l/\pi a) - 1}$, and the error of this approximation grows smaller as the deflection decreases. According to the cosine curve, however, the deflection h' is about $2a \sqrt{(l/\pi a) - 1}$; that is to say, the deflection deduced from the approximate solution usually considered satisfactory is about one half of the true value and becomes more and more nearly one-half as the deflection grows smaller. I shall show this fact more carefully by obtaining explicit expressions for the two deflections h and h' and working out a few numerical cases.

For the elastic curve, we have

$$\begin{aligned}
 l &= \pi a \left[1 + \left(\frac{1}{2} \right)^2 \left(\frac{h}{2a} \right)^2 + \left(\frac{1 \cdot 3}{2 \cdot 4} \right)^2 \left(\frac{h}{2a} \right)^4 \dots \right] \\
 \frac{l}{\pi a} - 1 &= \frac{1}{4} \left(\frac{h}{2a} \right)^2 + \left(\frac{1 \cdot 3}{2 \cdot 4} \right)^2 \left(\frac{h}{2a} \right)^4 \dots.
 \end{aligned}$$

Now assume

$$\left(\frac{h}{2a} \right)^2 = a_1 \left(\frac{l}{\pi a} - 1 \right) + a_2 \left(\frac{l}{\pi a} - 1 \right)^2 + a_3 \left(\frac{l}{\pi a} - 1 \right)^3 + \dots;$$

substitute the value of $(l/\pi a) - 1$ from the preceding equation, and equate coefficients of like powers of $[(l/\pi a) - 1]$. We thus determine the series

$$\begin{aligned}
 \left(\frac{h}{2a} \right)^2 &= 4 \left(\frac{l}{\pi a} - 1 \right) - 9 \left(\frac{l}{\pi a} - 1 \right)^2 + \frac{31}{2} \left(\frac{l}{\pi a} - 1 \right)^3 \\
 &\quad - \frac{185}{8} \left(\frac{l}{\pi a} - 1 \right)^4 + \frac{507}{16} \left(\frac{l}{\pi a} - 1 \right)^5 \dots.
 \end{aligned}$$

By applying the reasoning of Chrystal's Algebra, Vol. II., pp. 350-353, we may show that this converges at least when $[(l/\pi a) - 1] \leq \frac{1}{4}$.

Similarly for the cosine curve, we have

$$l' = \pi \sqrt{a^2 + h'^2} \left[1 - \left(\frac{1}{2}\right)^2 \frac{h'^2}{a^2 + h'^2} - \frac{1}{3} \cdot \left(\frac{1 \cdot 3}{2 \cdot 4}\right)^2 \left(\frac{h'^2}{a^2 + h'^2}\right)^2 \dots \right].$$

Hence

$$\begin{aligned} \frac{l'}{\pi a} &= \left[1 + \frac{1}{2} \frac{h'^2}{a^2} - \frac{1}{8} \frac{h'^4}{a^4} + \frac{1}{16} \frac{h'^6}{a^6} - \dots \right] \\ &\times \left[1 - \left(\frac{1}{2}\right)^2 \frac{h'^2}{a^2} \left(1 - \frac{h'^2}{a^2} + \frac{h'^4}{a^4} \dots \right) - \left(\frac{3}{8}\right)^2 \frac{h'^4}{3a^4} \left(1 - \frac{2h'^2}{a^2} \dots \right) \right. \\ &\quad \left. - \left(\frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6}\right)^2 \frac{h'^6}{5a^6} - \dots \right] \end{aligned}$$

and

$$\begin{aligned} \frac{l'}{\pi a} - 1 &= \frac{1}{4} \frac{h'^2}{a^2} - \frac{3}{64} \frac{h'^4}{a^4} + \frac{5}{256} \frac{h'^6}{a^6} \dots \\ &= \left(\frac{h'}{2a}\right)^2 - \frac{3}{4} \left(\frac{h'}{2a}\right)^4 + \frac{5}{4} \left(\frac{h'}{2a}\right)^6 \dots \end{aligned}$$

As before, we assume

$$\left(\frac{h'}{2a}\right)^2 = a_1 \left(\frac{l'}{\pi a} - 1\right) + a_2 \left(\frac{l'}{\pi a} - 1\right)^2 + a_3 \left(\frac{l'}{\pi a} - 1\right)^3 \dots$$

and substitute the value of $(l'/\pi a) - 1$ as given above.

Determining the coefficients as before, we obtain

$$\left(\frac{h'}{2a}\right)^2 = \left(\frac{l'}{\pi a} - 1\right) + \frac{3}{4} \left(\frac{l'}{\pi a} - 1\right)^2 - \frac{1}{8} \left(\frac{l'}{\pi a} - 1\right)^3 \dots$$

We have, then, two convenient series for h and h' , as $(l'/\pi a) - 1$ must be small, that is the force must be not much greater than it is when bending begins to occur. This happens when $l = \pi a = \pi \sqrt{EI/H}$, i. e., $H = \pi^2 EI/l^2$, Euler's column formula. As convenient numerical illustrations, then, we take $l = l' = \pi$, and a successively equal to $1/1.0001$, $1/1.001$, $1/1.01$, $1/1.05$ and $1/1.1$. It will be noticed that the last value of a gives a rather large deflection, beyond the point where the cosine curve would be considered a good substitute by any one.

We see that for small deflections, the error of the usual solution is nearly 100 per cent. It seems to me, therefore, that the cosine curve $y = h \cos x/a$, with h determined by a formula deduced from this equation, is not satisfactory as an approximate solution, in any case where the length of the bow or column is one of the given physical constants.

l .	a .	k (Deflection Elastic Curve).	k' (Deflection Cosine Curve).	k/k' .	k/l .
3.1416	$\frac{1}{1.0001}$	.03999	.019999	1.9994	.0127
"	$\frac{1}{1.001}$	0.1263	.06321	1.9981	.0402
"	$\frac{1}{1.01}$	0.3915	0.1988	1.969	.125
"	$\frac{1}{1.05}$	0.8065	0.4337	1.86	.257
"	$\frac{1}{1.1}$	1.034	0.596	1.73	.329

The difficulty is of course due to the fact that the deflection is of the same order of magnitude as the slope, which was considered negligible. Errors of this nature, due to dropping terms from a differential equation, probably exist in other physical problems; this one is unusual in that both the exact and the approximate equations can be solved in terms of known functions.

THE EFFECT OF PREVIOUS FILTERING UPON THE ABSORPTION COEFFICIENTS OF HIGH FREQUENCY X-RAYS.

BY S. J. ALLEN AND L. M. ALEXANDER.

A GREAT deal of interest has been created by the work of Rutherford and Barnes on the absorption of the high frequency X-rays from a Coolidge tube at voltages over 100,000. In that work only the so-called "end rays" were used, the great remainder being cut out by previous filtering. It was thought that in this way, the softer components being cut out first, the very small remainder would consist almost wholly of the most penetrating components of the heterogeneous bundle from the tube. They found minimum absorption coefficients for aluminium (.144) and lead (2.06) which did not change for voltages of 120,000 to 175,000. This result was contrary to the existing theory as to the relation between potential and frequency. This method of obtaining the absorption coefficients of the "end rays" is open to question, since in any case the coefficients so obtained only represent a certain "average," and not the hardest components.

Another very important point to remember is, that if the substance used as a filter has excited in it a characteristic secondary radiation, the emergent rays will on the average be softer than otherwise, since there will be superimposed on the primary rays a more or less large proportion

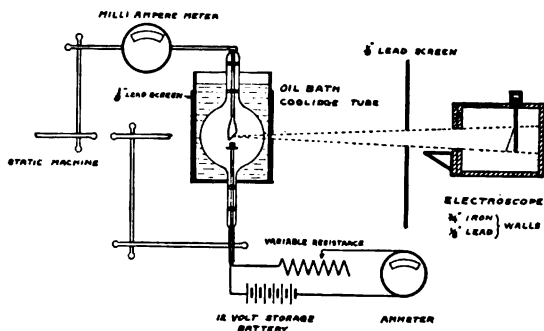


Fig. 1.

of softer secondary rays. If the characteristic radiation is very soft, compared to the primary it will have small effect, but if it is just a little

softer it will have a very large effect. There is every reason to believe that the high atomic weight elements have *K* radiations whose hardness is comparable to that of the high frequency primary rays used by Barnes and Rutherford, and that their end rays contained in the case of lead filtering a large proportion of lead characteristic *K* radiation. The present work of the authors' will bring this point out clearly.

The experimental arrangement is simple and is shown sketched in Fig. 1. The Coolidge tube was mounted in an oil bath to prevent danger of puncture. The walls of the glass vessel containing the oil were covered with $\frac{1}{8}$ in. of lead. The primary rays thus had to pass through about 2 in. of oil and $\frac{1}{2}$ in. of glass, before reaching the filter, which would remove partially some of the softer components.

The tube was excited by a large static machine, driven by an electric motor. The conductors were heavy brass tubes to prevent brush discharges.

The current through the heating coil was kept as constant as possible by a variable carbon resistance, controlled by a long insulated handle.

TABLE I.
 λ/ρ for Various Fillers.

Absorber and Atomic Number.	Filter, 12-inch Spark Gap.							Filter, 9-inch Spark Gap.	
	None.	Al.	Fe.	Sn.	Pt.	Pb.	Bi.	Pb.	Sn.
Carbon.....6	..	..	..	..	..	.12	..	.12	..
Aluminium.....13	.20	.158	.134	.12	.16	.16	.18	.16	.15
Iron.....26	..	..	..	.19	..	.48	..	.45	..
Nickel.....28	.85	.38	.33	.25	.40	.56	.55	.55	..
Copper.....29	..	..	..	..	..	.63	..	..	..
Zinc.....30	1.04	..	..	..	..	.65	..	.65	..
Silver.....47	..	1.38	.92	.87	1.47	1.80	..	2.1	..
Tin.....50	..	1.40	.96	.95	..	2.00	1.93	3.1	2.5
Tungsten.....74	4.50	..	2.42	2.10	1.87	3.60	..	..	..
Platinum.....78	4.70	3.67	3.15	2.40	..	3.50	..	3.9	4.3
Gold.....79	..	..	..	2.30	..	3.50	..	..	..
Lead.....82	3.95	3.07	2.83	2.40	2.64	2.30	2.91	2.2	3.2
Bismuth.....83	4.10	2.90	2.93	2.80	2.78	2.60	..	2.4	3.3

It was found that after about 20 minutes the static machine and conductors reached a constant voltage, and for several hours thereafter the tube gave quite steady readings. During a run the tube was never stopped, but continuous readings of the electroscope taken. A large number of readings were taken and all discordant ones discarded.

The electroscope consisted of an iron cylinder with its axis horizontal and parallel to the path of the rays. The walls were $\frac{3}{4}$ in. thick and the

end nearest to the tube 2 in. thick. A window to receive the rays was covered with a sheet of aluminium $\frac{1}{8}$ in. thick.

The walls were in addition covered with $\frac{1}{8}$ inch of lead. A lead screen placed between the tube and the electroscope, contained a window at which was placed the filter to be studied. The absorptive screens were placed directly in front of the window of the electroscope.

In all cases the filtering screen was made of sufficient thickness to

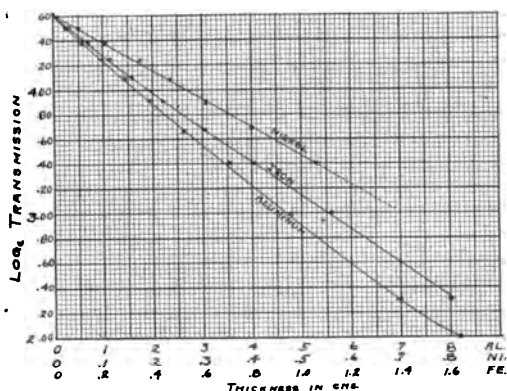


Fig. 2.

bring the initial ionization in the electroscope to the same value I_0 . If I_d is the ionization after passing through a thickness d , then the absorption coefficient

$$\lambda = \frac{\log_e I_0 - \log_e I_d}{d}.$$

The mass absorption coefficient = λ/ρ , where ρ = density.

The results with various filters and absorbers are given in Table I. Two sets are tabulated, one at 12-in. spark gap (about 120,000 volts), and the other at 9-in. spark gap (about 100,000 volts).

In Figs. 2 and 3 are plotted the absorption curves of Al, Fe, Ni, Pt, Au, and Pb, all tin filtered.

In Fig. 4 the values of λ/ρ , for tin, and lead filtered rays are plotted as a function of the atomic numbers.

For the purpose of bringing out the main characteristics of the results in Table I., it will be sufficient to discuss the values for tin, and lead filtered rays. It will be noticed that the values of λ/ρ for the tin filtered are with the exception of Pb and Bi much smaller than the corresponding ones for lead filtered. On the other hand the values of λ/ρ for Pb and Bi, filtered through lead are smaller than those filtered through tin. This is illustrated very clearly by the two curves in Fig. 4, the lead filtered

curve lying well above that of the tin filtered, except for lead and bismuth.

The results for the other filters fall into two classes, those less in atomic weight than tin behaving like tin, and those greater in atomic weight than platinum behaving like lead.

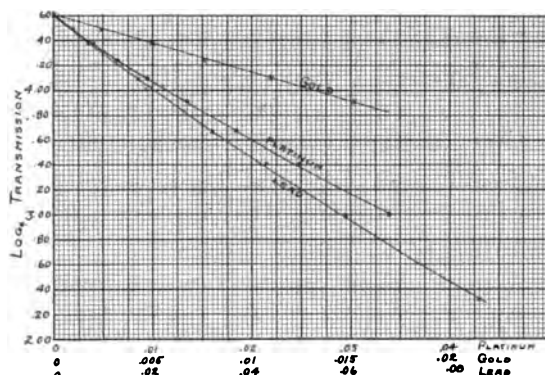


Fig. 3.

In other words, the rays after being filtered through Pt, Pb, and Bi, are softer than when filtered through Al, Fe, and Sn.

If we compare the results for 9-in. spark gap with those for 12-in.

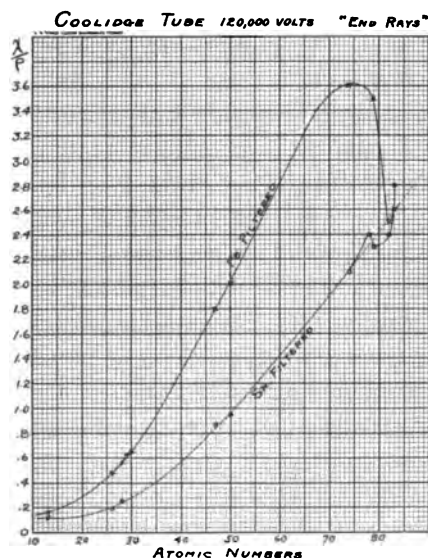


Fig. 4.

spark gap, we see that with a lead filter the rays are apparently not hardened appreciably in increasing the potential, but with the tin filter they show a decided hardening.

The explanation of these peculiarities is clear on the view expressed above. The primary rays in passing through the filters excite in them the characteristic K radiations which are softer than the primary, and therefore the emergent beam will consist in part of a hardened primary and in part of a softened secondary. As the filtering increases the proportion of secondary present will increase in the case of the high atomic weight atoms like lead, but will decrease in the case of lower atomic weight atoms like tin. For low atomic weight atoms like iron or aluminium, any K radiation if present would be too soft to have any effect. As a consequence the results filtered through tin are very much nearer the true values than those filtered through Pt, Pb or Bi.

It is evident from the above results that at a potential of 100,000 to 120,000 volts there is excited in Pt, Pb, and Bi their characteristic K radiations.

The values of λ/ρ for tin filtered indicate the hardest X-rays yet known. The value for aluminium (.12) is considerably smaller than the corresponding one (.144) given by Rutherford and Barnes. We think there is no reason to doubt that if the potential is raised still higher, harder rays can be produced even from a tungsten target.

The absorption curves for tin filtered are practically straight lines, but for lead filtered they show considerable curvature. This shows that the "end rays" for tin are nearly homogeneous, whilst for lead they are not, but contain some softer rays.

The fact that the values of λ/ρ for Pb, when filtered through lead are smaller than when filtered through tin is in accord with the well-known principle that an absorber of the same nature as the radiator always gives a smaller value of λ/ρ when in the neighborhood of the characteristic frequency of that substance.

The secondary rays from the various metals were also observed by direct reflection methods. The metal plate was placed in a horizontal plane and the primary rays projected on it at an angle of 45° . The electroscope was placed so as to receive the secondary rays at an angle also of 45° . The electroscope was carefully shielded by thick screens to prevent any of the primary rays from entering.

The amount of secondary radiation observed was very small in comparison with the primary, and a large proportion of it came from the air in the neighborhood of the electroscope. This amount was subtracted from the total to give the amount due to the radiator. The results are given in the following Table II.

Only a few results are shown for the higher potential as the Coolidge tube broke down at this point, and no measurements could be made on the absorption. Values of λ/ρ are given for the 8-in. spark gap.

TABLE II.

Reflector.	Relative Intensity.		λ/ρ , 8-in. Spark.		
	8-in. Spark.	12-in. Spark.	Al.	Sn.	Pb.
Carbon.....	114				
Aluminium.....	100	100	.25	7.4	5.3
Iron.....		57			
Nickel.....	41	41	.41	3.7	4.4
Copper.....	36				
Zinc.....	30				
Silver.....	100				
Tin.....	170	118	2.0	11.8	
Antimony.....	200				
Platinum.....	85				
Gold.....	76				
Lead.....	62		.38	4.0	2.4
Bismuth.....	45				

The results for intensity are given relative to aluminium in per cent. These values include the scattered radiation as well as the possible characteristic *K* radiation. All elements below silver will have only scattered radiation. It is seen that the low atomic weight atoms scatter more than the high, reaching a minimum about zinc. From silver up there is a steady and rapid increase, followed afterwards by a rapid decrease to bismuth. The absorption coefficients for tin show clearly the presence of the *K* radiation for that element. If the complete values for the 12-in. spark gap could have been obtained they would have undoubtedly shown the maximum displaced further to the high atomic weight atoms. All these results were taken without any previous filtering except through the glass walls and surrounding oil, thus allowing a large proportion of softer rays to act. If the rays had been filtered there would not have been enough secondary rays to measure, with the current through the tube limited to that given by the static machine.

The results shown in this paper can be briefly summarized as follows:

1. The absorption coefficients have been measured for the high frequency X-rays from a Coolidge tube after filtering through various elements, at a voltage of 120,000.

2. These absorption coefficients show clearly the presence of the *K* radiation of platinum, gold, lead and bismuth.

3. With a tin filter the values of the absorption coefficients for the "end rays" show harder rays than any that have been published up to this time, the value of λ/ρ for aluminium being .12.

4. Experiments on the scattering have been made, showing that the low atomic weight atoms scatter more than the high.

Addenda.—In an abstract of a paper presented before the American Physical Society, April, 1916, and published in the *PHYSICAL REVIEW*, Sept., 1916, Hull and Rice give values of the absorption coefficients of the high frequency X-rays from a Coolidge tube reflected from a rock-salt crystal. They clearly show the *K* characteristic of lead, which begins at a wave-length of 0.149 Å.U. At a wave-length of 0.147 Å.U., the value of λ/ρ for Al is 0.154, for Cu is 0.71 and for Pb is 1.50. At a wave-length of 0.122 Å.U. λ/ρ for Pb is 3.00. These values are in close agreement with those of the authors, which are of course the average over a short range of wave-lengths.

A CALORIMETRIC RESISTANCE THERMOMETER.

BY S. LEROY BROWN.

THE variation in the electrical resistance of metallic wires with a change in temperature affords an accurate means of measuring temperatures and temperature differences. Nearly all the development of the resistance thermometer has been confined, however, to platinum as the resistance element. Callender and Griffiths' improved methods of measurements and the work at the Bureau of Standards at Washington has made the platinum resistance thermometer a valuable instrument for moderate and high temperatures. The late work of H. C. Dickenson and E. F. Mueller¹ has produced a very sensitive calorimetric resistance thermometer in which the resistance element consists of about 10 cm. of fine (.02-mm.) platinum wire wound in a flat coil. This thermometer, when accurately calibrated, may be used to reproduce the hydrogen scale between 0 and 100° C. to within .002 or .003° C.

In a previous paper,² the author has pointed out the possibilities of utilizing the high temperature coefficient of resistance of the metallic oxides to make resistance thermometers. The paper referred to described oxide resistance thermometers made by clamping the solid oxide elements between the connecting wires and gave data which shows that such a thermometer is very sensitive and reliable as long as the contact resistance at junction with connecting wires is unaltered.

The present investigation has to do with an oxide resistance thermometer made by thrusting two wires into a globule of the molten oxide and allowing it to cool so that only a thin layer of the solid oxide separates the wires. Some difficulty may be experienced by the oxide cracking on account of unequal expansion of the oxide and connecting wires. Lead oxide is easily melted in a gas flame and, when fused with iron wires as described above, makes a reliable connection which is permanent when subjected to temperatures ranging from - 10° C. to 120° C. The resistance of a thermometer made as described may range from a few hundred ohms to several thousand ohms at room temperatures. The iron wires are soldered to larger copper wires near the oxide so that the

¹ Bulletin of Bureau of Standards, Vol. 3, No. 4.

² PHYSICAL REVIEW, February, 1915.

variation of resistance of the leads due to variable immersion is negligible and compensating leads are not necessary. The heat capacity is very small since the globule of lead oxide need not be larger than the head of a pin. The data given in this paper were taken with the thermometer proper enclosed in a tubular glass sheath but a thin metal sheath would allow the resistance to assume more quickly the surrounding temperature. All junctions, oxide to iron and iron to copper, are near together, thereby eliminating the possibility of unequal heating producing thermal electromotive forces.

The electrical resistance of the oxides decreases as the current through them is increased and therefore the oxide resistance thermometer must be excited with a constant current or so nearly constant that the variation in the current does not cause an appreciable change in the electrical resistance. For example, from curve 2 in Fig. 2 a change of $.005^{\circ}\text{C.}$ produces a resistance change of about $.01\text{ ohm}$ at room temperature. According to curve (29.3°C.) in Fig. 3, a change of current equal to 2.5 microamperes will produce about $.01\text{ ohm}$ change in resistance when the exciting current is one milliampere; or, a change in current of 2.5 microamperes will produce no greater change in resistance than $.005^{\circ}\text{C.}$ change in temperature under these conditions.

The plan of accurately controlling the strength of the exciting current is shown by reference to Fig. 1. The electromotive force across ab is

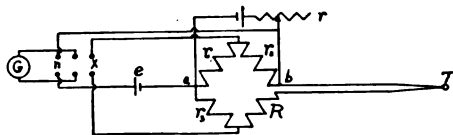


Fig. 1.

maintained equal to the electromotive force of the standard cell e by adjusting the variable resistance r and this equality is determined by throwing the galvanometer connections to n and balancing the electromotive force across ab against the electromotive force of the standard cell. The resistance from a to b is maintained constant; any increase or decrease of the resistance of thermometer T is compensated for by a decrease or increase at R . The bridge arms r_1 , r_2 and r_3 were so chosen that one milliampere of current flowed through the thermometer T while the electromotive force across the bridge was maintained equal to the electromotive force of a standard Weston cell.

The data in Table I. are shown graphically by the several curves in Fig. 2, each curve extending over a temperature range of about 30°C. , excepting no. 6 for which the temperature range is only from 25 to 28°C.

TABLE I.
Lead Oxide Resistance Thermometer.

Temperature.	Resistance.	Temperature.	Resistance.	Temperature.	Resistance.
-2.45° C.	247.76	25.25° C.	168.28	70.15° C.	101.21
-2.02	245.91	27.90	163.16	74.50	96.48
-1.30	242.91	28.80	161.46	82.30	89.36
-.55	239.66	29.30	160.38	88.00	84.81
0	237.61	30.80	157.42	92.25	81.66
2.40	227.78	32.384	154.26	99.25	76.98
4.30	221.06	35.75	148.33	99.36	76.91
6.72	213.16	45.50	132.18	108.50	71.66
10.00	203.66	51.40	124.04	112.85	69.46
15.55	190.10	58.95	114.48	120.00	66.36
19.67	180.51	66.00	105.91		

The steepness of these curves indicates the sensitiveness of the oxide resistance to temperature variations.

Observations 15-20 were taken by alternately subjecting the thermometer to the temperature of steam and ice. This thermometer has been used by students from time to time during the last year, has been subjected to as much mechanical strain as any other thermometer must

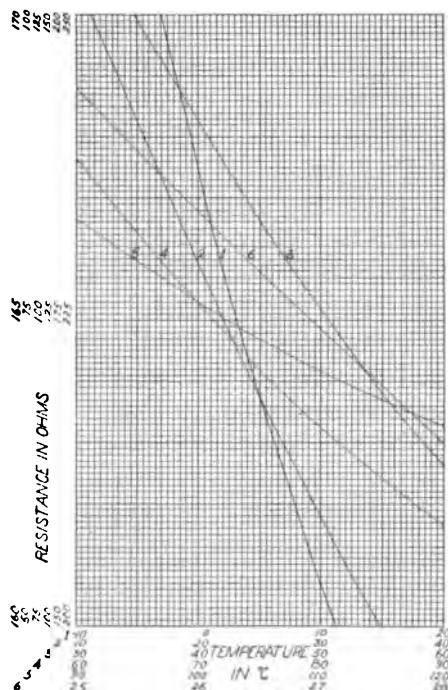


Fig. 2.

TABLE II.

Data Showing Permanency and Reproducibility of Lead Oxide Resistance Thermometer.

No.	Date.	Temp. °C.	Resistance.
1	Dec., 1915.....	99.30° C.	76.95
2	Dec., 1915.....	99.25	77.00
3	Dec., 1915.....	99.45	76.85
4	Dec., 1915.....	0	237.65
5	May, 1916.....	99.35	76.91
6	May, 1916.....	99.33	76.93
7	May, 1916.....	99.40	76.88
8	May, 1916.....	0	237.63
9	Sept., 1916.....	99.20	77.01
10	Sept., 1916.....	99.36	76.91
11	Sept., 1916.....	99.25	76.98
12	Sept., 1916.....	0	237.61
13	Nov., 1916.....	99.68	76.70
14	Nov., 1916.....	99.50	76.82
15	Nov., 1916.....	99.31	76.95
16	Nov., 1916.....	0	237.60
17	Nov., 1916.....	99.31	76.95
18	Nov., 1916.....	0	237.58
19	Nov., 1916.....	99.31	76.95
20	Nov., 1916.....	0	237.62

withstand, and has been subjected to temperatures ranging from $-10^{\circ}\text{C}.$ to $120^{\circ}\text{C}.$ Other lead oxide resistance elements have been heated to 200 or $250^{\circ}\text{C}.$ but a permanent change in their resistance was produced.

Fig. 3 shows the variation in the resistances at various temperatures with a change in the exciting current. The current variations which are permissible at the various temperatures in order that the error produced in the temperature measurement may be negligible must be obtained from the slopes of these curves. A variation of the current, at one milliamperere and near zero $^{\circ}\text{C}.$, not exceeding 1 point in 1,000 would produce an error not to exceed $.003^{\circ}\text{C}.$, and a variation of current at one milliamperere and near $100^{\circ}\text{C}.$, not exceeding 1 point in 1,000 would produce an error not to exceed $.0003^{\circ}\text{C}.$

The conductivity of the oxide elements made as described above may be decidedly asymmetric. In general the high resistance elements will have a quite different resistance depending on the direction of the current through them. The lead oxide resistance thermometer with which this data was taken showed very little difference in conductivity depending on direction of current. At room temperature, there was a difference of about .1 ohm when one milliamperere was reversed, .2 ohm when 2 milliampere and .3 ohm when 3 milliampere were reversed. The data were taken for the direction of current which gave the lowest resistance.

A change from very dry to very moist air surrounding the resistance element and a change of pressure of several centimeters produced no measurable change in its resistance. In all probability the greater part

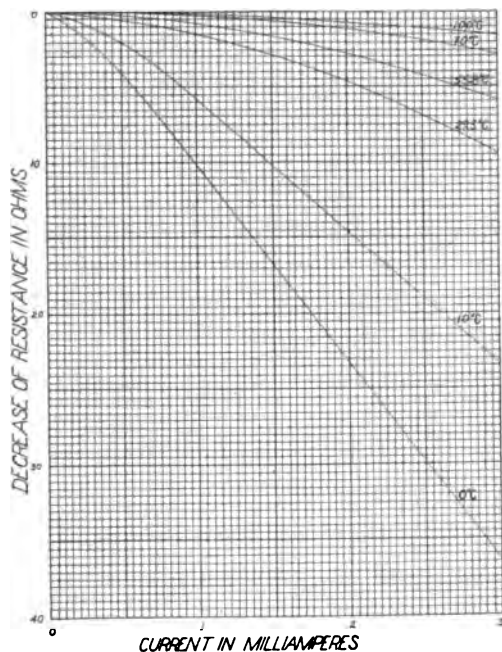


Fig. 3.

of the current was through a thin section of the oxide well inside the protecting shell of hard oxide and, therefore, external changes of moisture and pressure would not be effective.

SUMMARY.

1. A sensitive resistance thermometer for calorimetric work can easily be made which utilizes the high temperature coefficient of resistance of lead oxide.

2. This thermometer does not require compensating leads, has low heat capacity, and since the junctions are close together, unequal heating cannot produce thermal electromotive forces.

3. Temperature measurements can be reproduced with the oxide resistance thermometer to within a few thousandths of a degree between 0°C. and 100°C. The principle disadvantage as compared to the platinum resistance thermometer is that there is no simple relation between resistance and temperature. The calibration must be point by point comparison with a standard and the three fixed points, transition temperature of sodium sulphate, ice point, and steam point.

PHOTO-ELECTRIC POTENTIALS FOR EXTREMELY SHORT WAVE-LENGTHS.

BY P. E. SABINE.

THE following is a preliminary report of an investigation carried on in the Jefferson Physical Laboratory to determine the maximum photo-electric potential produced by light in the Schumann region of the spectrum as a function of the frequency.

Professor Millikan's work upon the voltage-frequency relationship in the case of the alkali metals has established the validity of the Einstein formula

$$V = \frac{h}{e} \nu - V_0$$

as a very close approximation of the facts for these metals. The results of Kadesh and Hennings¹ using the same method upon other metals give a value of h differing from that of Millikan by an amount considerably greater than the percentage error given by the latter. It is therefore desirable to extend the range of frequencies available for photo-electric measurements upon the other metals.

Moreover, since the magnitude of photo-electric currents increases rapidly with decreasing wave-length, the Schumann region of the spectrum offers an attractive field for photo-electric investigations, while the photo-electric cell ought to prove a valuable aid in Schumann spectroscopy.

The results so far obtained may be summarized as follows:

1. A monochromatic illuminator for use in the region between λ 2300, and λ 1250 has been constructed and calibrated.
2. Voltage frequency measurements have been made upon a single metal, zinc, using light of wave-lengths between λ 1800 and λ 1250.
3. It has been shown that the slope of the voltage-frequency line is quite independent of the surface condition of the emitting metal, provided these conditions remain constant during the measurements.
4. A direct study of the effect of stray light of shorter wave-lengths has shown that it is a source of large error in voltage-frequency measurements.

¹ PHYSICAL REVIEW, Sept., 1916, p. 221.

5. The maximum potentials attained by plates of zinc, cadmium and copper illuminated by light of wave-length λ 1250 have been determined. These measurements, taken with those of other experimenters, for longer wave-lengths give values for the slope of the voltage-frequency line very close to h/e .

APPARATUS.

The monochromator is a modification of Professor Lyman's vacuum grating spectrometer. The essential point of difference lies in the fact that the grating is mounted so that it slides along the focal circle, thus focusing different portions of the spectrum upon the slot through which light shines onto the metal plate in the photo-electric cell. Fig. 1 shows

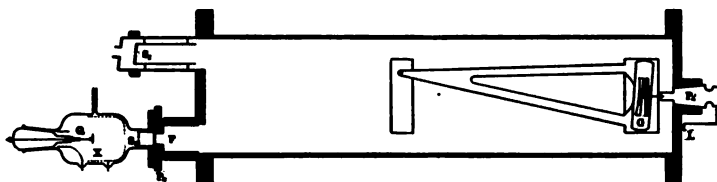


Fig. 1.

the general arrangement. The grating G is mounted upon a movable arm which is pivoted at the center of the focal circle. The slit S_1 and the slot S_2 are adjusted so as to lie upon the same circle. The grating is moved by means of a rack and pinion device. The rack is attached to the movable arm on which the grating is mounted. A small rod through the pinion is engaged by a slot in a sleeve at the end of the taper plug P_1 . The grating is shifted from point to point on the focal circle by turning P_1 . The position of the index I on a circular scale, attached to the outside of the face plate, determines the position of the grating upon the focal circle and hence the wave length that is incident upon the metal plate. The scale was calibrated by taking a series of photographs of the cadmium and aluminium spark spectra in the region between λ 2300, and λ 1854, and of the spectrum of the hydrogen discharge in the region from λ 1650 to λ 1280.

The photo-electric cell is cemented into the end of the conical plug P_2 . The cell is separated from the spectroscope by means of the fluorite window F . The receiving Faraday cylinder is of the usual type made of oxidized copper gauze. The illuminated plate X is a strip of metal 6×25 mm., placed well inside the receiving cylinder. It is mounted on the end of a light metal rod, sealed with Khotinski cement into the ground glass stopper which closes the cell. A guard ring Q prevents charges leaking from the receiving cylinder to the illuminated plate.

The latter is connected to a pair of quadrants of the electrometer. Both cell and spectroscope are exhausted by a Trimount Rotary Oil pump. The higher vacuum needed in the photo-electric cells is secured by means of charcoal and liquid air.

SOURCE OF LIGHT.

Considerable time has been spent in attempts to secure a suitable and convenient source of light. What is desired is a source, with a few strong lines well distributed in the region between λ 2300 and λ 1250, that shall be steady and require little attention. Such a combination of qualities has not yet been found in any one source. The metallic vapor arcs of zinc, calcium and cadmium enclosed in quartz have been tried. These spectra possess the requisite lines in this region, but so far it has not been possible to secure working conditions giving the necessary steadiness and reliability. Metallic arcs are difficult to start without heating. Moreover for continuous operation the pressure must be very low. Otherwise the metal soon oxidizes and the arc becomes unsteady. Without a fluorite window between the arc and the spectroscope a very slight leak in the latter soon produces this result. If a fluorite window is inserted, it shortly becomes covered with a film of the metallic vapor even when the arc is 15 cm. from the window and the space between is cooled by a water jacket around the lamp.

On the whole, the hydrogen discharge, while far from ideal, has proven the most satisfactory of the sources tried. It is objectionable in that, with a wide slit, its spectrum becomes practically continuous. Moreover when pure there is little or no radiation of wave-length longer than λ 1650. However, with a trace of air in the spectroscope, carbon bands appear in the region between λ 1650 and λ 2000, so that under these conditions a fairly wide range of frequencies is available. The discharge tube made of quartz was of the usual internal capillary type used by Lyman. It was designed with a view to securing the largest possible current.

The spectrometer case itself served as one electrode. The other electrode was attached to a taper plug of "invar" carefully ground into the quartz tube. The tube was water-jacketed and would carry a current of .2 ampere for 50 seconds without serious heating.

EXPERIMENTAL CONDITIONS.

Large photo-electric currents are desirable in determining maximum potentials. Although with the present form of apparatus only a small fraction of the energy of the source falls on the photo-electric plate, yet the effectiveness of these extremely short wave-lengths is so great that

fairly large currents were obtained. Thus for λ 1600, the most intense region of the hydrogen spectrum, with a slit .03 cm. wide a saturation current giving 10 mm. deflection per second was obtained. For λ 1250, currents one third to one half as great were produced under the same conditions. One millimeter deflection corresponded to .5 millivolt. For the most part electrical conditions were so steady that 1 mm. deflection in 50 seconds could be detected with certainty.

The "back current" due to light reflected from the illuminated plate to the walls of the cell was small. Generally such a current could not be detected at all even under a retarding potential one and a half volts greater than that required to produce zero current. In no case did it amount to more than .5 per cent. of the saturation current. This is probably to be explained by the low reflecting power of the emitting metals for light of short wave-lengths. Hulbert¹ has found that zinc and cadmium have small reflection coefficients in the extreme ultra-violet.

The error due to stray light from the grating, of wave-lengths shorter than that for which the illuminator was set, was found to be large. It will be discussed in a later section.

SURFACE CONDITIONS AND THE VOLTAGE-FREQUENCY RELATION.

No provision was made in these experiments for preparing surfaces in vacuo. Therefore, it is essential to know how the voltage-frequency relation depends upon the condition of the metal surface. The work of previous investigators seems to show that the only effect of changing surface conditions is a shift of the voltage-frequency line parallel to itself, without affecting its slope. For example Hughes² found that exposing a cadmium surface that had been distilled in vacuo, to the action of dry oxygen at a pressure of 14 cm. for 15 minutes, decreased the maximum potentials for all wave-length by the same amount.³ Millikan² also observed that the slope of the voltage-frequency line obtained with a sodium surface originally scraped in vacuo and then exposed to a pressure of .01 mm. agreed very closely with that obtained when the surface was scraped in the highest attainable vacuum and kept under this pressure while measurements were being made. To further test this point experiments were made using a zinc plate with widely different surface conditions. Table I. is typical of the results.

Surface I. had been scraped with a steel tool, and immediately placed in the photo-electric cell. The cell was exhausted in 15 minutes to .001 mm. and the pressure still further reduced by charcoal and liquid

¹ *Astrophysical Journal*, 42, No. 3, p. 205.

² *Phil. Trans. A.*, Vol. 212, pp. 205-226.

³ *Phys. Rev.*, March, 1916, p. 375.

TABLE I.

Surface.	Max. Volt. $\lambda 1280$.	Max. Volt. $\lambda 1600$.	δV .
I.....	5.44	4.32	1.12
II.....	4.64	3.51	1.13
III.....	4.92	3.79	1.13

air. Surface II. had been in the cell for four days at a pressure of .002 mm. and later exposed to the air for 15 minutes. Surface III. had been treated similarly to surface I., but had stood in the cell at low pressure for twenty-four hours. Other experiments in which the variation in the maximum voltage for a given frequency was not so great as those given, showed practically the same difference for the two wave-lengths, so that it seems certain that the change in maximum voltage for a given change in frequency is independent of the surface conditions of the metal, provided these conditions remain constant during the measurements.

SCATTERED LIGHT OF SHORT WAVE-LENGTH.

Millikan¹ has pointed out that the effect of stray light from the optical system giving the monochromatic illumination is in general to decrease the slope of the voltage-frequency curve, by raising the apparent potential due to the lower frequencies. The present attempt to determine the voltage frequency relation in the Schumann region of the spectrum bears out Millikan's suggestion in a somewhat striking manner. The results without color screens briefly stated are these. For wave-lengths $\lambda 1800$, $\lambda 1600$ and $\lambda 1250$ the voltage frequency curve was very accurately a straight line, with a slope however only a little more than one half as great as that called for by the Einstein formula. Table II. gives

TABLE II.

Wave-length.	Frequency.	Applied Voltage.	Max. Potential.
1800.....	1.66×10^{15}	3.82	4.88
1600.....	1.87×10^{15}	4.32	5.38
1250.....	2.40×10^{15}	5.44	6.50

the values obtained using a scraped surface of zinc. Other determinations with different surfaces gave virtually the same result so far as the slope of the line is concerned. The contact potential difference of 1.06 volts, between the zinc and the receiving cylinder was determined by measuring the positive potential applied to the latter that was necessary to produce the saturation current. These results are plotted ($\odot$) in

¹ PHYSICAL REV., March, 1916, p. 355.

Fig. 2. The results of Richardson and Compton for a similarly treated zinc surface are shown on the same graph. The slope of the line through the author's points in the Schumann region gives a value for h of 3.6×10^{-27} . That the potential measured for the two longer wave-lengths is larger than that corresponding to the maximum energy of

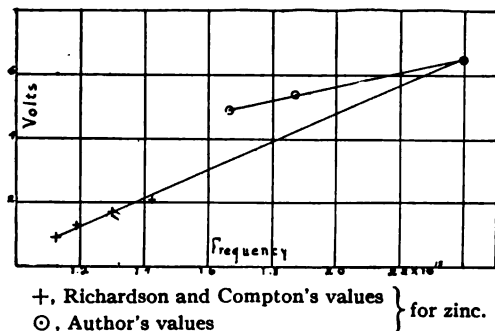


Fig. 2.

electron emission due to these wave-lengths was shown by the following experiment:

With the spectroscope so tight that the leak was less than .02 mm. in twenty-four hours, pure hydrogen was admitted. As has been stated, there are no lines in the Schumann region of the hydrogen spectrum of wave-length longer than about λ 1670. Examined spectroscopically the discharge showed no trace of carbon bands. Yet with the monochromator set for λ 1800, a photo-electric leak almost one tenth as great as that due to λ 1600 was observed. Applying retarding potentials, a voltage current curve was taken. This was found to coincide very closely with the curve due to λ 1600, the nearest and by far the most intense portion of the hydrogen spectrum, and to reach the axis of zero current at very nearly the same value of the retarding potential. In other words, one had the effect due to stray light alone, which in this case was found to be the same as the effect due to the strongest lines in the source of shorter wave-length than that for which the monochromator was set. It appears therefore that without color filters, the maximum potential determined is that at which this stray light effect becomes less than can be detected by the electrometer used and that with a sufficiently sensitive electrometer the voltage frequency line thus determined would be parallel to the frequency axis, at a voltage given by the shortest wave-length that is transmitted by the optical system employed. In the present experiment the trace of air in the spectroscope necessary to produce the radiation at λ 1800 served as a screen to reduce in a

measure the intensity of the stray light of shorter wave-lengths, thus giving a slope greater than zero and less than that due to the lines for which the monochromator was set.

It has not as yet been possible to undertake the task of finding specimens of quartz and colored fluorite to serve as color filters and of accurately determining the limits of their transparency. It has seemed worth while, however, to report this positive result concerning the effect of stray light, in view of the fact that this source of error must exist in all experiments in which color filters are not used.

MAXIMUM POTENTIAL FOR λ 1250.

The foregoing does not apply to the values obtained for the shortest wave-length used since the fluorite window between the cell and the spectroscope cuts out all wave-lengths shorter than λ 1250, the limit of transparency of fluorite. The maximum potential produced when the monochromator is set for this wave-length is therefore a measure of the kinetic energy of the fastest electrons liberated. Since this voltage is large compared with those produced by ordinary ultra-violet light, the percentage error in its determination is correspondingly small. The large range of frequencies between this and the long wave-length limit of most metals is sufficient to afford a good test of the validity of the Einstein formula. Careful determinations of the maximum potential acquired by zinc, cadmium and copper plates were made. In each case the contact potential difference was measured simultaneously by determining the positive potential applied to the receiving electrode necessary to produce saturation of the photo-electric current. The values are given in Table III.

TABLE III.

Metal.	App. Potential.	Contact Potential.	Max. Potential.	Mean.
Zinc.....	5.44	1.25	6.69	6.70
"	5.70	1.17	6.87	
"	5.16	1.35	6.51	
"	5.21	1.60	6.81	
"	5.00	1.63	6.63	
Cadmium.....	5.35	1.25	6.60	6.57
"	5.25	1.28	6.53	
Copper.....	6.05	.63	6.68	6.68
"	6.22	.47	6.69	

The two determinations for copper were made using the same surface, with an interval of about five hours between. Each of the other cases represents a fresh metallic surface. It is to be said that experimental

conditions were much more satisfactory when the measurements upon the cadmium and copper were made than when the earlier work with zinc was done. To make sure that surface conditions had not altered materially between taking the maximum and the contact potentials, the experimental procedure was as follows: First increasing accelerating potentials were applied and the upper part of the voltage current curve was taken to determine the applied voltage that just gave the saturation current. Immediately afterward retarding potentials were applied and the lower end of the current-voltage curve was taken to determine the applied potential just necessary to produce zero current. The saturation voltage was then again determined and the mean of the two values was assumed to be the contact potential difference between the emitting

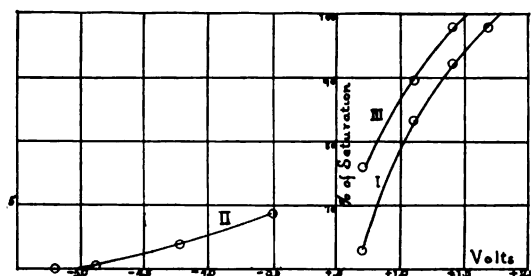


Fig. 3.

and receiving electrodes at the time the maximum potential was determined. The curves in Fig. 3 illustrate the method. They were taken in the order given. The shift of contact potential, .2 volt in the case illustrated was rather larger than usually occurred during a single set of measurements. The time between taking curves I. and III. was about two and a half hours.

The contact potential differences measured in this way with the metal surfaces in vacuo are in good agreement with values given by direct measurement. Thus the mean value for this difference between zinc and the walls of the copper oxide cylinder is 1.4. For copper against copper oxide, the mean C.P.D. is .55 volt. For zinc against copper we have .85 as compared with .84 volt determined directly by Hallwachs for fresh surfaces in air.

In order to use the results here obtained in connection with the longer wave-length determinations of previous observers one must take account of the contact potential difference between the emitting and receiving electrodes in these earlier experiments. Hughes in his work neglected this point entirely so that before using his values they must be corrected for the contact potential difference between the emitting metal and the

soot-blackened surface of the surrounding walls. One may get an approximate value for this correction as follows: Hallwachs¹ gives as the contact potential difference between zinc and lamp black 1.1 volts. For cadmium and lamp black this value would be .94. Hughes (l. c.) found that exposing his vacuum prepared surface of cadmium to the action of dry oxygen at a pressure of 160 mm. for 15 minutes lowered the maximum voltages for all wave-lengths by the same amount, .25 volt. Hence to compare Hughes's values with those for a surface prepared in air .25 volt must be subtracted. In Table IV. Hughes's values for cadmium corrected by adding .69 volt, *i. e.*, (.94 - .25) are given together with the value here obtained for λ 1250.

TABLE IV.

Wave-length.	Frequency.	Corrected Potential.	
2537.....	1.18×10^{15}	1.52	Hughes
2357.....	$1.33 \times "$	2.05	"
1849.....	$1.62 \times "$	3.13	"
1250.....	$2.40 \times "$	6.57	Author

These are plotted in Fig. 4. The straight line through the extreme points gives a value of h , equal to 6.58×10^{-27} .

In the case of zinc, the mean value of the maximum potential for λ 2537 obtained by Hughes is computed to be .70 volt. Making the

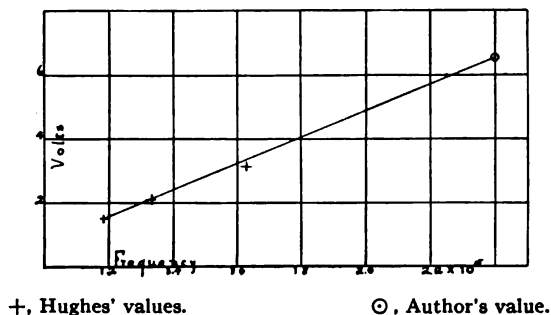


Fig. 4.

correction for contact potential and for the effect of air as in the case of cadmium, we have the potential which measures the maximum energy of electrons liberated by this wave-length

$$V = .70 + 1.1 - .25 = 1.55 \text{ volts.}$$

This with the maximum potential for λ 1250 gives

$$h = 6.71 \times 10^{-27}.$$

¹ Handbuch der Radiologie, Band III., p. 349.

The slope of the line in Fig. 2 in which Richardson and Compton's values are used, gives for the value of h , 7.23×10^{-27} .

It is scarcely possible to draw any conclusion from the data for the copper surface, since there are no very satisfactory figures for copper in the longer wave-length region. Richardson and Compton found that copper and bismuth gave unusually low values for the slope of the voltage frequency line. On the other hand, Hughes found that the slope of the bismuth line was very nearly the same as that for cadmium, while Kadesh and Hennings report that for a given frequency all of the metals examined by them including copper acquire practically the same maximum potential when emitting electrons to the same receiving electrode. The experiment with copper was tried only in view of Richardson and Compton's results, to ascertain whether the small slope of the voltage frequency line as determined by them for this metal showed itself in a correspondingly low voltage when the higher frequency was used. The results indicate that there is no reason for supposing that copper differs in any considerable degree from the other metals in the voltage frequency relation of the photo-electric effect.

Taken as a whole the values of the photo-electric potentials for λ 1250, when compared with the results of other investigators, while not affording any very accurate determination of the Planck constant, nevertheless give very good reason to believe that h/e is the slope of the voltage frequency line for zinc and cadmium. It is of course highly desirable that the measurements for both extremities of the available frequency range be made in a single set of experiments with proper precaution to eliminate the known large source of error due to stray light. The necessary changes in the present apparatus are being made for this purpose.

I acknowledge, with great pleasure, my obligation to Prof. Lyman, both for suggesting this research, and for his keen interest in its progress and constant help with the troublesome details of vacuum spectroscopy.

JEFFERSON PHYSICAL LABORATORY,
HARVARD UNIVERSITY,
September 1, 1916.

X-RAY EMISSIVITY AS A FUNCTION OF CATHODE
POTENTIAL.

BY DAVID L. WEBSTER.

IN the PHYSICAL REVIEW of June, 1916, I published an account of some experiments on X-rays of definite wave-lengths, characteristic and general, emitted by a rhodium target at definite potentials. One of the results of these experiments was to prove that as the potential was raised the general radiation of any given wave-length would appear first when the potential reached the value which would give a single cathode electron one quantum of that wave-length. This result was similar to that previously observed by Duane and Hunt,¹ and Hull,² for the case of tungsten. Furthermore, it appeared that above this potential the intensity of such rays would increase most rapidly at first and then more slowly, the graph soon becoming linear.

Evidently it should be of interest in connection with the problem of the mechanism of X-ray emission to find the emissivity of atoms struck by cathode electrons of a single velocity, as a function of that velocity. These data, however, do not give this emissivity directly, since some of electrons penetrate into the metal, and are slowed down by minor collisions before giving up the quantum of energy that appears necessary to produce any X-rays.

The ideal method of obtaining the true emissivity would be the use of an extremely thin target, which the electrons that did not lose quanta would penetrate with very little loss of speed. As such an experiment presents considerable difficulties in getting strong enough rays for spectrum measurements without melting the target, it seems desirable at present to get the information from known facts as to the loss of speed of electrons in a metal. Fortunately we are enabled to do this by the fact that the chance of any electron's losing a quantum is very small,³ so that when the potential is increased by a small amount dV almost all the electrons are reduced to the original V without losing a quantum.

¹ Duane & Hunt, *PHYS. REV.*, Aug., 1915.

² Hull, *PHYS. REV.*, Jan., 1916.

³ This point may be deduced from the remarkably low efficiency of an X-ray tube, as shown by Beatty's measurements. See Kaye, "X-Rays" Longmans Green & Co., 1914, p. 106; or Beatty, *Proc. Roy. Soc.*, Nov., 1913.

After this reduction, which occurs in a very thin surface layer, they will behave as though they were striking the target from the potential V . Consequently the increment of intensity by the addition of dV is simply the intensity from this surface layer, and the problem before us is that of finding the chance of emission in one atom in terms of the thickness of this layer and the increment of intensity.

Experimental Laws and Assumptions.—A. As a starting point for this calculation I shall take the law first deduced theoretically by Sir J. J. Thomson¹ and confirmed by the experiments of Whiddington,² that the kinetic energy of a cathode particle at a depth x (measured along its path) is given by the formula

$$(1) \quad e^2 V^2 - e^2 V_x^2 = ax$$

where V is the potential on the tube, V_x the potential that would give the energy it has at the depth x , and a is a constant of the material of the target. This law does not, of course, apply to an electron which has already lost an X-ray quantum, but since such electrons form a very small percentage of the whole, it seems safe to apply it to all as a first approximation.

B. From the results of my experiments quoted above, it appears that the intensity of radiation from rhodium can be represented within the limits of experimental error by the formula

$$(2) \quad I(V, \nu) = k(V - Q) + l(1 - e^{-q(V-Q)})$$

where $I(V, \nu)d\nu$ is the energy radiated per unit time and per unit of cathode ray current in the frequency interval $d\nu$, V the cathode ray potential, Q the quantum potential $h\nu/e$, and k , l , and q are functions of ν . For example, in the curve for "General radiation .645 to .661 Å." in Fig. 1 of the paper quoted, $k d\nu = 0.0331$ ionization unit per watt, $l d\nu = 0.055$ ionization unit per milliamper, and $q = 0.62$ kilovolt⁻¹, while in the curve for ".571 to .587 Å.," the numbers are 0.0465, 0.065 and 0.77 respectively. The ionization unit used here may be considered as a unit of rate of emission of energy, whose mechanical equivalent varies with frequency but not otherwise. The exact form of the functions k , l and q is not known, since the measured ionizations give comparisons of actual intensity only at a constant frequency and also since the range of frequencies tested was small. Doubtless k increases continuously with ν , and the l term is always comparatively small except near $V = Q$. The spectrum curves for tungsten at constant potentials

¹ "Conduction of Electricity Through Gases," 1906, p. 383.

² Camb. Phil. Soc., Proc., Dec., 1911; Proc. Roy. Soc., Apr., 1912.

published by Hull,¹ and Hull and Rice,² as well as curves for molybdenum which he has shown me indicate the same general type of law for these materials; though these curves are adapted primarily to the study of I as a function of ν rather than V and do not give any test of the details of equation (2).

Whatever may be said as to the generality of equation (2) as an exact statement, one fact made certain by the work of Duane and Hunt, and

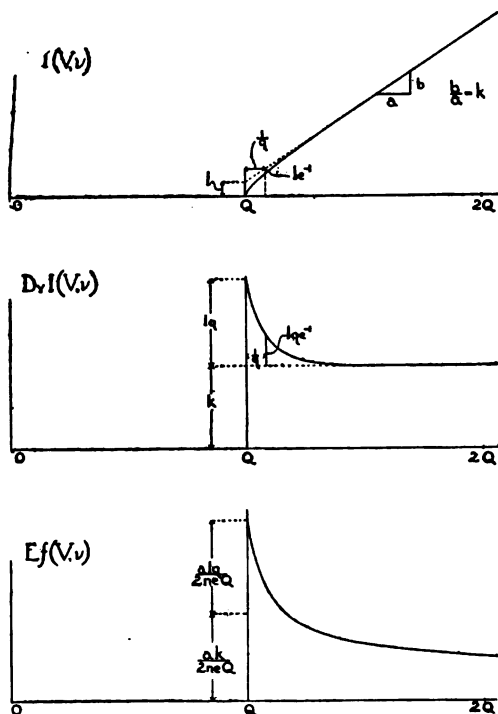


Fig. 1.

Hull, as well as my own, is that the slope of the I, V graph jumps from zero to a finite value at $V = Q$, and another is that in my experiments on rhodium at least the slope is greater at a voltage within one or two per cent. above this potential than at any higher one.³ These facts, especially the former, are the really essential features of assumption B.

C. Since no X-rays of a given frequency are started by an electron having less than a whole quantum, the energy emitted at each start is

¹ A. W. Hull, Amer. J. of Röntgenology, Dec., 1915.

² Hull and Rice, Proc. Nat. Acad., Mar., 1916.

³ The slight rounding off at the bottoms of these curves is due, as noted in the other paper, to the width of the slit and consequent lack of homogeneity of the rays. It disappears as the slit is narrowed down.

most probably equal to or greater than $h\nu$; and it is evidently equal to or less than the energy of an electron eV_x . These facts, together with the known tendency of radiant energy to become collected in quanta, as shown in the photo-electric effect, lead to the assumption C, that the energy emitted is $h\nu$ itself.

D. It will be assumed that the number of atoms penetrated by an electron in a given distance is independent of V_x and that the probability of starting X-rays of frequency ν to $\nu + d\nu$ in any one of them is $f(V_x, \nu)d\nu$. The problem is that of finding how this function depends on V_x .

Calculation of $f(V, \nu)$.—From the above assumption it is evident that if the potential on the tube is increased from V to $V + dV$, the number of atoms penetrated by an electron in losing energy down to the original V can be expressed as

$$(3) \quad ndx = n \frac{e^2}{a} \{ (V + dV)^2 - V^2 \} = \frac{2ne^2}{a} VdV$$

where n is the number of atoms per unit distance. Therefore if E is the energy emitted by an atom radiating with frequency ν , if excited at all by this electron, the total energy started by an average electron from these atoms with frequencies between ν and $\nu + d\nu$ is

$$(4) \quad \frac{2ne^2}{a} Ef(V, \nu) Vd\nu dV.$$

Dividing by e one obtains the rate of emission per unit of cathode ray current, from these atoms, which, as we have pointed out above, is the increase of $I(V, \nu)d\nu$ with the increase of potential dV . Therefore

$$(5) \quad Ef(V, \nu) = \frac{a}{2neV} D_\nu I(V, \nu) = \frac{a}{2ne} \frac{k + lqe^{-q(V-Q)}}{V}$$

The presence of the unknown function E causes some ambiguity in this result, but as we have seen, E is most probably equal to $h\nu$. On the other side of the equation are k , l and q , functions of ν requiring further investigation. For the cases quoted above, k and lq are nearly equal, and q is such that the exponential term disappears at a potential about 20 per cent. above Q . Graphs of I , $D_\nu I$, and f (assuming $E = h\nu$), for wave-length 0.579 Å. are shown in the accompanying figure.

Conclusions.—From equation (5) it appears that the probability of starting a quantum of X-rays from an atom penetrated by a cathode particle is zero for all energies below the quantum value, at which it rises discontinuously to a maximum. A further increase of energy of the cathode particle decreases this probability. One might expect by analogy with collisions such as those to which Thomson's law (1) relates,

that the chance of losing energy would be less at very high velocities than at low ones, as indicated by (5). Nevertheless the present case is different, in that (5) represents not an average loss of energy to an atom, but a probability of starting a definite frequency of radiation that is emitted only in definite quanta.

In ordinary mechanical systems, and in the systems assumed in dispersion theory, the frequency of oscillation is determined entirely by the constants of the oscillator. It is therefore reasonable to expect that the frequency of the X-ray oscillators must be determined similarly, and that the existence of the short wave limit of the constant potential spectrum means that for some unknown reason no oscillation can occur unless the cathode particle imparts *at least* a quantum of energy to the oscillator. That is, the collision must be within certain limits of obliquity. Obviously these limits will become narrower as the energy of the colliding electron is reduced toward the quantum value. This assumption would therefore make the chance of radiation approach zero, rather than the maximum given by equation (5). Notwithstanding this, the existence of the discontinuous rise in f , followed by a slow decline, is a consequence of the part of the intensity-voltage law of which we are most certain.

An assumption that fits the facts much better is that radiation occurs when the energy imparted to the oscillator is not greater than a quantum, but almost exactly equal to a quantum. In this case, using the equations of Thomson's theory of the slowing of cathode particles, one may readily show that if the force between the oscillator and the cathode electron is an inverse square repulsion only, the chance of transferring energy within a definite small fraction above or below the quantum is inversely proportional to the energy of the cathode electron whenever the latter exceeds the quantum. The same result would follow if the collision were between two hard spheres, though not for some other laws of repulsion such as an inverse cube. While the assumptions and the data are now too uncertain to justify a long mathematical theory on this basis the similarity of this result to equation (5) makes it reasonable to expect that some such assumptions may later be found correct. This means that quantum collisions obey laws that differ from those we are accustomed to even more than one might expect from the mere existence of a short wave limit of the constant potential spectrum.

Summary.—At any rate the above calculations prove the following points:

A. That if radiation occurs in quanta, the probability of its occurrence with any one frequency in any one atom struck by a cathode electron

rises from zero to a finite value when the energy of the cathode electron is increased past the quantum.

B. That for the case of rhodium at least the probability decreases with any further increase of energy of the cathode electron.

C. That these laws of quantum collisions are radically different from those of collisions which merely slow down the electron slightly without producing X-rays.

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AN ATTEMPT TO DETECT A CHANGE IN THE EMISSIVE
PROPERTIES OF PLATINUM AND OF TUNGSTEN
AT INCANDESCENCE WITH A CHANGE IN
THE METHOD OF HEATING

BY A. G. WORTHING.

IN this work the emissive properties at incandescent temperatures obtained by electric currents have been contrasted with those occurring when the same temperatures are obtained by the absorption of radiation. A possible difference in the general interrelations of the conducting electrons and the atoms for the two methods of heating might be expected to lead to a change in the emissive properties. For both cases it is necessarily true that much of the heat is brought to the radiating layer by thermal conduction. In very thin incandescent ribbons only will the amounts brought thus by conduction differ appreciably. Together with this difference, there will also occur relatively high current densities for the electrical heating method. Though it seemed improbable that one would be able to detect a change resulting from the considerations just stated, it seemed worth while to make the test.

The general procedure consisted in measuring with an optical pyrometer the red and blue brightnesses of very thin incandescent ribbons for the two methods of heating. The terms red and blue brightnesses refer to those values obtained when certain absorption screens are used at the pyrometer eyepiece. The corresponding effective wave-lengths of transmission in the present work were approximately 0.665μ and 0.470μ . The blue brightnesses associated with a given red brightness for the two heating methods might be expected to be different or the same depending on whether the effect sought for is real or not. Failure to detect a change by the plan outlined, however, is not conclusive for the red and blue emissive powers in going from one method of heating to the other might both increase or decrease in such a fashion that for the same red brightness, which would then occur at different temperatures, there would occur the same blue brightness.

In the final test on platinum, a bolometer strip between 1μ and 2μ thick was mounted just in front of a thick ribbon of the same material.

The thin strip could evidently be heated by either of the two methods. For the electrical method the current densities were of the order of 500 amps. mm². The temperatures maintained were in the neighborhood of 1,600° K.

In the final test on tungsten, a thin hollow cylinder made from some sheeting about 0.02 mm. thick was mounted concentrically about a spiral of heavy wire, in an evacuated lamp bulb. For the electrical method the current density used was of the order of 150 amps. mm². The temperature maintained was approximately 1850° K.

For both platinum and tungsten the results were negative, the certainty of this being considerably greater in the case of the latter, a consequence of the steadier conditions of operation for this metal. In the case of tungsten the results indicated that, for a given red brightness, the blue brightnesses do not differ with the method of heating by more than 0.5 per cent. In the case of platinum the corresponding value was of the order of 1.5 per cent.

NELA RESEARCH LABORATORY,

NATIONAL LAMP WORKS OF GENERAL ELECTRIC COMPANY,

NELA PARK, CLEVELAND, OHIO,

November 24, 1916.

SPONTANEOUS GENERATION OF HEAT IN RECENTLY
HARDENED STEEL. II.

BY CHARLES F. BRUSH.

IN April, 1915, I had the honor of presenting to The American Philosophical Society a paper on "Spontaneous Generation of Heat in Recently Hardened Steel,"¹ of which the present paper is a continuation. It was shown that a specimen of carbon tool steel, and also a specimen of "high-speed" tungsten-chromium steel after hardening by quenching at a high temperature, spontaneously generated heat in appreciable quantity for at least several weeks, the rate of generation steadily diminishing.

Fig. 1 is a diagram of the apparatus employed. *A*, *B* represent two large silvered Dewar vacuum jars selected to have very nearly equal thermal insulating efficiency. They are supported in a wooden rack inside a thick copper cylinder *C* packed in granulated cork in a wooden box *E*. *D* is a paper extension of *C*, packed with layers of felt by removal of which and the loose copper cover of *C* easy access is had to the Dewar jars. The copper cylinder weighs 52 pounds and its functions are, by reason of its large thermal capacity and high conductivity, to protect the Dewar jars from any rapid change of temperature, and from temperature stratification.

The box *E* is surrounded by a much larger wooden box *F* lagged with felt. By means of a long resistance wire and thermostat, not shown, the air space between the boxes may be maintained at constant temperature.

A' is an airtight cylinder of thin copper, six inches high and two and a half inches in diameter, provided with an open half-inch axial tube also of copper. A small round opening at the top of *A'* permits the introduction of a weighed quantity of water, after which the opening is tightly corked to prevent any change of temperature by evaporation of the water. *B'* is another copper cylinder just like *A'* except that it has a removable top to permit the introduction of the substance whose thermal behavior is to be investigated. The high thermal conductivity of these copper cylinders prevents temperature stratification within them. The

¹ Proc. Am. Phil. Soc., Vol. LIV., No. 217, May-July, 1915.

Dewar jars are filled above the copper cylinders with layers of felt, and granulated cork, and covered with waxed cardboard carefully sealed on to prevent temperature difference inside the jars which would follow unequal loss or gain of moisture by the felt and granulated cork. A small thin glass tube, flanged at top and closed at bottom, is located

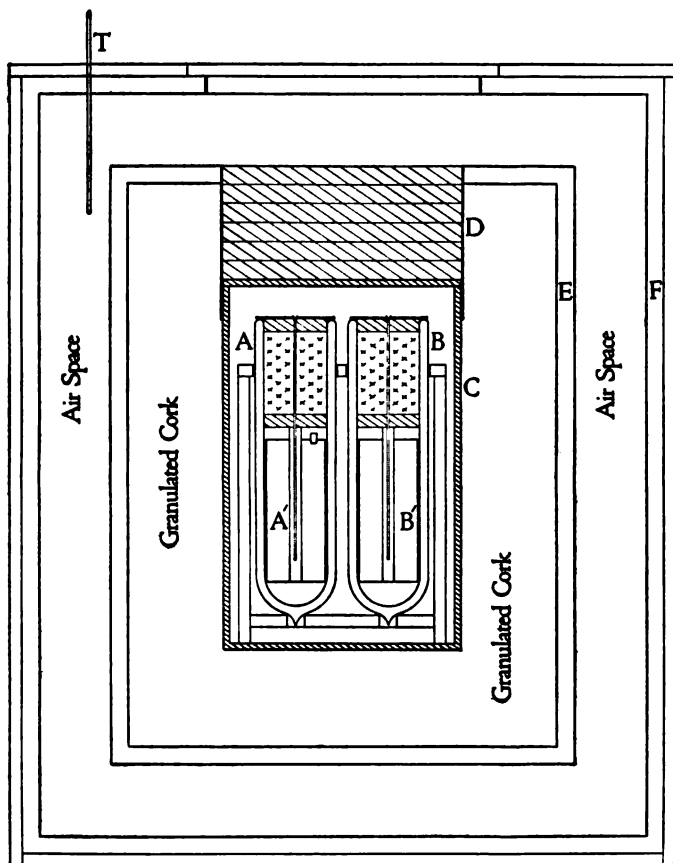


Fig. 1.

in the axis of each Dewar jar and extends from the waxed cover nearly to the bottom of the inclosed copper cylinder. The glass tubes contain the ends of thermo-electric couples of fine constantan, copper and iron wires, one iron-constantan and one copper-constantan junction at the bottom of each tube. The leading-out wires are copper, and connect the thermo-couples with a reflecting galvanometer having the customary reading telescope and scale. Careful calibration has shown that 55 scale divisions of the galvanometer indicate one degree C. temperature-difference between A' and B' , and that temperature-difference and

galvanometer deflection are very closely proportional throughout the range used.

In the following experiment *A'* and *B'* were removed from the Dewar jars and allowed to attain equal room temperature. Twelve half-inch, round bars of tool steel, five inches long and with machined surfaces were hardened by heating to high "cherry-red" in a reducing atmosphere of a gas furnace and quenching in cold water. The bars then had a thin and strongly adhering coating of black oxide. They were next stirred in a large quantity of water at room temperature, to acquire that tem-

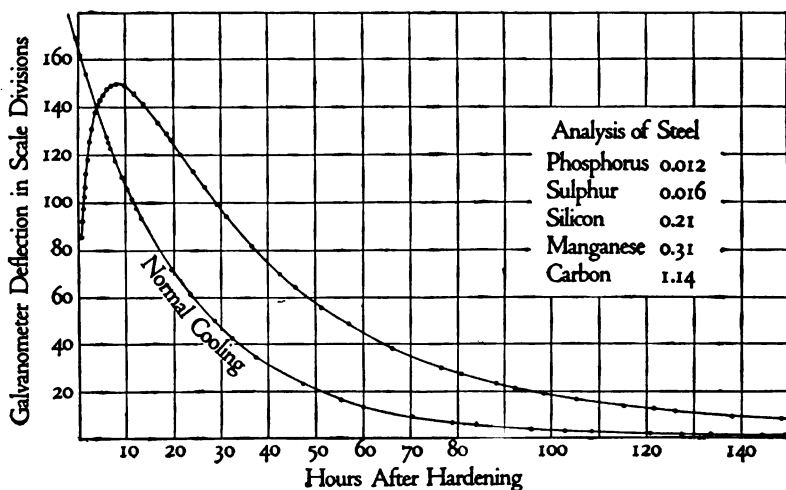


Fig. 2.

perature, wiped dry, and oiled with heavy, neutral mineral oil to prevent generation of heat by further surface oxidation, wiped free of excess of oil and placed in the copper cylinder *B'*. A weighed quantity of water, also at room temperature, just sufficient to equal the steel bars in thermal capacity had already been placed in *A'*. The whole apparatus was then assembled as quickly as possible, and galvanometer readings commenced.

The upper curve in Fig. 2 shows the progress of heat generation in the carbon tool steel bars during the first 150 hours after hardening.

The "normal cooling" curve was obtained five or six weeks after the other, and when the generation of heat had very nearly ceased. For this purpose the steel bars were removed, warmed a few degrees, and replaced; then galvanometer readings were made from time to time as before. This curve is plotted in a location convenient for visual comparison with the heating curve, but otherwise might just as well be plotted further to the right. For studying the generation of heat at any point in the upper curve, the curve of normal cooling may be plotted where it will intersect the upper curve at the desired point.

The tungsten-chromium steel, consisting of the same number of bars of the same dimensions as in the first experiment, was treated in the same manner.

Fig. 3 shows the curve of heat generation in the tungsten-chromium steel, and the curve of normal cooling located with respect thereto as in Fig. 2.

The former paper showed that carbon steel when hardened remains

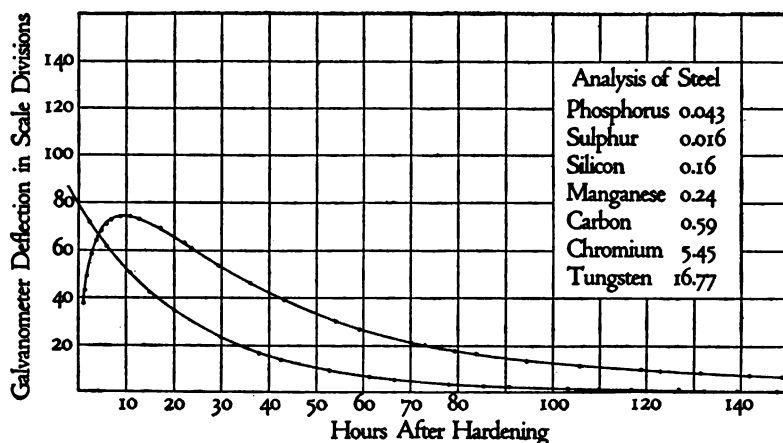


Fig. 3.

very considerably expanded, and that it shrinks progressively when "tempered" to various stages and finally annealed.

A long thin bar of high-carbon steel was found to shrink spontaneously after hardening, to such extent that its progress was easily measured and followed for many days. The spontaneous shrinkage of this steel is plotted in Fig. 4.

It was further shown that while spontaneous generation of heat was probably accompanied by spontaneous shrinking in the specimens of steel examined, yet the shrinking could not possibly be the prime cause of the heat, because the work represented by the heat generated was hundreds of times more than sufficient to effect by compression the reduction in volume that probably occurred. This conclusion, that shrinking was not the cause of the generation of heat, has been questioned; but the more recent experiments, some of which show *no* shrinking during the generation of heat, confirm it.

Six or eight months ago Sir Robert Hadfield wrote an expression of interest in the subject outlined above, and after some correspondence sent two specimens of nickel-chromium steel with the request that they be tested for spontaneous generation of heat after hardening. He also

invited the preparation of a paper for the Faraday Society embodying the results of the tests. Each specimen consisted of twelve half inch round bars five inches long, with machined surfaces, so that results obtained would be quantitatively comparable with those of the other kinds of steel formerly examined.

Specimen *A* was gradually heated in a gas furnace, with reducing atmosphere, while its magnetic susceptibility was observed from time to time by means of a simple induction apparatus devised for the purpose.

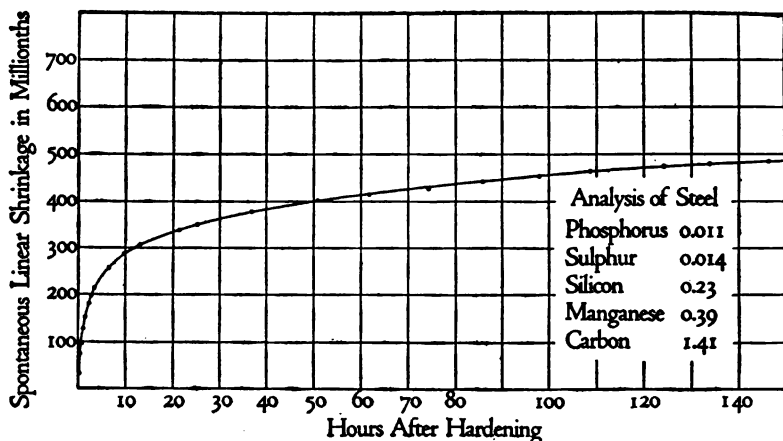


Fig. 4.

Susceptibility remained substantially constant until cherry-red heat was reached, then soon rose ten per cent. or more and again remained constant, at the higher value, for several minutes, then fell off rapidly to nothing. This was taken to indicate complete decalescence, and the proper temperature for hardening. The furnace was immediately opened and the 12 bars quenched separately in water and crushed ice, then stirred in much water at room temperature, wiped dry, oiled and placed in the calorimeter (Fig. 1) all as in the earlier experiments first described.

Fig. 5 shows the progress of heat generation following this first hardening of specimen *A* of the Hadfield nickel-chromium steel. Rapid generation of heat is indicated in the early stages, especially during the first hour, and maximum temperature, the point at which gain of heat was balanced by normal cooling, was reached in about three and a half hours. Obviously the rate of generation had diminished very much by that time and was falling fast, though it still retained considerable value at the end of 150 hours. In general, the curve of "First-Hardening" clearly indicates that the rate of heat generation diminished steadily from the moment of quenching the hot steel.

After a week of cooling, the same bars were again placed in the furnace and heated to a temperature considerably higher than before by continuing the heating fifteen minutes after complete loss of magnetic susceptibility. They were then quenched and treated as formerly.

The other curve of Fig. 5 shows the effect of the higher quenching temperature. Generation of heat was at first much slower than before, the far lower maximum temperature requiring eight hours to reach; but the generation was much better sustained, as indicated also by the

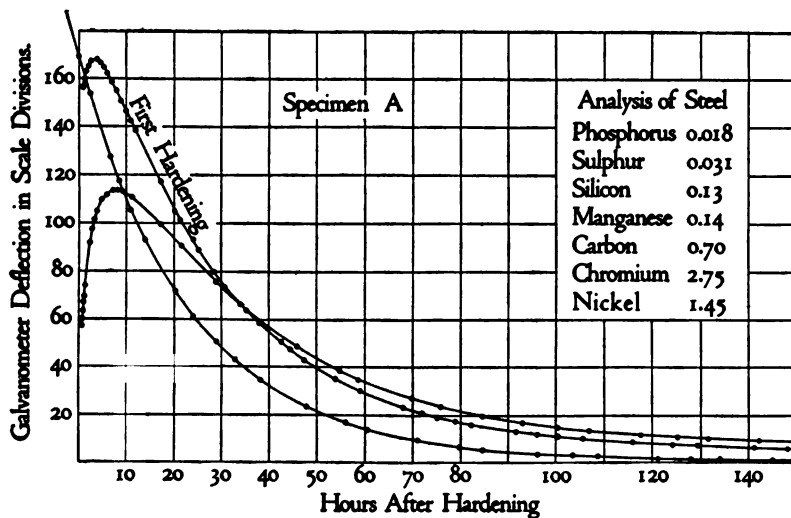


Fig. 5.

crossing of the curves after 35 hours. The total amount of heat generated was not greatly different in the two cases.

Specimen *B* of the Hadfield steel was treated in "First-Hardening" precisely like specimen *A*; but in second hardening was not heated quite so far above the temperature of decalescence.

Fig. 6 shows progress of heat generation in specimen *B* after first and second hardening. The total amount of heat generated was much greater after the second hardening than after the first; *B* differing greatly from *A* in this respect. Probably this was due to the different carbon, chromium and nickel content as shown in the analyses of the two specimens.

The effect on specimen *B* of a moderate increase in quenching temperature was so great that it was thought worth while to quench it a third time, at a temperature somewhat *lower* than in "First-Hardening," and the temperature of complete recalescence was chosen; so that while the quenching temperature of first hardening was just above the critical

temperature of decalescence, that of the third hardening would lie just below the critical temperature of recalescence. In this way light might be thrown on the *cause* of the spontaneous generation of heat.

In pursuance of this idea specimen *B* was again slowly heated in the gas furnace until complete loss of magnetic susceptibility was reached. This was the quenching temperature of first hardening. Then the supply of gas and air were so far reduced that the furnace very gradually cooled. In about twenty minutes the steel commenced to regain its magnetic susceptibility, and a few minutes later had completely regained it. The bars were immediately quenched and treated as formerly.

I had expected to find that the spontaneous generation of heat would

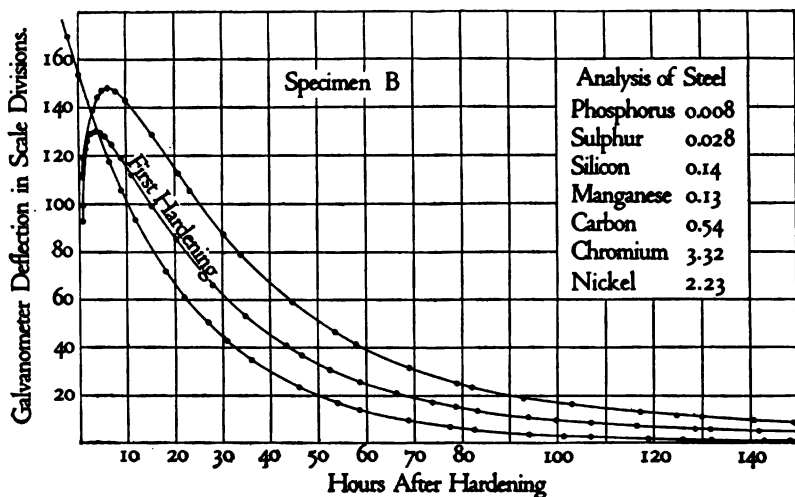


Fig. 6.

be greatly modified, or possibly eliminated by thus quenching the steel below the critical temperature, because true hardening could not have taken place. But it was not so. On the contrary, fully three fourths as much heat was generated as followed the first hardening, and its curve (not plotted) was parallel throughout with that of "First-Hardening."

We may safely conclude then, that the spontaneous generation of heat, at least in the case of specimen *B*, was not associated with the phenomenon of recalescence.

It has been suggested that some, at least, of the spontaneously generated heat found in all my experiments may have been due to chemical action of residual moisture, or oil or both, on the metal or its coating of oxide.

To meet this criticism I again slowly heated specimen *B* until it began to lose its magnetic susceptibility. This was above the temperature

of the previous quenching, and presumably obliterated its effects; it also destroyed the oil, and oily compounds if any, of the previous operation. Then the furnace was closed and, with its contents, allowed to cool four hours. The steel bars were then removed from the furnace, still too hot to handle, allowed to air-cool half an hour longer, stirred in much water at room temperature, wiped dry, oiled and placed in the calorimeter just as formerly. The slow leakage of air into the hot furnace after closing had provided the steel bars with a thin coating of black oxide having the same appearance as that which followed the previous quenchings. The aim of all this procedure was to reproduce as faithfully as possible all the conditions which obtained in previous experiments *except hardening*.

In all former experiments the temperature of the steel had risen considerably during the assembling of the apparatus immediately following the oiling of the bars, as shown by the curve sheets. But in this case the first galvanometer readings were slightly *minus*, showing that the steel was about one fiftieth of a degree colder than the balancing water in the other Dewar jar; and several days passed before approximate equality of temperature was reached.

Hence we may safely say that not the slightest trace of heat was generated in the steel, and that the heat generation observed in former experiments was surely due to hardening.

Before commencing the experiments with specimens *A* and *B* above detailed, a test bar of each lot was prepared by machining its ends slightly convex and polishing, so as to permit of accurate length-measurements by means of a micrometer caliper easily manipulated to constant end-pressure, and easily read to one twenty-thousandth of an inch. The test bars were mechanically held always in the same position between the caliper jaws.

Measurements of the test bars were made after various treatments, and the results are shown in Tables I. and II. The error of measurement does not exceed one unit in the last decimal.

It will be observed that in neither case was there any spontaneous

TABLE I.
Specimen A.

Length of Test Bar.	First Hardening.	Second Hardening.
After hardening.....	5".0274	5".0354
After spontaneous shrinking.....	.0274	.0342
After tempering to light straw.....	.0263	.0293
After tempering to light blue.....	.0255	.0293
After annealing.....	.0222	.0250

TABLE II.
Specimen B.

Length of Test Bar.	First Hardening.	Second Hardening.
After hardening	5".0180	5".0151
After spontaneous shrinking	.0180	.0143
After tempering to light straw	.0172	.0093
After tempering to light blue	.0161	.0091
After annealing	.0139	.0063

shrinking after the first hardening, thus demonstrating that the heat generation which then occurred was not due to shrinking.

In both specimens total shrinkage was about twice as great after second hardening as after first hardening, but was divided between the several treatments quite differently. Thus, comparing second hardening shrinkages with first: Considerable occurred spontaneously (none in first); about five times as much followed tempering to light straw, and little or none followed the second tempering, to light blue, though this caused considerable shrinking in both cases of first hardening.

In connection with the first tempering, to light straw color, in both cases of second hardening, an interesting phenomenon was observed, as follows: In all cases of tempering to color, the bar was sandpapered bright, very slowly and uniformly heated until the desired color appeared and then quenched in water at room temperature to arrest progress of tempering. But after this secondary quenching the bars continued to shrink measurably for several hours. The final, stable measurements are given in the tables. Why no further shrinking occurred when the bars were tempered to light blue, is not entirely clear; seemingly, the large shrinkage which followed first tempering, to light straw color, brought about a condition sufficiently stable to withstand further change by the moderately higher temperature required for tempering to light blue.

In conclusion, I cannot at present do better than quote from the closing paragraph of the former paper. "I am led to regard the hardened steel as being in a condition of very great molecular strain somewhat unstable, especially at first. Spontaneous relief of a small portion of the strain causes generation of heat until stability at room temperature is reached. Any considerable rise of temperature, as in tempering, permits further spontaneous relief of strain, or molecular rearrangement, doubtless accompanied by more generation of heat, and so on until annealing is reached."

CLEVELAND, O.,
October, 1916.

THE MEASURABLE QUANTITIES OF PHYSICS.

BY RICHARD C. TOLMAN.

INTRODUCTION.

AS any branch of science develops, its considerations become more and more deductive in their nature, the ideal form of scientific exposition apparently being one in which all the important relations are logically derived from a small number of independent general principles or postulates and all the terms used are defined with the help of a small number of indefinables.¹ The time is already ripe for a much more comprehensive and systematic treatment of the field of mathematical physics than has hitherto been attempted, and the completion of this task would make it possible to derive all the equations of mathematical physics from a few consistent and independent postulates, and to define all the quantities occurring in these equations in terms of a small number of indefinables. The purpose of this article is to discuss from a somewhat general point of view the nature of the quantities which occur in the equations of mathematical physics and to consider a set of indefinables for their definition. We shall thus hope to help in the preparation for that more complete systematization of mathematical physics which is undoubtedly coming.

We shall find it possible in agreement with the work of others to distinguish two general classes of quantity, those having *extensive* and those having *intensive* magnitude, and shall consider the methods necessary for measuring these two quite different kinds of magnitude. We shall then discuss the method of choosing a set of fundamental kinds of quantity, which will serve as indefinables for defining all other kinds of quantity, and shall find that the *number* of independent kinds of quantity which must be taken as fundamental is apparently independent of the particular set that is chosen. Contrary to previous practice, however, we shall find it desirable to choose as fundamental, only quantities which have *extensive* magnitude, since we shall then be able to introduce a

¹ The term "indefinables" is used in its technical sense, and by it we shall understand certain entities which are not defined in the particular discussion undertaken, but for the purposes at hand are assumed rather to be matters upon which there is general agreement. This does not of course in any way prejudice the possibility or desirability of their definition in some more fundamental treatment.

number of important simplifications. Besides this set of fundamental kinds of quantity, we shall also find it necessary to have a set of indefinable operations for use in the definition of quantities, and we shall briefly consider these further indefinables and the definitions which can be obtained with their help. Making use of our definitions of the different kinds of quantities we shall then briefly consider the choice of units for their measurement. In conclusion we shall discuss the impracticability of trying to reduce the number of fundamental kinds of quantity and shall also consider what kind of developments in science will necessitate a further increase in their number.

§ I. ON MAGNITUDE AND QUANTITY.

In accordance with the so-called absolute theory of magnitude as presented by Russell¹ we shall consider *magnitudes* as indefinable *terms* which are capable of entering into the *relations* of "*greater than*" and "*less than*." The relations "*greater than*" and "*less than*" are *asymmetrical* and *transitive* and are taken as indefinable. Every magnitude bears a peculiar relation to some particular concept such that we say it is the magnitude of that concept; and magnitudes are said to be of the same kind when they bear this relation to the same concept. Only magnitudes of the same kind can enter into the relations of greater and less.

Using the language of this theory we may speak of magnitudes of pleasure, magnitudes of temperature, magnitudes of volume, etc., each of the concepts pleasure, temperature, volume, etc., having a set of magnitudes of the same kind associated with it. And we can say that one magnitude of volume is greater or less than some other magnitude of volume, but cannot relate it of course in this way with a magnitude of temperature or of pleasure.

Quantities are to be regarded as magnitudes which have been particularized by the specification of spatial or temporal conditions. Thus the statement of the temperature in a particular beaker at a particular time, is a quantity, this quantity having a definite magnitude as one of its attributes. Thus we see that magnitude is a more abstract conception than quantity. We shall say that one quantity is greater than another when its magnitude is greater. We shall say that two quantities are equal, however, when they have the *same* magnitude. To speak of equal magnitudes would be meaningless.

¹ Bertrand Russell, "The Principles of Mathematics." Cambridge, 1903.

§ 2. EXTENSIVE AND INTENSIVE MAGNITUDE.

If we regard the different kinds of quantities that are used by physicists, we find that they fall with reasonable lack of ambiguity into two general classes, those having a certain additive nature so that a given quantity can be regarded as being the sum of a number of smaller quantities of the same kind, and those which have no such additive properties. We say that quantities of the first class have *extensive* magnitude, and that quantities not having an additive nature have *intensive* magnitude.

The distinction between these two kinds of quantity can be made clearer by examples. Volume is typical of quantities having extensive magnitude. A given volume may be regarded as composed of smaller volumes into which it can be mentally divided, and similarly quantities of length, mass, energy, entropy, etc., will be seen on consideration to have extensive magnitude. Temperature, density, magnetic permeability, dielectric inductivity, etc., are quantities which we cannot successfully regard as being the sum of smaller quantities of the same kind and such quantities are said to have intensive magnitude.

In order to decide whether a given quantity has extensive or intensive magnitude it is sometimes helpful to see if the simultaneous presence of two systems, each having a definite quantity of the kind in question, can be regarded as giving a larger system with twice the quantity; if so that kind of quantity has extensive magnitude. For example, consider a system composed of two pieces of platinum, each having a mass of 10 grams; this gives us a larger system of 20 grams, and we say that mass has extensive magnitude. On the other hand, if each piece of platinum had a temperature of 100° absolute, we should not say that the temperature of the total system was 200° ; indeed the larger system also would have the same temperature 100° and we say that temperature has intensive rather than extensive magnitude. As another example, two pieces of platinum, each with a density of 21 grams per cubic centimeter, does not give us a larger piece of platinum with a density of 42 grams per cubic centimeter, and density also is a quantity having intensive magnitude.

Whether this classification of quantities into those having extensive and intensive magnitude is perfectly unambiguous is perhaps difficult to decide. The classification is usually simple, however, and like many other classifications of science this is one which would still have great value even if there should turn out to be borderline quantities of such a nature that it was hard to apply the criterion.

§ 3. THE MEASUREMENT OF PHYSICAL QUANTITIES.

The method of measuring quantities which have extensive magnitude is more or less obvious. Since these quantities have an additive nature, we may pick out more or less arbitrarily some particular quantity as having unit magnitude and then find experimentally how many of these unit quantities must be taken together in order to obtain a quantity of the same magnitude as the one in question. Thus we can measure the length of a building by finding how many foot rules must be laid end to end in order to get a length of the same magnitude.

In the case of quantities with extensive magnitude, however, we may distinguish for purposes of measurement two quite different groups. In the first class we may put those quantities which can be regarded as composed of a finite number of distinct and identical parts. For the measurement of these quantities the most natural unit quantity to choose will be one of these individual parts, and the measurement of the quantity will be a process of *counting* the number of these discrete units necessary for the composition of the whole system. For example, the quantity of apples in a barrel may be measured by counting the number present. Quantities of this kind are often used in the natural sciences, as for example in the correlation of the pressure of a gas with the *number* of molecules it contains or in statements as to the *number* of petals in a given species of flower. It should be noticed that quantities of this simple kind which are measured by counting are associated with a discrete series of magnitudes which can be put into one-to-one correspondence with the ordinal numbers.

Quantities having extensive magnitude and belonging to the second group cannot be regarded as the sum of a finite number of parts, and in this case the magnitudes of a given kind form a continuous series and can only be put into one-to-one relation with the whole series of real numbers. Volume is an example of such quantities, it not being in accordance with our present ideas of space to consider a volume as composed of a finite number of parts which could not be further divided. For the measurement of extensive magnitudes of this kind, the choice of unit will usually be somewhat arbitrary, but the additive nature of all quantities with extensive magnitude will permit the construction from unit quantities of a whole having the same magnitude as the quantity to be measured, and the number of unit quantities used will be the desired measurement. In making such measurements, in general of course it will not turn out that our unit quantity is of just the right magnitude to fit, so to speak, an integral number of times into the quantity to be measured, but it will not be necessary to discuss here the

well known methods of approximation which must then be employed.¹

As for quantities with intensive magnitude, since they do not have an additive nature, their measurement must be brought about by some device in which the magnitudes to be measured are put into a one-to-one correspondence with a series of quantities having extensive magnitude. For example, we might try to measure quantities of pleasure by correlating pleasurable experiences with the money spent in obtaining them. In fact such a correlation is tacitly but pretty generally adopted in a commercial age, and in spite of obvious defects it certainly has the advantage of definiteness. In the case of temperature, a typical physical quantity with intensive magnitude, various methods of measurement have been devised, such for example as the correlation of magnitudes of temperature, with the varying lengths of a thread of mercury, or the varying volumes occupied by a perfect gas or other quantities having extensive magnitude. In our discussion of fundamental and derived quantities we shall find a very natural method of correlating the physical quantities having intensive magnitude with those having extensive magnitude, since we shall find it possible to define all our physical quantities in terms of a set all of which have extensive magnitude. This procedure, which has not hitherto been adopted, seems to be a distinct advance in the direction of simplicity and fundamentality.

§ 4. ON FUNDAMENTAL AND DERIVED QUANTITIES.

It is not necessary to take all of the many different kinds of quantity used by the physicist as indefinable, since most of them can be defined in terms of a few which may be taken as fundamental. Thus it is customary to regard a quantity of velocity as a quantity of length divided by a quantity of time, a quantity of volume as the product of three quantities of length, a quantity of density as a quantity of mass divided by a quantity of volume, a quantity of momentum as a quantity of mass multiplied by a quantity of velocity, etc. In definitions of this kind the operations "multiplied by," "divided by," etc., may be taken as further indefinables and we shall present a brief discussion of them in a following section.

¹ There is one case in which the method for measuring quantities with extensive magnitude described above seems to fail. Time intervals are quantities which have extensive magnitude, since we can obviously regard a twenty-minute interval as composed of two ten minute intervals following one another. It is obvious, however, that, for us, a direct experimental comparison of one time interval with another is impossible, and hence we cannot measure a time interval by fitting together unit time intervals until we obtain the magnitude in question. For this reason some artificial method of measuring time, such as counting the beats of a pendulum, has to be devised. A direct comparison of time intervals might be possible to an observer with a chrono-synoptic point of view.

The relation between a derived quantity and the fundamental kinds of quantity in terms of which it is defined is customarily symbolized by physicists by a statement of the so-called dimensions of the quantity in question. Thus velocity is said to have the dimensions of length divided by time and this is symbolized by the formula $[lt^{-1}]$, and momentum which is defined as mass times velocity is said to have the dimensional formula $[mlt^{-1}]$. *The dimensional formula of a quantity may be regarded as a shorthand statement of the definition of that kind of quantity in terms of the kinds of quantity chosen as fundamental, and hence also as a partial statement of the "physical nature" of the quantity in question.*

§ 5. CRITERIA FOR THE CHOICE OF FUNDAMENTAL QUANTITIES.

In choosing a set of fundamental kinds of quantity, it will be desirable to keep in mind the following considerations, namely, (1) that the number of kinds chosen shall be *sufficient* (and not redundant) for the part of physical science that is to be handled; (2) that all of the kinds chosen as fundamental shall have *extensive* magnitude; and (3) that the choice shall be such as to further the requirements of simplicity.

Sufficiency.—For the body of physics which is at present well established we seem to need five kinds of fundamental quantity for defining all the quantities in use. The ones usually chosen for this purpose are those of length l , time t , mass m , magnetic permeability μ , and temperature T , although, as we shall later see, a somewhat different set of five will be more satisfactory. These five indefinables, together with the indefinable relations "multiplied by" and "divided by," etc., of which we have already spoken, are sufficient and also apparently necessary for defining all the quantities we need at present.¹

The reason that just five different kinds of fundamental quantity are necessary seems to be because the science of physics is at the present time considering five fundamentally different kinds of "thing," namely, space, time, matter, electricity, and entropy (or "degree of run-downness"). The nature of these different kinds of thing will perhaps be clearer if we point out that we can arrange the physical sciences in a hierarchy such that each successive member introduces the consideration of one additional kind of "thing." Thus the science of *geometry* considers one kind of "thing" space, *kinematics*, which is the science of moving points, lines, planes, etc., introduces the further "thing" time, *mechanics*, which considers moving bodies rather than immaterial points and lines, introduces the new "thing" matter, *electrodynamics*, which

¹ It was first pointed out by Sir A. W. Rücker, Proc. Phys. Soc., London 10, 37 (1888) that *five* kinds of quantity are necessary, it having previously been assumed that length, time and mass would be sufficient for the whole of physics.

discusses the further laws of motion when the bodies may carry charges of electricity, introduces electricity as a further "thing," while *thermodynamics*, which considers systems composed of such a large number of individual units that we cannot follow the behavior of the separate parts but only the statistical results of their interactions, introduces a new "thing" entropy or "degree of run-downness."

Extensivity of Fundamental Quantities.—Our second criterion for the choice of fundamental quantities was that they should all have extensive magnitude. This is desirable since we shall then be able, as we have already seen, to measure all our fundamental kinds of quantity by a simple process of fitting unit quantities together until we reach the magnitude of the quantity which we are measuring. Moreover, since all further kinds of quantity will be directly defined in terms of this fundamental set, we shall then be in a position to measure all the derived quantities by determining the magnitudes of the fundamental quantities on which they depend. This will be particularly important in the case of derived quantities having intensive magnitude, since otherwise we should have to devise some purely arbitrary method of correlating the intensive magnitude, involved with quantities having extensive magnitude. These considerations alone are enough to make us reject permeability (or dielectric inductivity) and temperature as suitable fundamental quantities.

Simplicity.—Our third criterion for choosing fundamental quantities was simplicity, which includes the following considerations: (1) that the kinds of quantity should be psychologically simple to grasp; (2) that they should permit straightforward definitions of the desired derived quantities; and (3) that they should bear as simple a relation as possible to the five different kinds of "thing" of which we have spoken in a previous paragraph.

Let us consider an actual set of fundamental kinds of quantity which fulfills the requirements.

§ 6. FUNDAMENTAL QUANTITIES.

For *spatial considerations* we shall choose *length* as the fundamental kind of quantity. This is the choice ordinarily made by physicists. It has extensive magnitude, is an idea relatively simple to grasp, permits a straightforward definition of such quantities as area and volume, and bears a simple relation to the corresponding fundamental "thing"—space. We see that volume would be a less successful choice, since its three-fold nature makes it a psychologically less simple concept, and furthermore the definition of length in terms of volume would be much less straightforward than the reverse procedure.

For *temporal considerations* we shall choose *time interval* as the fundamental kind of quantity, a choice which is obvious in the light of the criteria which we have laid down.¹

For *mechanical considerations* we shall make the usual choice of *mass* as the fundamental kind of quantity. This choice conforms to all the criteria that we have laid down; in particular we should not overlook the intimate relationship between the "thing" matter and mass, the latter often being considered indeed as a measure of the quantity of matter. The close connection between mass and energy found in the newer mechanics is also significant.

For *electrical considerations* we shall choose *quantity of electric charge* as fundamental. This is superior to the usual choice of magnetic permeability (or dielectric inductivity) not only because it has extensive magnitude but also because of greater psychological simplicity and a more direct relation to the fundamental "thing" electricity. It is interesting to note that the recent experimental work of Millikan and others has indicated that electric charge is in the group of quantities already spoken of which have *discrete* magnitudes. The fundamental unit of electric charge is that of one electron and the most natural method of measuring a charge would be to count the number of electrons which it contains. Such a procedure presents of course experimental difficulties, and since we usually deal with charges containing a large number of electrons we can if desired treat these quantities as though they had a continuous series of magnitudes using any convenient unit. This is analogous to measuring apples by the peck instead of by counting.

For *thermodynamic considerations* we shall choose *entropy* as the fundamental kind of quantity, although the choice has previously been temperature. Perhaps this choice of entropy can not be justified on the ground of psychological simplicity since temperature certainly seems to most persons a simpler idea. Entropy, however, does bear a simpler relation to the kind of "thing" considered in thermodynamic considerations since it may be regarded as a quantitative measure of the "degree of run-downness" of a system. Entropy has, moreover, extensive magnitude since the entropy of a system is the sum of the entropies of its parts, while the temperature of a system is certainly not the sum of the temperatures of its parts.

§ 7. ON THE INDEFINABLE OPERATIONS.

Before proceeding to the actual definition of the derived quantities of physics, in terms of the fundamental quantities, we must first turn

¹ We have already pointed out the experimental difficulty of fitting unit quantities of time together.

our attention to the indefinable *operations* which enter into the definitions.

We have already seen that quantities are to be regarded as magnitudes which have been particularized either spatially or temporally or both. It should be noticed now that in the case of length, which is one of our five fundamental kinds of quantity, a spatial particularization of a magnitude of length gives us a *vector* quantity, since a complete particularization will state not merely the location in space but the direction and sense of the quantity of length in question. For this reason, in defining derived quantities in terms of fundamental quantities we find that we need to include among our operations the two types of vector multiplication.

Our indefinable operations will then be multiplication, division and differentiation and the two types of vector multiplication which lead respectively to the inner and outer product. Multiplication may be indicated by a simple juxtaposition of the quantities involved, thus ab ; division by the superposition of dividend and divisor, thus a/b ; differentiation in the familiar way, thus da/db ; the formation of the inner product by a small dot between the quantities involved, thus $a \cdot b$; and the formation of the outer product by the interposition of a small cross, $a \times b$.¹

We shall take these operations for our purposes as indefinable, of course, however, without in the least objecting to their being defined in terms of something simpler in some more fundamental discussion. Certain characteristics of the five operations may be noted. In the case of multiplication one of the multipliers must be a scalar quantity, in the case of division the divisor must be a scalar, and we shall only carry out differentiation with respect to scalar quantities. In forming the inner or outer product both quantities must be vectors (or pseudo scalars). In accordance with Lewis² the outer product of two lines will be the parallelogram which they determine rather than a line perpendicular thereto, and the outer product of three lines will be the corresponding parallelopiped. It should also be particularly noticed that these five operations are to be considered as performed upon actual quantities, not upon numbers which are used to represent the magnitudes involved.

§ 8. THE DEFINITION OF DERIVED QUANTITIES.

We are now in a position to proceed to the definition of the quantities of physics in terms of the five fundamental quantities and the five

¹ Vector quantities are indicated by heavy Clarendon type.

² Lewis, Proc. Amer. Acad., 46, 165 (1910).

indefinable operations. It will be sufficient to give one or two examples of such definitions for each of the general classes of geometrical, kinematical, mechanical, electrical and thermodynamic quantities.

As an example of *geometrical* quantities consider area. Two lines running from a given point determine a parallelogram, and we define the quantity of area of the parallelogram as the outer product of the two quantities of length, thus $A = l_1 \times l_2$. In accordance with the rules of vector analysis, the numerical measure of the area A will be the product of the numerical measures of the two lines multiplied by the sine of the included angle.

This example illustrates an important characteristic of derived quantities, namely, that they are themselves particularized spatially or temporally. This is of course perfectly proper since we have from the start looked upon quantities as particularized magnitudes. And hence it need not surprise us that two magnitudes of length which have a definite location in space and in time should determine a magnitude of area which is also definitely located in space and in time.

As an example of *kinematical* quantities we may take velocity. The quantity of velocity of a moving point will be defined as the limit approached by the quantity of length travelled by the quantity of time required as we make the time interval smaller and smaller, thus

$$v = \lim_{t \rightarrow 0} \left(\frac{l}{t} \right) = \frac{dl}{dt}.$$

Here again we see that derived quantities are themselves particularized spatially or temporally.

As examples of *mechanical* quantities we may define the quantity of momentum of a particle as its quantity of mass multiplied by its quantity of velocity, $M = mv$, and the quantity of force acting on the particle as the differentiation with respect to the time of the quantity of momentum $F = (d/dt)(mv)$. In the definition of work we introduce for the first time the use of the inner product. The quantity of work done by a force acting at a given point being the inner product of the quantity of force by the quantity of length through which the point is displaced, thus $W = F \cdot l$, or in the case of variable forces $dW = F \cdot dl$. In accordance with the rules of vector analysis, the numerical measure of the work is equal to the product of the numerical measures of the force and the displacement multiplied by the cosine of the included angle.

As a single example of *electrical* quantities, we define the electric potential at a point as the work done in bringing up a small charge to the point in question from an infinite distance divided by the charge itself.

The test charge is supposed taken small enough so that it will not itself disturb the electrical field.

As the most important example of thermodynamic quantities, we define the quantity of temperature of a given system by the equation $T = dQ/dS$ where dS is the infinitesimal change in the quantity of entropy of the system corresponding to the absorption of heat energy dQ , quantities of energy having already been defined in terms of mechanical quantities.

Without presenting any further complete definitions we give here for future reference a table of the "dimensions" of physical quantities

TABLE I.
Dimensions of a Few Important Quantities.

	Dimensions.		Dimensions.
<i>Geometrical Quantities:</i>		<i>Electrical Quantities:</i>	
Length, l	$[l]$	Charge, e	$[e]$
Area, A	$[l^2]$	Potential.....	$[ml^2t^{-2}e^{-1}]$
Volume, V	$[l^3]$	Capacity.....	$[m^{-1}l^{-3}e^2]$
Angle.....	$[l^0]$	Inductivity, K	$[m^{-1}l^{-3}e^2]$
<i>Kinematical Quantities:</i>		Current, I	$[lt^{-1}e]$
Time, t	$[t]$	Resistance, R	$[mlt^{-1}e^{-2}]$
Velocity, v	$[lt^{-1}]$	Electric field strength, E	$[mlt^{-2}e^{-1}]$
Acceleration, a	$[lt^{-2}]$	Magnetic field strength, H	$[t^{-1}l^{-1}e]$
<i>Mechanical Quantities:</i>		Magnetic pole strength, M	$[ml^2t^{-1}e^{-1}]$
Mass, m	$[m]$	Permeability, μ	$[mle^{-2}]$
Density.....	$[ml^{-3}]$	Magnetic induction, B ...	$[mt^{-1}e^{-1}]$
Momentum, M	$[mlt^{-1}]$	<i>Thermodynamic Quantities:</i>	
Force, F	$[mlt^{-2}]$	Entropy, S	$[S]$
Energy, U	$[ml^2t^{-2}]$	Temperature, T	$[ml^2t^{-2}S^{-1}]$
Energy Density, u ...	$[mt^{-1}l^{-2}]$	Specific heat.....	$[m^{-1}S]$
Pressure, p	$[mt^{-1}l^{-2}]$		
Elasticity of compression, $E = V(dp/dV)$	$[mt^{-1}l^{-2}]$		

which are shorthand restatements of the way the derived quantities depend on the fundamental ones and may be looked upon as statements of the physical nature of the quantities in question. The expressions given in the table are not completely satisfactory since they do not distinguish between the different kinds of product nor take account of the frequent necessity of using differential quantities.¹ They are, however, sufficient for many purposes.

¹ It was first pointed out by W. Williams, Proc. Phys. Soc. Lond., 11, 257 (1892) that ordinary dimensional formulæ do not state all that we should be glad to have them since they omit any reference to the vector nature of some of the quantities. He proposed to remedy this by using different symbols for distances in different directions.

§ 9. UNITS OF MEASUREMENT.

Units for Fundamental Quantities.—In picking out units for the measurement of the five fundamental kinds of quantity considerations as to convenience in magnitude, ease of comparison, permanence and reproducibility of the standards and generality of use will ordinarily be the determining factors. And in agreement with these considerations, the system which is universally adopted by scientists takes as its five fundamental units the centimeter, the second, the gram, the so-called magnetic unit of electric quantity and the centigrade degree. As pointed out by Planck,¹ however, this particular choice has been unduly influenced by the accidents of our purely terrestrial civilization. It would seem to the writer a rational choice to pick out the three units of length, time and mass to satisfy the three conditions that light should travel unit distance in unit time, that two electrons unit distance apart should repel each other with unit force, and that there should be unit energy in an energy quantum having unit frequency, to take as unit charge of electricity that of the electron, and to make unit increase in entropy that which occurs per molecule when a perfect gas is expanded without doing external work to e times its original volume.²

Units for Derived Quantities.—The units for measuring derived quantities may be chosen if desired without any reference to the fundamental units, on the basis of considerations of convenience or familiarity. In case the derived quantity has intensive rather than extensive magnitude some more or less artificial correlation of the magnitude in question with quantities having extensive magnitude will then have to be used, as has been done in the case of our ordinary temperature scale.

A more rational device, however, for the measurement of derived quantities will be to give the unit the magnitude which is obtained by combining unit quantities of the fundamental kinds in the way indicated by the dimensional formula. Such a choice simplifies the theory of measurement of derived quantities since it is then possible to measure

¹ "Theories der Wärmestrahlung," Leipzig, 1913.

² This choice of units would give the numerical value of unity to the velocity of light c , to the constant K in Coulomb's law for the repulsion of electric charges, to the constant h of the quantum theory and to the "gas constant for one molecule," $k = R/N$. Making these constants unity, will result in considerable simplification in numerical calculations.

This choice of units differs from that of Planck in that he also makes the gravitation constant equal to unity. With such a choice, however, it is not possible to take the charge of the electron as unity and at the same time have the constant of Coulomb's law equal to unity, and it seems preferable to sacrifice the gravitational constant rather than that of Coulomb's law, since the author's development of the theory of similitude has indicated that the gravitational constant may not be very fundamental. See Tolman, *PHYS. REV.*, 3, 246 (1914); Nordström, *Öfversigt af Finska Vetenskaps-Societens Förhändlingar*, Afd. A, 57, 1 (1914-15).

the fundamental quantities upon which the derived quantity depends and combine the numbers obtained in the way indicated by the dimensional formula. In the case of quantities having intensive magnitude this is particularly significant since all the fundamental kinds of quantity have been picked out so as to have extensive magnitude, and we obtain in this way a natural rather than an arbitrary correlation between the intensive magnitudes to be measured and a set of extensive magnitudes.

On the Confusion of Quantity and Unit.—The fact that it has become usual to pick out the units for derived quantities in the way indicated has sometimes led to an unfortunate confusion as to the real significance of dimensional formulæ. Thus there has grown up the practice of speaking of the dimensions of a unit when what is really intended is the dimensions of the quantity involved. It certainly seems best, however, to use the dimensional formula of a quantity as a shorthand restatement of its definition in terms of the fundamental kinds of quantity. The dimensional formula is thus a symbol for the physical nature of the derived quantity and a recapitulation of the *necessary* relation between different kinds of quantity rather than the statement of a relation between units which we find convenient.

This opportunity for confusion has been particularly troublesome in the case of the fundamental kinds of quantity, since the desirability of making the units for the different fundamental kinds of quantity depend upon each other in the way indicated has sometimes led to an unfortunate desire to try to make the dimensions of the *quantities* involved depend upon each other in the same way that we have found convenient for the *units*.

The most common proposal of this kind has been to try to give time the same dimensions as length. This has arisen partly, as we have just pointed out, because of the real desirability of making the unit of time depend on the unit of length, and also partly because the recent developments connected with the theory of the relativity of motion have shown the frequent convenience of plotting kinematical events in a four-dimensional space in which the time axis is taken at right angles to the three axes of ordinary space, which has apparently led to the conclusion that time and space must then have the same physical nature and hence should have the same dimensions.

It seems to the writer, however, erroneous, to give time and length the same dimensions. We have already pointed out that a relation between units of measurement which may be found convenient should not be confused with that necessary relation between kinds of quantities which is symbolized by dimensional formulæ. And as to the possibility

of plotting time and distance along axes at right angles to each other this is no more significant than the possibility of plotting the pressures and volumes of a perfect gas in a two-dimensional space, a procedure which certainly does not make pressure have the dimensions of volume. Indeed the fact that, for the most successful plotting of kinematical events, it is necessary to use a non-Euclidean space in which the properties along the time axis are quite different from those along a space axis should have further emphasized the difference between the physical nature of space and time and hence the necessity for difference in dimensions.

In particular we may also point out that the assignment of the same dimensions to time and length would greatly reduce the value and specificity of the principle of the dimensional homogeneity of physical equations.¹ Consider as a single example, the use of the principle of dimensional homogeneity to determine the way in which the centripetal force f which constrains a particle to move in a circle depends on the mass of the particle m , its velocity v and the radius of the circle r . We have

$$f = \phi(m, v, r),$$

where ϕ is the function whose form we wish to determine. Now it is evident that the desired combination of m , v , and r must have the physical nature of a force and hence the dimensions of force. The three quantities involved have the dimensions $[m] = [m]$, $[v] = [lt^{-1}]$, $[r] = [l]$ and it is evident that the only combination of these which has the dimensions of force, $[mlt^{-2}]$, is $[mv^2/r]$ and hence the desired equation must be of the form

$$f = k \frac{mv^2}{r},$$

where k is some purely numerical constant which turns out to be unity with the usual choice of units.

Suppose now, however, we had given time and length the same dimensions l . Then we should have had $[f] = [ml^{-2}]$, $[v] = [l^0]$, $[r] = [l]$, and we see that any one of an infinite number of combinations m/r , mv/r , mv^2/r , mv^n/r , etc., would satisfy the requirements of dimensional homogeneity, and such an identification of the dimensions of length and time would be disastrous to the import of the principle of dimensional homogeneity.

I am inclined to believe that this same confusion of the dimensions of a *quantity* with what might be called the dimensions of the *unit* which

¹ For a discussion of the principle of dimensional homogeneity, see for example the writer's article, *PHYS. REV.*, 6, 219 (1915).

we have chosen for its measurement is a contributory cause for a number of criticisms which have been made on the principle of similitude.¹

Thus I am quite willing to agree with Professor Bridgman² when he says that "the number of fundamental *units* we are to use is in large part a matter of convenience. We can get along with fewer units if we introduce corresponding definitions which are in accord with experimental facts." I could not agree with him, however, if he should conclude from this that the number of fundamental kinds of *quantity* is also reduced, merely by setting up convenient relations between their units. And this would seem to be the basis for what seems to me his erroneous belief that the reasoning used by the theory of similitude is nothing but ordinary dimensional reasoning based on a set of dimensions with only one fundamental kind of quantity. If this contention of Professor Bridgman's were correct, then as will be seen by an examination of the transformation equations involved, we should not only have to ascribe to time the dimensions of length, but to mass the dimensions of the reciprocal of a length, to electric charge the dimensions of angle, and to temperature the dimensions of the reciprocal of a length. These startling conclusions would certainly disagree with our idea of the dimensions of a quantity as a shorthand restatement of its definition and hence as an expression of its essential physical nature, and if such a set of dimensions were adopted, the principle of dimensional homogeneity would become meaningless, since this principle merely expresses the obvious necessity that all the terms in an equation connecting physical quantities shall have the same physical nature.

By way of digression, it may be pointed out from quite a different angle, that the principle of similitude most certainly contains elements foreign to the theory of dimensions, since as was shown in my first paper it permits the prediction that Newton's law of gravitation is not a complete statement of the experimentally determinable facts. If the principle of similitude is true, we are led either to the conclusion that "gravitational mass" is not the same thing as ordinary mass, or to the conclusion that there is a gravitational mechanism of such a nature that the gravitational constant is not independent of other physical quantities. We have perhaps already received the first hint of experimental evidence confirming this prediction if the elaborate work of Shaw³ showing the effect of temperature on gravitation should prove to be conclusive. Gravitational work is also being planned in this laboratory. A theory

¹ Tolman, *PHYS. REV.*, 3, 246 (1914); *Ibid.*, 4, 145 (1914); *Ibid.*, 6, 219 (1915); *Ibid.*, 8, 8 (1916).

² Bridgman, *PHYS. REV.*, 8, 423 (1916).

³ Shaw, *Roy. Soc., Phil. Trans.*, 216, 349 (1916).

should certainly be evaluated not only on the basis of its success in correlating known facts but also for its success in pointing the way to further research.

§ 10. FUTURE DEVELOPMENTS.

In conclusion it will not be out of place to consider certain possibilities of future development in the theory of quantities.

Increase in the Number of Fundamental Kinds of Quantity.—As science becomes more thoroughly rationalized in its more complex fields we shall undoubtedly increase the number of fundamental kinds of quantity. This may take place and indeed has done so in the past in two different ways.

In the first place, as physics develops we may discover new fundamental kinds of "thing" to which we shall ascribe objective existence in the external world and shall then need quantities for measuring this new kind of "thing." Thus in the past only three kinds of quantity, those of mass, length and time, were taken as fundamental, and it was only with the later development of electrical science that we conceived of another kind of "thing" besides matter, namely, electricity, as existing in the world of space and time. To the writer, as pointed out above, it does not seem at all impossible, for example, that we may in the future find "gravitational mass" to be a different kind of thing from ordinary mass, and only accidentally proportional to it in the materials with which we have hitherto experimented. If this should prove to be the case we shall then need a new fundamental kind of quantity for measuring the new kind of thing "gravitational mass."

A second possibility of increasing the number of fundamental kinds of quantity can arise from the desirability of looking at phenomena from a new point of view, with the consequent *invention* of a new fundamental kind of thing. Thus thermal phenomena can apparently all be explained by a consideration of the behavior of the individual atoms and molecules or other elements of which material systems are composed, and as long as we look at these systems from such a microscopic point of view (for example that of the kinetic theory) we do not need to introduce other fundamental kinds of quantity than those of mass, length, time and electricity. But when we wish to look at these systems from a macroscopic point of view, that is, to neglect the behavior of individual molecules and obtain laws which express the statistical result of their interactions, we objectify a new fundamental kind of quantity, entropy or "degree of run-downness." A bridge between the two points of view is given by the Boltzman-Planck equation connecting entropy S

and probability of configuration W ,

$$S = k \log W.$$

It should also be noted that as other fields of science besides physics in the narrower sense come into the field of mathematical analysis we have to introduce further kinds of quantity some of which will probably be most conveniently taken as fundamental and other as derived. Thus in chemistry we need, for example, different kinds of quantity for measuring amounts of each chemical species, and in biology we begin to find the introduction of new kinds of quantity when, for example, the vitality of a given strain of bacteria is correlated with the time it takes for a definite inoculation to produce a definite amount of chemical change. And in the distant future we may even hope that the speculations of economists, and sociologists will at least regard exact quantitative investigation as a desired goal and such developments will also necessitate the introduction of new kinds of measurable quantity.

Possibility of Decreasing the Number of Fundamental Kinds of Quantity.

—In building up any logical system it is of course desirable to choose only a small number of propositions as postulated and only a small number of concepts as indefinable. For this reason it would be desirable to decrease the number of kinds of quantity that we take as fundamental and indefinable, provided this did not itself introduce complexities. At the present time, however, the writer can see no possibility of reducing the number of fundamental kinds of quantity for the accepted body of physics below five, without, indeed, producing a complete alteration in the nature of our system of physical concepts. The reason why there must be five fundamental kinds of quantity is because the physicist has chosen to make use of five fundamental kinds of concept, namely space, time, matter, electricity, and entropy or probability of configuration. In terms of these five kinds of "thing" he can define other "things" in the physical world. Thus he can define a quantity of velocity as a quantity of length divided by a quantity of time, or a quantity of momentum as a quantity of mass multiplied by a quantity of velocity, but if he tries to claim that a quantity of time is the same thing as a quantity of length, or that a quantity of mass is the same thing as the reciprocal of a quantity of length he will meet difficulties that are insurmountable.

PROCEEDINGS
OF THE
AMERICAN PHYSICAL SOCIETY.

AN EXTENSION OF THE MAYER EXPERIMENT.¹

BY R. R. RAMSEY.

THE experiment consists in giving the bicycle balls which are floated on mercury a rotation and thus better representing the modern idea of atomic structure than is done by the floating needles. The apparatus consists of a wooden tray with a platinum electrode in the middle and four platinum electrodes, one in each corner. When a current of electricity is sent into the mercury through the middle electrode and out at the corner electrodes which are connected in multiple, we have an approximately radial current flowing at right angles to the magnetic field of the magnet which plays the part of the positive nucleus. This causes the mercury to rotate and the balls are carried with the mercury. The motion of the balls is not simply the rotation of the Mayer figures. The centrifugal force and the fact that the angular velocity decreases as the radius increases causes the motion of any one ball to be very complicated. With two balls the paths of the balls approximate to rotating ellipses, the balls taking turns at the center. If the balls represent electrons the various motions and perturbations may be said to be the source of the various radiations of the atoms. A ball coming from the outside into the system produces a great disturbance. This disturbance may be said to be the source of the X-rays. At times a ball starts out on a tangent and seems to try to escape. A tray was made in which a ring of iron was imbedded, thus causing the magnetic field of the magnet to vary from a maximum at the center and pass through a minimum and then increase to a maximum over the ring, as we move along a radius. By this means a ball which starts on a tangent is held over the ring. If the balls represent alpha particles then we have an illustration of the disintegration of a radio-active substance. From five to ten disintegrations can be represented in this manner.

INDIANA UNIVERSITY,
Nov. 11, 1916.

¹ Abstract of a paper presented at the Chicago meeting of the Physical Society, December 2, 1916.

THE MAGNETIC PROPERTIES OF Fe, Ni, AND Co ABOVE THE CURIE POINT,
AND KEESOM'S THEORY OF MAGNETIZATION.¹

BY EARLE M. TERRY.

THE paper gives the final results of an investigation of the magnetic properties of iron, nickel and cobalt from the temperature at which ferro-magnetism disappears up through the melting point, a preliminary report of which was made at the Chicago meeting of last year. The work has been carried out in a good vacuum, the high temperatures being obtained by means of a special tungsten furnace. The results show that Curie's law,

$$\chi \cdot T = \text{const},$$

where χ is the susceptibility and T the absolute temperature, holds in the case of nickel only, and here in but a restricted region. For β iron, β cobalt and β nickel in the lower temperature region, the $1/\chi$, T curves all show a distinct concavity upward instead of being straight lines, as demanded by Curie's law. For γ iron, the susceptibility varies only slightly with temperature but the concavity of the χ , T curve is downward instead of upward as required by the law. At the γ - δ transformation point an increase in χ of approximately 50 per cent. occurs above which the χ , T curve is a straight line with a large negative slope.

In passing through the melting point, the susceptibility for iron decreases rapidly, while for nickel there is no change whatever, but for cobalt the χ , T curve shows a slight depression.

These results are in marked contrast to those of Weiss and Foëx who found Curie's law to hold in all cases. Keesom's quantum theory of magnetization, including the assumption of a zero-point energy, gives χ , T curves of the same general character as those obtained by the author but they depart from the asymptote less rapidly than required by experiment. An attempt has been made to obtain a better fit by modifying the theory by the assumption of a larger zero point energy, but it has been found that even when the latter is taken twenty times that of the theory, the agreement with experiment is still unsatisfactory.

PHYSICAL LABORATORY,
UNIVERSITY OF WISCONSIN.

A. C. AND D. C. CORONA IN HYDROGEN.¹

BY JOHN W. DAVIS.

THE corona discharge between concentric cylinders when the space between the cylinders is filled with hydrogen presents some characteristic differences from the discharge in air.

¹ Abstract of a paper presented at the Chicago meeting of the Physical Society, December 2, 1916.

1. The discharge in hydrogen does not start and stop at the same voltage.
2. The current obtained with a given voltage impressed between the two cylinders depends on whether that value of voltage was obtained by an approach through lower or higher values.
3. The discharge takes place at decidedly lower voltages in hydrogen than in air.
4. There is a wide difference in the critical electric intensity at the surface of the inner cylinder, depending on whether this cylinder is positive or negative.
5. The character of the volt-ampere characteristics with the inner cylinder positive and the inner cylinder negative are totally different.
6. The corona discharge in hydrogen may be used for the rectification of alternating currents.
7. The appearance of the corona, spark and arc are different in air and hydrogen.
8. There is apparent evidence of ionization potential gradients at the surface of both the inner and outer cylinders, as is shown by the appearance of light at both of these surfaces.

PHYSICAL LABORATORY,
UNIVERSITY OF ILLINOIS.

THE KINETIC THEORY OF NON-SPHERICAL RIGID MOLECULES.¹

BY YOSHIO ISHIDA.

THIS paper deals with the kinetic theory of a system of non-spherical rigid molecules according to the method of Boltzmann's integro-differential equation.

With use of the general notion of invariants of encounters, the conditions of conservation during encounter, which are independent of any supposition regarding the frequency of various types of encounter, are shown to lead to a kind of extended hydrodynamical theory, which includes not only the familiar equations of mass, force and energy but in addition what may be called the equation of gyroscopic disturbances.

Introduction of a plausible hypothesis regarding the frequency of encounters leads to the conclusion that for the case of equilibrium a system of centrally symmetric rigid molecules would satisfy the same relation of pressure, volume and temperature as the ideal monatomic gas.

RYERSON LABORATORY,
UNIVERSITY OF CHICAGO,
November 17, 1916.

¹ Abstract of a paper presented at the Chicago meeting of the Physical Society, December 2, 1916.

THE PHYSICAL REVIEW.

NATURAL AND MAGNETIC ROTATORY DISPERSION IN THE INFRA-RED SPECTRUM.

BY L. R. INGERSOLL.

INTRODUCTION.

IN spite of the large amount of investigation which has been carried out on the rotation of the plane of polarization by naturally active substances for wave-lengths of the visible spectrum and to a certain extent in the ultra-violet, apparently nothing has hitherto been done on the infra-red side save in the case of the one substance, quartz.

Hussel,¹ using the phosphoro-photographic method of Lommel made measurements of the rotatory dispersion of this crystal to wave-length 1μ , while Carvallo,² Dongier³ and others,⁴ using modifications of the original method of Desains,⁵ extended this spectral region to beyond 2μ . In none of these investigations, however, were other substances studied, possibly because of the difficulties involved in measuring the much smaller rotations usually found in materials other than quartz.

By a slight modification of the method and apparatus already developed and used by the writer⁶ for measuring magnetic rotation in the early infra-red it has been found possible to determine the rotation of naturally active substances as well, in this spectral region, and with an accuracy at least comparable with that of the best visual observations. This is sufficient to handle even substances of small rotatory power, and accordingly the rotatory dispersion of several organic liquids and solutions,

¹ A. Hussel, Wied. Ann., 43, p. 498 (1891).

² E. Carvallo, Comptes Rendus, 114, p. 288 (1892).

³ R. Dongier, Comptes Rendus, 125, p. 228 (1897) & 126, p. 1627 (1898).

⁴ G. Moreau, Ann. Chim. Phys., 30, p. 433 (1893), and A. Hupe, Wied. Ann. Beib., 19, p. 501 (1895).

⁵ P. Desains, Pogg. Ann., 128, p. 487 (1866).

⁶ Phil. Mag. (6), 11, p. 41 (1906), and 18, p. 74 (1909). Phys. Rev., 23, p. 489 (1906).

as well as of quartz, has been determined for the spectral region in which it is possible to work with this method, viz., from about the D lines to a little beyond 2μ . In some cases supplementary visual observations have been made to carry the series farther towards the blue end of the spectrum.

Magnetic rotation measurements on these substances have also been made at the same time to add further evidence on Wiedemann's law of the proportionality of magnetic and natural rotations for different wave-lengths. This law, first enunciated by G. Wiedemann¹ in 1851 as the result of experiments on turpentine for a series of five Fraunhofer lines, has been the subject of some controversy. Disch² concluded from experiments on ethyl valerate and quartz that it holds approximately for the visible spectrum, but more recently Darmois³ finds that for a number of compounds of the terpene series, at any rate, the law does not hold at all. He concludes that Wiedemann's relation, then, was the result of pure chance and has no longer any significance. Very recently, however, Lowry⁴ in the course of an extremely careful investigation of the rotatory dispersion of quartz finds that the law applies with great accuracy for this material over the range of spectrum ($\lambda = .43\mu$ to $.67\mu$) which he has been able to investigate. He suggests that its validity may be confined to crystals and that active liquids may constitute exceptions.

It becomes, then, a matter of considerable interest to test this law further, over the greatest possible spectral range and with an experimental arrangement which shall afford identical conditions (of wave-length, temperature, part of crystal used, etc.) for each series of measurements.

Temperature coefficients of both natural and magnetic rotation have also been determined in one or two cases in the present work, with the view of attempting to decide whether or not these coefficients depend on the wave-length. Soret and Sarasin⁵ found an apparent increase of the natural coefficient for quartz in the shorter wave-lengths. Molby⁶ has also recently found indications of the same thing over the range from room to liquid air temperatures. On the other hand, v. Lang,⁷ Sohncke⁸ and Gumlich⁹ have not been able to draw any such conclusions, their results on this crystal pointing to an independence of wave-length.

¹ G. Wiedemann, Pogg. Ann., 82, p. 215 (1851).

² J. Disch, Ann. d. Phys., 12, p. 1153 (1903).

³ E. Darmois, Ann. Chim. Phys., 22, p. 495 (1911).

⁴ T. M. Lowry, Phil. Trans., 212, p. 261 (1913).

⁵ Soret and Sarasin, Sur la Polarisation Rotatoire du Quartz, Geneva, 1882.

⁶ F. A. Molby, Phys. Rev., 31, p. 291 (1910).

⁷ V. v. Lang, Sitzungsber. d. k. Ges. d. Wiss. zu Wien, 71, p. 707 (1875).

⁸ L. Sohncke, Wied. Ann., 3, p. 516 (1878).

⁹ E. Gumlich, Wied. Ann., 64, p. 333 (1898).

It has seemed worth while, then, to supplement these observations with measurements over the six-fold greater spectral region which the writer has investigated, and to extend them to the magnetic rotation as well; also to test one or two other substances in addition to quartz.

METHOD AND APPARATUS.

The method need be outlined only briefly as it has—save for the slight but essential modification which admits of its application to the measurement of natural rotations—been already described in previous papers by the writer.¹ The general disposition of the apparatus is evident from

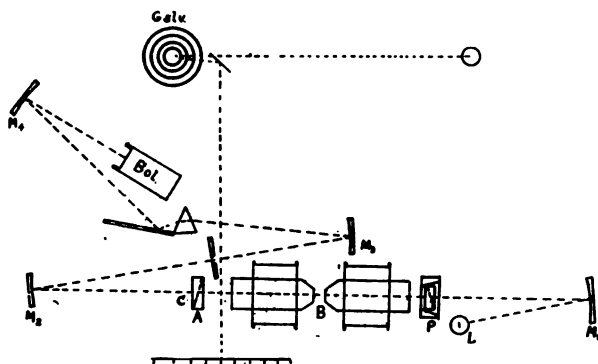


Fig. 1.

Fig. 1. Light from the flat strip tungsten lamp L ,² after reflection from M_1 is polarized by P , transmitted down the axis of the electromagnet, and, after division into two beams by the (double image prism) analyzer A , falls on the slit of the spectrometer. The two images on the slit, of mutually perpendicular planes of polarization, form two spectra on the strips of the differential bolometer which is connected with a sensitive Thomson galvanometer. If the polarizer is oriented so that its plane makes an angle of 45° with each of the principal planes of the analyzer the intensity of radiation on each strip of bolometer will be the same and the galvanometer will show no deflection: but if a rotation is produced, as by a cell with sugar solution placed at B , a galvanometer deflection will result which may be used to measure such rotation. The magnetic rotation produced by exciting the electromagnet is obviously measurable in the same way.

¹ Loc. cit., particularly Phil. Mag., 18, p. 77 (1909).

² These lamps were specially made and furnished for this investigation through the kindness of Director E. P. Hyde of the Nela Research Laboratory. They give a line source many times more brilliant than the Nernst glower formerly used and of great constancy. Without them the present work would have been practically impossible.

A *modification of the method* as already mentioned will be seen to be necessary for the present work from the following considerations: Magnetic rotation, being produced by merely exciting the magnet, does not involve in the process of measurement any shift of the substance under test, *i. e.*, its removal from the field. Natural rotation, on the other hand, obviously cannot be produced or reversed in this simple way and it would seem to be necessary to remove the test cell entirely to get the zero from which the rotation is to be measured. This, however, must be avoided if possible as it is almost certain to involve slight shifts of the beam of light and a consequent lack of uniformity accompanied by a spurious galvanometer deflection.

To avoid this, the polarizer, which was mounted in a large divided circle, was arranged so that it could be rotated exactly 90° between stops. Fig.

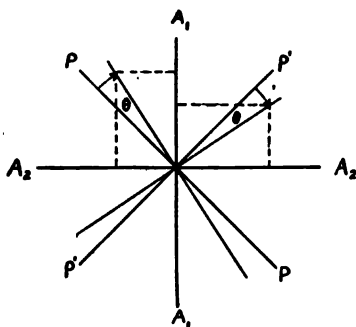


Fig. 2.

2 shows almost at a glance the results of such rotation. The original plane of polarization P being rotated by the substance through the small angle θ , gives unequal components on the two principal planes of the analyzing double image prism. A rotation of the polarizer to P' will exactly interchange these two components and a doubled galvanometer deflection will result. The proc-

ess is analogous to reversing the direction of the field in measuring magnetic rotations.

To interpret these galvanometer deflections in terms of degrees and minutes of rotation by the active substance, the polarizer was turned by a measured amount—usually about 3 degrees—and a comparison of the resulting galvanometer deflection with that above noted gave at once an accurate measurement of the (magnetic or natural) rotation for any chosen wave-length. It should be noted that while this was the general method the procedure had to be varied somewhat for rotations as large as those encountered in quartz.

Sources of error incidental to the bolometric method have been discussed at some length in the previous articles and only some special precautions need be mentioned here.

A large (9 cm. high) 45° prism of dense glass gave somewhat higher dispersion than that used in most of the writer's previous experiments, although energy requirements preclude very great advances along this line. A test was made in the case of quartz of the errors due to slit and

bolometer widths by exaggerating the effect; the conclusion reached was that the maximum error from this cause did not exceed $\frac{1}{2}$ per cent. The wave-length scale of the prism had originally been determined by double dispersion and comparison with rock salt. Repeated careful checks of the position of a number of infra-red solar absorption lines indicated that the probable error in the wave-length scale should be less than $.01\mu$.

To eliminate a variety of errors, each measurement was preceded by a blank run with the cell or crystal in position *C* where it should, of course, produce no deflection, any effect shown here was applied as a small correction. Some precautions had to be used in this connection, particularly in the case of quartz, because of the slight polarizing action of the glass prism of the spectrometer. On the magnetic side careful tests for spurious rotations indicated that these did not in any case exceed 0.0006° .

Perhaps the best indication of the general accuracy of the results comes from the comparison which was made in several cases of the rotation measured bolometrically for the wave-length of the *D* lines with the same rotation measured with sodium light by the customary visual method using a tri-field polarizer. The agreement in every case was within 1 per cent.

RESULTS.

Six representative substances were tested, including a crystal, two organic liquids, two aqueous and one alcohol solution. One of the

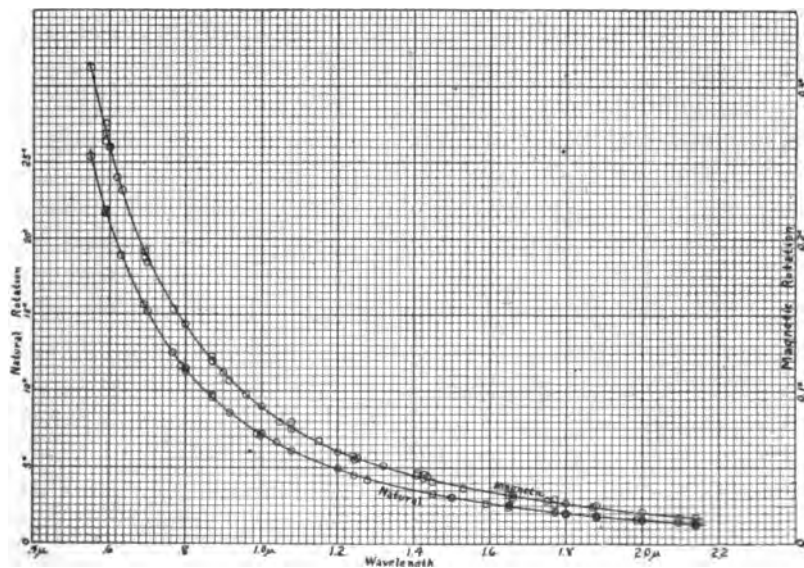


Fig. 3.

Rotary Dispersion Curves for Quartz.

solutions (tartaric acid) shows anomalous rotatory dispersion in the visible spectrum.

Quartz.

The sample tested was a clear piece of right-handed quartz 5.030 mm. thick. The rotation for the *D* line and 20° C. was found to be 21.739° per mm., in excellent agreement with the value of 21.728° found by Lowry. The actual measurements are plotted in Fig. 3 and are seen to lie with few exceptions very well on the curve. The tabulated values were read from the smooth curve (plotted on a large scale).

TABLE I.

Quartz—Rotation per Mm. at 20° C.

Wave-length, μ	Natural Rotation.					Magnetic Rotation, Observed.	Mag. <i>R</i> Nat. <i>R</i> .
	Observed.		Calculated.		Temp. Coeff.		
	Present Work.	Others.	Lowry For- mula.	Gumlich For- mula.			
.5495	25.403°	21.728° (Lowry)	25.201°	21.719°	.000164 (39°—129° C.)	0.3120°	0.0123
.5893	21.739		21.728			.2694	.0124
.600	20.914		20.915			.2595	.0124
.700	15.139		15.124			.1860	.0123
.800	11.432	6.18 (Carvallo)	11.443	6.20	.000185 (21°—141° C.) .000152 (20°—141° C.)	.1425	.0125
.900	8.873		8.953			.1119	.0126
1.00	7.121		7.189			.0890	.0125
1.08	6.123		6.122			.0752	.0123
1.10	5.904		5.893			.0724	.0123
1.20	4.918		4.911			.0603	.0123
1.30	4.101		4.151			.0516	.0126
1.40	3.515		3.549			.0448	.0127
1.45	3.272	3.43 (Carvallo)	3.295	3.41	.000156 (20°—125° C.)	.0418	.0128
1.50	3.030		3.055	.0389		.0128	
1.60	2.662		2.669	.0341		.0128	
1.70	2.348		2.341	.0303		.0129	
1.715	2.304	1.83 (Moreau)	—	2.28	.000166 (20°—113° C.)	—	—
1.77	2.153	2.28 (Carvallo)	2.144			.0277	.0129
1.80	2.070		2.035			.0266	.0128
1.82	2.020	2.22 (Dongier)	2.017			.0259	.0128
1.90	1.827	1.60 (Carvallo)	1.835	1.55		.0234	.0128
2.00	1.614		1.636			.0205	.0127
2.10	1.431		1.466			.0179	.0125
2.14	1.372		1.404			.0169	.0123

Dispersion Formulæ.—Of the numerous formulæ which have been suggested to represent the natural rotatory dispersion of quartz we shall consider only two. The one used by Gumlich¹ is of the type

$$R = \frac{A}{\lambda^2} + \frac{B}{\lambda^4} + \dots$$

to five terms. These five constants naturally permit a close fitting of calculated to experimental results and this Gumlich has done, using his own results in the visible spectrum, Soret and Sarasin's for the ultra-violet and Carvallo's and Dongier's infra-red observations quoted in Table I., in determining the constants.

Lowry² finds that the formula

$$R = \frac{11.6064}{\lambda^2 - \lambda_1^2} + \frac{13.42}{\lambda^2 - \lambda_2^2} - \frac{4.3685}{\lambda^2},$$

where

$$\lambda_1^2 = 0.010627; \quad \lambda_2^2 = 78.22,$$

λ being expressed in microns, represents his measurements in the visible spectrum with great accuracy. Using this formula *without changing the constants* in any way, the figures in Table I. were calculated. It will be noted that the agreement with the writer's observed values is exceedingly good throughout the range of the spectrum, the average divergence indeed being only a fraction of a per cent.

Wiedemann's Law.—Magnetic rotations for a field of approximately 9,400 gaussess were made with the quartz in the same position as for the naturally active work. The ratios in Table I. show some variation, it is true, but part of this, at any rate, may be accounted for by the fact that the magnetic rotations were small and the probable error consequently larger than for the other rotations. There is certainly no consistent variation with wave-length and on the whole the results point strongly to the validity of Wiedemann's law for quartz over the whole spectral range investigated.

Temperature Coefficients.—The experimental arrangements for determining temperature coefficients of rotation were somewhat simpler than perhaps would have been the case if this had been the primary object of the investigation. The quartz was mounted in a simple brass cell, heated by a winding of resistance wire, the temperature being determined with a mercury thermometer. The results in Table I. vary somewhat but show no definite relation between coefficient and wave-length. Their mean value is in agreement with Gumlich's determination of 0.000165

¹ Loc. cit., p. 350.

² Loc. cit., p. 287.

(0°–100° C.) for the visible spectrum. Because of the very small changes involved the coefficient of magnetic rotation proved very hard to determine with any accuracy. The figures obtained, however, indicate a value somewhat higher than for the natural rotation.

Organic Liquids.

Liquids were tested in a cell 36 mm. diameter and 9.54 mm. thick, formed by drilling a hole through a plate of glass of this thickness. The sides were of thin plate glass which had been optically tested for strains. While they had no effect on the natural rotation as measured, correction had to be made, of course, for their magnetic rotation. It may be of interest to note that ordinary finely sifted Portland cement proved the most suitable substance for fastening the parts of the glass cell together. When adjoining surfaces were ground to give a good adhesion to the cement these cells proved entirely satisfactory for all the liquids and solutions tested.

Limonene and pinene were the organic liquids selected for test. The (dextro¹) limonene was furnished through the kindness of Professor Kremers and Dr. Wakeman, of the pharmacy department, to both of whom the writer has been greatly indebted during the course of this investigation. Its density at 20° C. was .845 and boiling point 175° C. The (*d* – α) pinene was kindly furnished by Dr. Schorger, of the Forest Products Laboratory. It was distilled from Port Orford cedar, boiling point 155° C., density at 15° C. .863, and it had one of the highest recorded rotations for this substance. It was redistilled the day before testing.

The results are shown in Table II., the natural and magnetic rotations having been read from the smooth curves. The ratios of the two show no consistent variation with wave-length, indicating that Wiedemann's law holds as well for these liquids as for quartz. It is of interest to note in this connection, however, that the temperature coefficients for the two rotations may be quite different as shown by the results on limonene. This difference is exaggerated if the coefficients of specific rotation are calculated, taking account of the change in density with temperature; the specific natural rotation coefficient of this substance averages .0012 with the magnetic less than one fourth as much. Both show much greater and more irregular variations with wave-length.

The specific natural rotations listed in Table II. are expressed in the usual way in terms of a decimeter thickness and unit density.

¹ It is to be noted that what the chemist calls a dextro rotation is really a left-handed twist when viewed from the customary standpoint of the physicist, *i. e.*, along the path of the light. Thus right-handed quartz and dextro limonene really rotate in opposite directions.

TABLE II.

Optical Rotation of Two Terpenes at 22° C.

Wave-length, μ	Limonene.						Pinene.			
	Observed Natural Rotation.	Temp. Coeff. (25° — 120° C.) Nat. R.	Specific Rotation.	Observed Magnetic Rotation.	Temp. Coeff. (25° — 120° C.) Mag. R.	Mag. R. Nat. R.	Observed Natural Rotation.	Specific Rotation.	Observed Magnetic Rotation.	Mag. R. Nat. R.
.50	13.915°		173.0°	3.571°		.257	5.716°	69.90°	3.252°	.569
.5893	9.665		120.1	2.475		.256	3.968	48.48	2.291	.577
.60	9.220		114.6	2.363		.257	3.818	46.59	2.196	.575
.70	6.482	-.00201	80.56	1.639	-.00114	.253	2.708	33.09	1.463	.540
.80	4.950	-.00206	61.55	1.229		.248	2.066	25.24	1.119	.541
.90	3.825		47.54	0.938		.245	1.606	19.61	0.870	.542
1.00	3.024	-.00205	37.61	0.747	-.00114	.247	1.286	15.71	0.702	.546
1.10	2.500		31.09	0.622		.249	1.058	12.93	0.573	.542
1.20	2.059	-.00201	25.60	0.522		.253	0.876	10.73	0.495	.564
1.30	1.734		21.57	0.445		.257	0.742	9.06	0.433	.584
1.40	1.509		18.76	0.383		.254	0.640	7.82	0.365	.570
1.50	1.325	-.00192	16.48	0.342	-.00117	.258	0.542	6.62	0.303	.560
1.60	1.191	-.00192	14.81	0.308	-.00123	.259	0.500	6.11	0.273	.547
1.90	0.870		10.82	0.232		.266	0.408	4.99	0.232	.567
2.00	0.780		9.70	0.208		.267	0.380	4.64	0.222	.583

Absorption Bands.—Both limonene and pinene show marked absorption bands at 1.2μ and 1.4μ while the extinction at 1.8μ is too great to allow of reliable measurements in this region. Theories of optical activity agree that absorption which is electronic in character may produce anomalies in the rotatory dispersion curve. Drude associates such anomalies only with those electrons which vibrate along spiral paths, but the recent theory proposed by Gray¹ and experimentally verified for solutions of neodymium nitrate, assumes that any absorption band belonging to an electron in an active molecule may be accompanied by an anomaly.

The fact that limonene and pinene show no such anomalies may, however, be explained in several ways. The dispersion may not have been great enough to detect them, although this is considered unlikely as the bolometer subtended less than $.02\mu$ in this part of the spectrum. A more reasonable assumption is that these absorption bands are not primarily electronic in their origin and hence would not be expected to be accompanied by anomalies in dispersion.

¹ Frank Gray, *PHYS. REV.*, 7, p. 472 (1916).

Solutions.

Three solutions were experimented with: a 20 per cent. (by weight) aqueous solution of cane sugar (density at 22° C., 1.081); a 28.62 per cent. aq. solution of tartaric acid (density at 22° C., 1.140); and a 34.70 per cent. solution of camphor in absolute alcohol (density at 20° C., 0.845). The last two were chosen because they (*i. e.*, same percentage solutions) had been previously examined by Nutting¹ in his study of ultra-violet rotatory dispersion. All show dextro-rotations in the spectral region examined in the present work.

The results are shown in Table III. The specific rotations (*i. e.*,

TABLE III.

Optical Rotation of Three Solutions.

Wave-length. μ .	Tartaric Acid (20° C.).			Cane Sugar (22° C.).			Camphor (20° C.).		
	Observed Natural Rotation.	Specific Rotation.	Mag. R Nat. R	Observed Natural Rotation.	Specific Rotation.	Mag. R Nat. R	Observed Natural Rotation.	Specific Rotation.	Mag. R Nat. R
.45	0.365°	11.71°	—	—	—	—	—	—	—
.50	0.387	12.42	—	—	—	—	—	—	—
.55	0.369	11.84	—	—	—	—	—	—	—
.60	0.318	10.21	5.75	1.288°	62.48°	1.42	1.098°	39.26°	1.51
.70	0.227	7.28	5.68	0.922	44.68	1.40	0.725	25.92	1.64
.80	0.181	5.80	5.51	0.700	33.93	1.42	0.528	18.87	1.71
.90	0.147	4.71	5.34	0.543	26.32	1.44	0.388	13.87	1.75
1.00	0.118	3.78	5.34	0.428	20.76	1.47	0.308	11.01	1.80
1.10	0.098	3.15	5.31	0.351	17.02	1.48	0.245	8.75	1.89
1.20	0.082	2.63	5.25	0.293	14.21	1.55	0.194	6.93	2.13
1.30	0.067	2.15	—	0.246	11.93	1.52	0.164	5.86	2.16
1.40	—	—	—	—	—	—	0.145	5.18	2.17

the rotations per decimeter divided by the product of the density and percent (by weight) strength of solution) for tartaric acid and cane sugar are in good agreement with the values listed by Landolt in his *Optische Drehungsvermögen*, but the camphor is somewhat low. The curve of the natural rotatory dispersion of tartaric acid is shown in Fig. 4. The other curves are of the same general character as those for limonene or quartz and show no anomalies. Tartaric acid has long been known to exhibit anomalous rotatory dispersion and Nutting found that in solutions of this concentration the rotation changes sign at $\lambda = .38\mu$ and becomes negative for shorter wave-lengths and with much larger values. The interesting point about this curve in Fig. 4 is that the

¹ P. G. Nutting, *PHYS. REV.*, 17, p. 1 (1903).

anomalous character disappears entirely after $\lambda = .6\mu$ and the dispersion is quite similar to that of the other substances for longer wave-lengths.

The ratio of magnetic and natural rotations is seen to be less constant for these solutions than for the former substances. Camphor shows the greatest variation from Wiedemann's law, although tartaric acid would unquestionably prove a striking exception for wave-lengths shorter

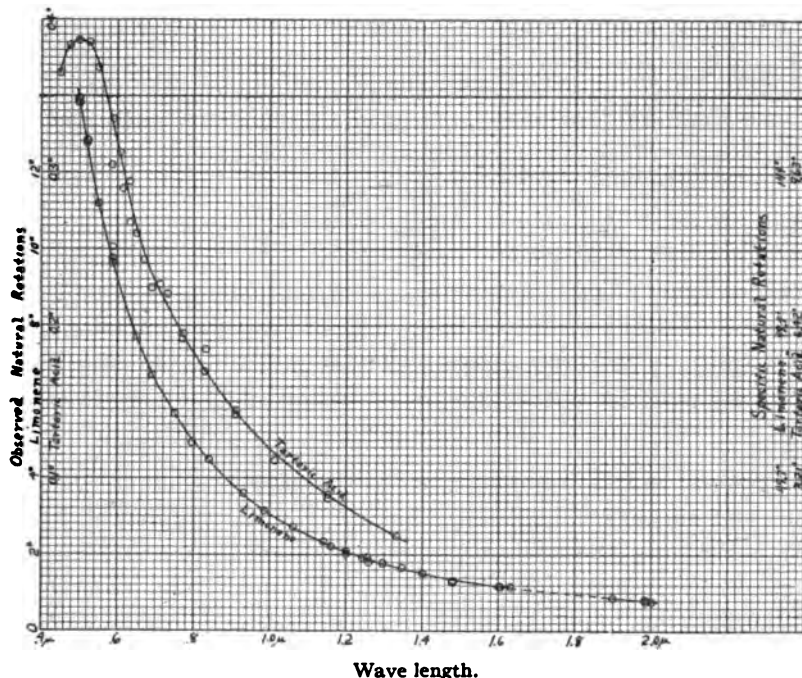


Fig. 4.

Rotatory Dispersion Curves for Limonene and Tartaric Acid.

than $.6\mu$ had the magnetic rotation been investigated in this region. These shorter wave-length points were obtained by visual measurements with a 20 cm. tube which did not lend itself readily to magnetic investigations. The magnetic rotatory dispersion of both tartaric acid and cane sugar solutions in the spectral region investigated is, however, practically identical with that of pure water and may be safely assumed to be such in this region also. In discussing Wiedemann's law for solutions some writers (vide Darmois, loc. cit.) attempt to separate the magnetic rotations of the solvent and solution, but the writer is sceptical of the value of such calculations and has not made use of them here.

Dispersion Ratios.—The relative dispersions of the various substances tested may be seen at a glance from the following table in which the

rotation for wave-length $.6\mu$ is called unity in each case. It will be noted that in practically all cases the rotations decrease in value more rapidly than the inverse square of the wave-length.

TABLE IV.

Wave-length, μ .	Inverse Square.	Quartz.		Limonene.		Pinene.		Cane Sugar.		Tartaric Acid.		Camphor.	
		Nat. R.	Mag. R.	Nat. R.	Mag. R.	Nat. R.	Mag. R.	Nat. R.	Mag. R.	Nat. R.	Mag. R.	Nat. R.	Mag. R.
.60	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
.80	0.563	0.547	0.549	0.537	0.520	0.541	0.510	0.544	0.543	0.569	0.543	0.481	0.547
1.00	0.360	0.340	0.343	0.328	0.316	0.337	0.320	0.333	0.344	0.371	0.344	0.281	0.335
1.30	0.213	0.196	0.199	0.188	0.188	0.194	0.197	0.191	0.204	0.211	—	0.149	0.213
1.60	0.141	0.127	0.131	0.129	0.130	0.131	0.124	—	—	—	—	—	—
2.00	0.090	0.077	0.079	0.084	0.088	0.099	0.101	—	—	—	—	—	—

In conclusion the writer takes pleasure in acknowledging his indebtedness to the Rumford Fund. A large share of the apparatus with which these measurements were carried out was originally purchased with grants from this source.

SUMMARY.

1. The natural and magnetic rotatory dispersions of quartz, limonene, pinene, and solutions of cane sugar, tartaric acid and camphor have been determined over a range of wave-lengths from the visible to beyond 2μ . The curves are all regular save in the case of tartaric acid in the shorter wave-lengths.

2. The natural rotation for quartz over the whole spectral range agrees very well with the formula used by Lowry, with constants determined by him from visible spectrum observations.

3. The temperature coefficients of rotation as determined for quartz and limonene show no definite dependence on wave-length. The natural and magnetic rotation coefficients have quite different values, however, in the case of limonene.

4. Wiedemann's law of the proportionality of natural and magnetic rotations holds with a fair degree of approximation for quartz and the two terpenes over the whole spectral range investigated. It does not hold as well for the solutions, the variation being greatest in the case of camphor.

PHYSICAL LABORATORY,
UNIVERSITY OF WISCONSIN,
January 10, 1917.

THEORETICAL CONSIDERATIONS ON THE NATURE OF
METALLIC RESISTANCE, WITH ESPECIAL REGARD
TO THE PRESSURE EFFECTS.

BY P. W. BRIDGMAN.

IN a recent number of the Proceedings of the American Academy¹ I have given data for the effect of pressures to 12,000 kg. and temperatures between 0° and 100° C. on the electrical resistance of 22 metals. A detailed discussion of the nature of the results and methods of measurement and calculation will be found in that paper. The most important results follow. (1) The temperature coefficient of resistance is nearly independent of pressure; this holds for pressures high enough to compress the metal in many cases to less than its volume at 0° Abs. Another way of stating the same fact is that the pressure coefficient is independent of temperature. (2) The relative pressure coefficient of resistance becomes less at higher pressures. (3) The curvature of the resistance-pressure curves is in most cases greater at lower temperatures. (4) Two metals, bismuth and antimony, show an abnormal positive pressure coefficient of resistance. The anomaly of bismuth was known before, but that of antimony is new.

The purpose of this paper is to present a view of the nature of metallic conduction which had its origin in an attempt to bring into line the facts at high pressures, but which also accounts for other facts not intimately connected with the pressure effects. In brief, this view of the nature of conduction regards all metals as naturally perfect conductors in the sense that the electrons may pass without resistance from atom to atom when the atoms are in contact at rest. The precise mechanism of this free transfer need not be further specified for our purposes; we may if we like suppose the electrons capable of freely penetrating the substance of the atom, in much the same way as recently suggested by Professor Hall.² If the atoms become too much separated, however, the electrons encounter difficulty in passing from atom to atom, and if the separation is too great, are almost entirely unable to pass. In a solid at absolute zero the electrons pass freely from atom to atom, thus giving perfect conductivity, but as the temperature is raised, the atomic centers become separated, and if the separation passes a certain critical value, the electrons encounter

¹ P. W. Bridgman, *Proc. Amer. Acad.*, 52, 571-646, 1917.² E. H. Hall, *Proc. Amer. Acad.*, 50, 67-103, 1914.

difficulty, and electrical resistance makes its appearance. In a solid under given conditions there must be a certain number of pairs of atoms separated by more than the critical distance, or we may say, there are a certain number of "gaps." The resistance is proportional to the number of gaps. Now the number of gaps evidently is closely connected with the mean amplitude of oscillation of the atoms. Our first task, in attempting to cast this view of the nature of conduction into quantitative form, is to find how the amplitude changes with pressure and temperature.

The following deduction of the variation of amplitude and frequency is valid only at moderate temperatures, where the classical statistical mechanics and quantum theory are not essentially in conflict. For these temperatures, everything may be obtained from three equations. We assume in the first place that the vibration of an atom is simple harmonic, and that the energy of a degree of freedom is equal to the energy of a degree of freedom of a gas atom at the same temperature. This gives as a first equation

$$\nu^2 \alpha^2 = \text{const } \tau, \quad (1)$$

where ν is the average frequency of atomic vibration, and α is the average amplitude. The second assumption concerns the nature of the restoring force under which the atoms execute their harmonic vibrations. If the volume remains unaltered, it is natural to suppose that in most cases the average restoring force for a given displacement will not be affected by such changes as changes of temperature. We therefore write as the second equation

$$\left(\frac{\partial \nu}{\partial \tau} \right)_v = 0. \quad (2)$$

These two relations are not enough to completely determine ν and α as functions of pressure and temperature. As the third equation I will adopt Grüneisen's relation¹

$$\frac{1}{\nu} \left(\frac{\partial \nu}{\partial p} \right)_s = \frac{1}{C_p} \left(\frac{d\nu}{d\tau} \right)_p. \quad (3)$$

These three relations must all be recognized as only approximate. At low temperatures $\nu^2 \alpha^2$ increases more rapidly than τ , and at high temperatures there may be further increase. The frequency cannot be a function of volume only unless at constant volume the average restoring force on an atom is a linear function of atomic displacement. But it seems inevitable that as the amplitude of oscillation becomes greater the restoring force, due to contact between atoms, must increase more

¹ E. Grüneisen, *Ann. Phys.*, 39, formula 23, p. 276, 1912.

rapidly than the displacement, resulting in an increase of frequency as temperature is increased at constant volume. The existence of an effect of this nature is demonstrated by the existence of a positive thermal expansion. For if the restoring force were truly linear, the expansion must probably vanish, as I have suggested in connection with effects found in liquids at high pressures, and as Debye¹ has suggested more recently. Equation (2) is, therefore, only a first approximation. Grüneisen's relation, equation (3), is in accordance with an equation of quantum theory, obtained by assuming Planck's relation between energy and entropy. Planck finds that the entropy is a determinate universal function of ν/τ . Grüneisen's assumption is more general than the relation obtained from quantum theory, in that he assumes entropy a function only of ν/τ without assuming anything at all about the special form of the function; in fact he does not even assume that the function is of the same form for different substances. Since the particular form of the function demanded by quantum theory seems to suit the facts at atmospheric pressure fairly well, we are reasonably safe in assuming Grüneisen's less restricted form.

From equations (2) and (3) above we obtain immediately $(\partial\nu/\partial p)_\tau$, and $(\partial\nu/\partial\tau)_p$. These are the derivatives which we want, because in this work p and τ are taken as the independent variables. We have, from (2)

$$\left(\frac{\partial\nu}{\partial\tau}\right)_p \equiv \left(\frac{\partial\nu}{\partial\tau}\right)_p + \left(\frac{\partial\nu}{\partial p}\right)_\tau \left(\frac{\partial p}{\partial\tau}\right)_p = 0,$$

and from (3)

$$\frac{1}{\nu} \left(\frac{\partial\nu}{\partial p}\right)_\tau \equiv \frac{1}{\nu} \left(\frac{\partial\nu}{\partial p}\right)_\tau + \frac{1}{\nu} \left(\frac{\partial\nu}{\partial\tau}\right)_p \left(\frac{\partial\tau}{\partial p}\right)_\tau = 0.$$

But

$$\left(\frac{\partial p}{\partial\tau}\right)_p = - \left(\frac{\partial\nu}{\partial\tau}\right)_p / \left(\frac{\partial\nu}{\partial p}\right)_\tau, \quad \text{and} \quad \left(\frac{\partial\tau}{\partial p}\right)_\tau = \frac{\tau}{C_p} \left(\frac{\partial\nu}{\partial\tau}\right)_p.$$

Substituting and solving, we obtain

$$\frac{1}{\nu} \left(\frac{\partial\nu}{\partial\tau}\right)_p = \left(\frac{\partial\nu}{\partial\tau}\right)_p^2 / C_p \left(\frac{\partial\nu}{\partial p}\right)_\tau, \quad (4)$$

and

$$\frac{1}{\nu} \left(\frac{\partial\nu}{\partial p}\right)_\tau = \left(\frac{\partial\nu}{\partial\tau}\right)_p / C_p. \quad (5)$$

Now differentiating (1) logarithmically, and eliminating $(1/\nu)(\partial\nu/\partial\tau)_p$ and $(1/\nu)(\partial\nu/\partial p)_\tau$, we have immediately

$$\frac{1}{\alpha} \left(\frac{\partial\alpha}{\partial p}\right)_\tau = - \left(\frac{\partial\nu}{\partial\tau}\right)_p / C_p, \quad (6)$$

¹ P. Debye, Wolskehl. Conference, Göttingen, 1914, p. 19-60.

and

$$\frac{1}{\alpha} \left(\frac{\partial \alpha}{\partial \tau} \right)_p = \frac{1}{2\tau} - \left(\frac{\partial v}{\partial \tau} \right)_p^2 / C_v \left(\frac{\partial v}{\partial p} \right)_\tau. \quad (7)$$

These equations give, therefore, the variation of both ν and α in any direction in terms only of quantities of thermodynamic nature.

The numerical magnitudes of these quantities first claim attention. The relative pressure coefficient of amplitude is negative, and its magnitude is in general six to nine times the linear compressibility. The amplitude increases with increasing temperature, approximately as $1/2\tau$, since the term $(\partial v/\partial \tau)_p^2 / (C_v(\partial v/\partial p)_\tau)$ is small. The frequency increases with increasing pressure, the rate of increase being the same as the rate of decrease of the amplitude, and the frequency decreases with increasing temperature, but the rate of decrease is small compared with the rate of increase of amplitude.

TABLE I.

Metal.	$-\frac{1}{\nu} \left(\frac{\partial \nu}{\partial \tau} \right)_\tau$	$\frac{1}{\nu} \left(\frac{\partial \nu}{\partial \tau} \right)_\tau$	$\frac{C_v}{\text{Kg. Cm. per Cm}^3}$	$\frac{1}{\alpha} \left(\frac{\partial \alpha}{\partial \tau} \right)_p$	$-\frac{1}{\alpha} \left(\frac{\partial \alpha}{\partial \tau} \right)_\tau$	$-\frac{1}{\nu} \left(\frac{\partial \nu}{\partial \tau} \right)_p$
In.....	(?) 2.0×10^{-4}	.013	15.2	2.39×10^{-4}	8.6×10^{-4}	55.5×10^{-4}
Sn.....	1.86	.051	16.8	1.92	3.03	8.3
Tl.....	2.26	.092	15.4	2.06	5.96	24.3
Cd.....	2.06	.090	19.2	2.04	4.7	20.4
Pb.....	2.28	.0879	14.0	2.07	6.27	24.3
Zn.....	1.66	.078	26.8	1.97	2.91	13.7
Mg.....	2.84	.078	17.7	1.96	4.4	12.1
Al.....	1.44	.074	23.1	2.00	3.19	16.5
Ag.....	.99	.055	24.1	1.96	2.30	12.9
Au.....	.628	.0429	24.8	1.95	1.73	11.8
Cu.....	.735	.051	34.2	1.94	1.49	10.3
Ni.....	.42	.0375	40.0	1.88	.94	8.4
Co.....	? .50	.037	37.0	1.91	1.00	7.5
Fe.....	.58	.0375	34.6	1.90	1.08	7.0
Pd.....	.53	.0355	29.4	1.92	1.21	8.1
Pt.....	.272	.0292	28.5	1.94	1.02	10.9
Mo.....	.45	.0108	24.7	1.84	.44	1.13
Ta.....	.52	.0237	22.8	1.88	1.04	4.85
W.....	.265	.0101	27.6	1.84	.37	1.39

In Table I. are collected the numerical values of the changes of amplitude and frequency for the metals whose pressure coefficient I have measured. In the table are also included the fundamental thermodynamic data used in the calculations. There is often considerable uncertainty as to the best experimental values to choose. I have in general used for thermal expansion the most consistent values from the last edition of Landolt and Börnstein and for compressibility the values of

Richards. A more detailed discussion of the manner of choice of these data may be found in the American Academy paper.

With the numerical values of the changes of amplitude at hand, we are now in a position to search for a connection between the changes of resistance and amplitude. It turns out that there is a very simple approximate relation between these two quantities, which I offer as the most important result of this paper. The relation is that the change of resistance with temperature or pressure or any other thermodynamic variable is twice the relative change of amplitude under the same conditions. It is perhaps worth mentioning that I was led to search for a relation between resistance and amplitude by the view I have outlined of the nature of resistance, and that the view was not suggested by observing the relation.

Consider first the variation of resistance with temperature. In order to simplify this first discussion, we consider the temperature coefficient at constant volume, thus avoiding a very small correction for changing volume. From the formulas already given, it follows immediately that

$$2 \frac{1}{\alpha} \left(\frac{\partial \alpha}{\partial \tau} \right)_v = \frac{1}{\tau}.$$

Now the temperature coefficient of most metals is nearly the reciprocal of the absolute temperature, so that we have here an approximate expression for the change of resistance in terms of the change of amplitude. The temperature coefficient as ordinarily given, however, is the coefficient of the observed resistance at constant pressure. The coefficient at constant volume is more nearly equal to $1/\tau$ than is the coefficient at constant pressure. In Table II. are shown the temperature coefficients at constant volume of the specific resistance at 0° C. In comparing these values with others it must be remembered that we are giving here the specific resistance, and not the observed resistance, and also that we are listing the instantaneous coefficient at 0° and not the average coefficient between 0° and 100° .

The temperature coefficient at constant pressure involves a correction for changing volume, as already suggested. If the view of conduction given above is justified, we would expect that decreasing the number of atoms would, other things being equal, increase the resistance. The increase is proportional to the change of linear dimensions, that is to $\frac{1}{3}(1/v)(\partial v/\partial \tau)_p$, because an increase of length and an increase of one of the cross sectional dimensions work in opposite directions on the resistance. It turns out that to the same degree of approximation to which $(1/w)(\partial w/\partial \tau)_v$ is equal to $2(1/\alpha)(\partial \alpha/\partial \tau)_v$, $1/w(\partial w/\partial \tau)_p$ is also equal to

$2(1/\alpha)(\partial\alpha/\partial\tau)_p + \frac{1}{3}(1/v)(\partial v/\partial\tau)_p$. It is hardly worth while to give a special table for this other temperature coefficient. The correction for volume averages about $\frac{1}{3}$ per cent.

TABLE II.

Metal.	$\frac{1}{w} \left(\frac{\partial w}{\partial \tau} \right)_r$	$\gamma = \frac{1}{w} \left(\frac{\partial w}{\partial p} \right)_r$		Ratio of γ calc. by Grüneisen's Formula to γ Obs.	$\left(\frac{\partial \tau}{\partial p} \right)_w$ $\left(\frac{\partial \tau}{\partial p} \right)_a$
		$\gamma_{\text{Obs.}}$	$\frac{\gamma_{\text{Calc.}}}{\gamma_{\text{Obs.}}}$		
In.....	.00360	-12.9×10^{-4}	1.39	1.063	0.875
Sn.....	402	11.1	0.597	0.420	1.62
Tl.....	463	13.9	0.91	0.813	0.932
Cd.....	357	11.3	0.894	0.643	1.21
Pb.....	350	15.2	0.875	0.671	1.23
Zn.....	389	5.95	1.066	0.718	0.99
Mg.....	380	6.4	1.547	1.094	0.72
Al.....	409	4.64	1.48	1.157	0.665
Ag.....	384	3.83	1.288	0.973	0.800
Au.....	369	3.33	1.132	0.877	0.949
Cu.....	414	2.45	1.315	0.985	0.738
Ni.....	605	1.72	1.09	1.197	0.568
Co.....	358	1.10	1.975	1.328	0.573
Fe.....	545	2.60	0.905	0.807	0.815
Pd.....	308	2.16	1.205	0.792	1.057
Pt.....	367	2.07	1.030	0.865	1.007
Mo.....	432	1.48	0.601	0.338	1.443
Ta.....	290	1.66	1.356	0.814	1.007
W.....	313	1.37	0.598	0.305	1.354

We next compare the pressure coefficient of resistance at constant temperature with the change of amplitude with pressure at constant temperature, corrected for the effect of changing volume as just explained. The pressure coefficient computed in this way is given by the formula

$$\frac{1}{w} \left(\frac{\partial w}{\partial p} \right)_r = 2 \frac{1}{\alpha} \left(\frac{\partial \alpha}{\partial p} \right)_r + \frac{1}{3} \frac{1}{v} \left(\frac{\partial v}{\partial p} \right)_r.$$

The last term, which is the correction factor for changing volume, amounts to from 5 to 10 per cent. of the total effect. In Table II., Column 4, the ratio of the observed value of the pressure coefficient of specific resistance at 0° C. and 0 kg. to the value computed as above is shown.

The agreement is not as good as could be desired, but it should be remembered that the classical free electron theory did not give even the right sign to the pressure effect, to say nothing of the numerical magnitude, and that there has been only one other theory proposed which has

attempted to account for the pressure effect. This other theory is that of Grüneisen,¹ who modifies the theory of Wien,² who regards the electrons as moving with constant velocities independent of the temperature. The formula of Grüneisen is

$$\frac{1}{w} \left(\frac{\partial w}{\partial p} \right)_\tau = \frac{1}{u} \left(\frac{\partial u}{\partial p} \right)_\tau - \frac{1}{N} \left(\frac{\partial N}{\partial p} \right)_\tau - \frac{1}{v} \left(\frac{\partial v}{\partial p} \right)_\tau - \frac{1}{C_p} \frac{1}{v} \left(\frac{\partial v}{\partial \tau} \right)_p \left[1 + \frac{1}{w} \left(\frac{\partial w}{\partial \tau} \right)_p \tau \right],$$

where u is the velocity of the free electrons, and N is their number per cm³. In column 5 of Table II. the ratio of the values computed by Grüneisen's formula to the observed values are listed for purposes of comparison. It will be seen that on the whole the new formula agrees better with experiment than does that of Grüneisen. Grüneisen has discrepancies amounting to a factor of three for both Mo and W, whereas the greatest discrepancy of the new formula is by a factor of two for Co, which was known to be impure. There is also this difference between the new formula and that of Grüneisen, apart from its much greater simplicity. Grüneisen's formula does not give the pressure coefficient in terms only of other quantities of non-electrical nature, but, besides the first two terms which are small, introduces the temperature coefficient of resistance, which must be determined by experiment. It is of course true that the temperature coefficient is approximately equal to $1/\tau$, so that the last factor in Grüneisen's formula might have been written 2, but the formula would then have lost its theoretical significance, and become merely empirical. Furthermore, the agreement with observation would not have been so good if this simplification had been made, the departures of the temperature coefficients from equality with $1/\tau$ playing a real part in the numerical values given by the formula.

It is interesting to notice that by a combination of Grüneisen's formula with that suggested above, an empirical relation may be obtained much better than either. The relation is

$$\gamma_O/\gamma_{Obs} + \gamma_{Obs}/\gamma_B = \text{Const.}$$

where $\gamma = (1/w)(\partial w/\partial p)_\tau$ and the subscripts indicate whether the coefficient is observed or calculated according to Grüneisen's or my formula. The "constant" varies for the 19 normal metals from 1.54 to 1.97, or a range of 14 per cent. about the mean, whereas the variation in Grüneisen's formula, omitting Co, is 53 per cent., and that of the proposed formula above, omitting Co, is 37 per cent. It is curious, however, that the formula cannot be reversed, that is solved for γ_{Obs} to give an approxi-

¹ E. Grüneisen, *Verh. D. Phys. Ges.*, 15, 186-200, 1913.

² W. Wien, *Columbia Lectures*, 1913, 29-48.

mately correct value. It will be found, if one tries to operate with a fixed average value of the constant, that some substances give imaginary values for γ_{Obs} .

A result of some interest to be deduced from the expressions above for temperature and pressure coefficient is that the resistance is approximately constant along a line of constant amplitude. Column 6 in Table II. shows the ratio of $(\partial\tau/\partial p)_w$ to $(\partial\tau/\partial p)_a$. This ratio of course should be unity if resistance were strictly constant at constant amplitude.

We may summarize the results of these numerical calculations by the statement that the change of specific resistance with either pressure or temperature, and therefore with any other thermodynamic variable, is given by

$$\frac{1}{w}dw = 2\frac{1}{\alpha}d\alpha + \frac{1}{3}dv,$$

to at least as good a degree of approximation as that of any other formula. The term containing the change of volume is comparatively unimportant, so that we may say that *the relative change of specific resistance in any direction is approximately equal to twice the corresponding change of average amplitude of atomic vibration*. The agreement with the pressure coefficient is better than that of the only previously proposed formula, and the temperature coefficient is correctly reproduced in the majority of cases. I know of no attempt to account for the departures of the temperature coefficient from $1/\tau$ except that of Wien, and his attempt to bring the departures into connection with the average frequency of atomic vibration cannot be considered a success.

The theoretical significance of this general relation becomes immediately obvious from the viewpoint outlined above that the resistance is proportional to the number of gaps between atoms. If the number of gaps is proportional to the square of the mean amplitude, then the change in the relative resistance will be proportional to twice the relative change of mean amplitude, plus a correction term for changing volume. Now it is easy to see how the number of gaps may be proportional to the square of the mean amplitude. A gap is formed when two neighboring atoms each have an amplitude enough greater than the mean so that the maximum distance of separation, when they are at opposite parts of their swing, shall exceed a critical value. That is, a gap is only produced by the chance coincidence in two neighboring atoms of two amplitudes both larger than the mean. If we suppose the amplitudes are distributed at random, the chance that there be two such coincident large amplitudes in two neighboring atoms is evidently proportional to the square of the chance that a single atom have an amplitude large enough. If then, the

chance that a single atom have an amplitude of the requisite magnitude be proportional to the mean amplitude of oscillation of all the atoms, and this is certainly the simplest assumption that can be made, the connection becomes at once obvious. Of course in any actual case the conditions are much more complicated than we have supposed above; the amplitude of a single atom need not exceed a definite critical value before it can under any conditions form a gap with a neighboring atom, but the amplitude necessary to form a gap will depend on the relative phases of the two atoms. Such modifying features as this will not change the general result that the number of gaps is proportional to the square of the number of atoms with amplitude above a certain value, but will only change the numerical value of the factor of proportionality.

It is evident that these general considerations must be modified by many factors that vary from substance to substance. Consider first the manner of distribution of amplitudes about the mean. This is a subject which has not yet been worked out, but which can perhaps be solved in the not too distant future by such methods as those initiated by Born¹ in his recent book. Whatever the precise manner of distribution will turn out to be, we may be sure that it will not be Maxwell's distribution. I have obtained independent evidence on this point from the time rate of transition of polymorphs.² If one does assume Maxwell's distribution, the variation of number of gaps with mean amplitude may be worked out, and will be found proportional to some higher power of the mean amplitude than the second. This is sufficient to show that the manner of variation of number of gaps depends on the manner of distribution of amplitudes about the mean. The manner of distribution may well be expected to vary from substance to substance.

It is obvious, if all amplitudes are concentrated in a narrow band about the mean, that as temperature is increased from 0° Abs. there will be a definite temperature at which a finite number of gaps will suddenly appear, and the metal will suddenly become conducting, as do tin, lead, and mercury. This suggestion as to the sudden appearance of resistance must not be taken too seriously at present until it has been settled that this discontinuity is not due to a polymorphic transition, but in any event the new point of view leaves open such a possibility. If, on the other hand, the distribution is not so sharp, the appearance of resistance will not be so sudden, but in any event it is pretty evident that the resistance will at first, after it has once appeared, increase more rapidly than the increase of temperature itself. This is of course true for most pure metals; at low temperatures the resistance increases approximately as the

¹ M. Born, *Dynamik der Krystallgitter*, Teubner, 1915.

² P. W. Bridgman, *Proc. Amer. Acad.*, 52, 80, 1916.

fourth power of the temperature. The fact that the manner of variation of resistance with temperature is much the same for all metals at low temperatures suggests that at low temperatures, at least, the manner of distribution of amplitudes must be much the same for all substances. At higher temperatures individual differences in the behavior of resistance appear, and it seems not unlikely that similar differences in the manner of distribution should also appear.

Another important consideration is that of frequency. We have entirely neglected this factor in the considerations above. If frequency has no effect, then the effective number of gaps is not changed by a change of frequency. This means that if we double the frequency of oscillation we have thereby halved the time during which two atoms are separated by the critical distance during a single encounter, or doubling the rapidity of mean oscillation doubles the rapidity of every stage of the motion. This seems the most natural hypothesis, but judging by analogy with the collisions of elastic systems, it is by no means a necessary hypothesis. If the law of action between atoms is different for different kinds of atoms, there may be differences in the proportional length of time occupied in a collision. It does not seem unlikely that there should be such variations, and probably some of the discrepancies are to be explained by this neglected factor. The fact, however, that we have obtained as good agreement as we have probably means that the atoms of all normal metals are much alike. It ought to be possible to obtain information about the law of force between atoms by considerations of this character.

Perhaps the most striking factor that has been neglected is the effect of change of volume, apart from the effect on the number of atoms in unit volume. If the atoms are of finite size, and if this size remains constant, it is evident that when volume is decreased at constant amplitude, for example, the number of gaps and therefore the resistance should decrease. If the total volume occupied by the atoms is a large fraction of the total volume of the solid, such an effect should be rather large. According to Wien's theory, on the other hand, the resistance should increase instead of decrease as volume is decreased at constant amplitude. As a matter of fact, there seems to be no such effect; for some metals the resistance decreases with decreasing volume at constant amplitude, and for others the resistance increases. In any event the effect is small; in fact, to a first approximation, resistance is constant for all normal metals at constant amplitude, as has been shown in Table II. The point is an important one, and seems to me to indicate that under pressure the atom undergoes some sort of distortion so as to become effectively smaller. The same conclusion is strongly suggested by the fact that at pressures

high enough to compress the volume to less than its value at 0° Abs. the temperature coefficient is practically unaffected. This would seem to indicate that the solid in some way remains similar to itself as pressure is increased, and this might be brought about by such a decrease of volume of the atoms themselves as to keep pace with the change of volume of the entire solid.

Another possibility to be kept in mind is that the boundary of the atom which determines the elastic collisions may not be the same as the boundary which determines the gaps for the passage of electrons. The latter may be inside the former, much in the same way as the effective radius of the atom as determined from dielectric measurements has been found to be less than the effective radius as found from kinetic theory. In fact, this view of the nature of conduction demands fundamentally a certain amount of interpenetration of the outer parts of the fields of force surrounding the atoms when they are in contact under normal circumstances in a solid, or else a very considerable distortion of the atom under changes of temperature. Or again, the atoms may not be absolutely separated in order to form a gap, but may be merely in contact over a smaller area, thus affording less chance for the passage of electrons.

In view of all these neglected factors it does not seem strange that the variations from the formula are as large as they are, but rather that they are no larger. It must be one of the next tasks of this theory to attempt to bring the variations from agreement with the formula into connection with the neglected factors, and obtain if possible a closer approximation to the observed values.

This is as far as I have been able to get quantitatively according to this view point. Qualitatively, however, many other facts may be brought into line if we consider that at high temperatures the gaps begin to function in a new way. Consider a metal kept at constant volume as temperature increases. If the volume is initially small, the number of gaps will increase, as already explained, as the average amplitude of oscillation is increased. But if the initial volume is large, the atoms may be separated so far that the electrons cannot pass freely from atom to atom under ordinary conditions, but can pass only when two atoms are brought especially close together during a collision of unusual violence. The number of such favorable opportunities will evidently increase as the mean amplitude increases. An increase of amplitude functions in two ways, therefore; at small volumes it increases the resistance by increasing the number of gaps, and at large volumes it decreases the resistance. If the metal is heated at constant pressure, being allowed to expand, there will be a progression from the first state of affairs toward the second. We

may represent this by saying that effectively the atoms of the metal are divided into two classes; those of the first class are the ones to which we have already applied our formulas, and their resistance increases with increasing amplitude, while those of the second class tend to decrease in resistance as mean amplitude increases. The atoms of the first class have a low specific resistance, and those of the second class a high resistance. As the metal is heated the resistance will be affected in three ways; there will be an increase of resistance of the atoms of the first class, an increase of resistance due to the passage of atoms from the first to the second class, and a decrease of resistance of the atoms of the second class.

We now enumerate the various kinds of effect that can be brought into line by these considerations. This enumeration will include the facts that have already received quantitative treatment. (1) The temperature coefficient of resistance of solid metals at constant pressure is very nearly equal to $1/\tau$. It is, however, greater than $1/\tau$, and for nearly all metals becomes increasingly greater at higher temperatures. (2) The temperature coefficient of most liquid metals, on the other hand, is much less than $1/\tau$, and for some metals (Cd and Zn) may actually be negative. (3) The only liquid metal for which measurements have been made, mercury, has a negative temperature coefficient of resistance at constant volume. (4) At constant volume the temperature coefficient of solid metals becomes increasingly less than $1/\tau$ as temperature is raised, and if the same tendency persists will eventually become negative. (5) At very low temperatures some metals show a sudden drop of resistance to sensibly zero. (6) The pressure coefficient of resistance at constant temperature of nearly all pure metals is negative. The pressure coefficient of the only liquid metal measured, mercury, is much larger than that of solids. (7) The relative pressure coefficient at constant temperature decreases with increasing pressure. (8) The decrease with pressure of pressure coefficient at constant temperature is less at higher temperatures for most metals, but for a few metals with low melting point is greater. (9) If a metal is stretched by a force in tension, the resistance along the line of stretch is increased, but is decreased at right angles. (10) Two metals, Bi and Sb, have a positive pressure coefficient. This may be brought into direct connection with their abnormally small thermal expansion and the fact that both expand on freezing. (11) The high specific resistance of alloys compared with that of their constituents is what one would expect. The negative temperature coefficient of some alloys may also be explained by the same mechanism, and possibly the positive pressure coefficient.

We now consider these points in detail. (1) The temperature coef-

ficient of solid metals, both at constant pressure and constant volume has already been discussed. One would be tempted to explain the fact that the coefficient is greater than $1/\tau$ by the passage of atoms from the first to the second state on raising temperature, but this explanation is seen not to be justified on considering that the coefficient at constant volume is also too large, and that the discrepancy at constant pressure is no greater than at constant volume. An explanation of the variations of temperature coefficient is much to be desired; as yet there seems to be no suggestion as to any possible connection with other physical properties. As already remarked, Wien's attack does not seem fruitful. Possibly a suggestion may be found in a distortion of the atoms with increasing violence of vibration, even at constant volume, which effectively decreases their volume, and so increases the number of gaps and the resistance. The upward curvature of the resistance curves, on the other hand, is probably to be explained by the increasing number of atoms in the second condition.

(2) The temperature coefficient of liquid metals, which are to be thought of as largely in the second condition, would be expected to be less than that of a solid because the temperature coefficient of atoms in the second class is by themselves negative. In any actual liquid the negative temperature coefficient of the second-class atoms is modified by the atoms which are passing from the first to the second class, and which produce an effect of the opposite sign. If the total number of atoms in the second class is large enough to outweigh this other effect, the coefficient will be negative, as it is for liquid Cd and Zn. In any event, we expect the coefficient for the liquid to be less than for the solid, and in general to be less than $1/\tau$. The accompanying table shows this.

TABLE III.

Temperature Coefficient of Resistance of Liquid Metals.

Metal.	Temp. °C.	Temp. Coeff.	$\frac{1}{\tau}$	Coefficient of Solid Extrapolated.
Na.....	100°	.00330	.00268	.00348
K.....	100	.00342	.00268	.00369
Al.....	657	.00051	.00107	
Cu.....	1100	.00037	.00073	
Sn.....	500	.00048	.00115	
Au.....	1100	.00045	.00073	
Hg.....	100	.00094	.00268	
Pb.....	500	.00044	.00115	
Bi.....	271	.00042	.00184	.0020±

Only two of the metals, K and Na which have low melting points, have

a coefficient greater than $1/\tau$, and for all the metals the coefficient of the liquid is less than that of the solid at the same temperature. The coefficients of the solids listed in the table were obtained by an extrapolation; the coefficients of the solids not listed are in all cases greater than $1/\tau$.

(3) If a liquid metal is heated at constant volume, the relative number of atoms in the two classes is probably little affected, and we would expect the temperature coefficient to be negative, if the number of atoms in the second group is larger than those of the first, as we suppose it is for a liquid. This is verified by the only liquid metal for which the data exist, liquid mercury. This observation was first made by Barus.¹

(4) One meaning of a pressure coefficient of resistance for solids independent of temperature is that as temperature is increased at constant pressure (say atmospheric) the temperature coefficient at constant volume becomes more nearly negative, and if the same tendency persists will ultimately become negative, as it is for liquid mercury. This indicates that at higher temperatures the solid metal approaches a condition in which most of its atoms are in the second group, and is exactly what one would expect.

The fact that there is a tendency toward the behavior of the liquid may be seen on considering that the thermal expansion and the compressibility of a metal are both little affected by temperature at constant pressure, and that the relation between temperature and resistance is to a first approximation linear. This means that at high temperatures the same increment of pressure and temperature will be required to maintain volume constant as at low temperatures. The same increment of temperature will produce the same increment of actual resistance, but the decrement of resistance under a constant increment of pressure will increase, because the relative pressure coefficient is constant, while the actual resistance, on which the coefficient is calculated, becomes higher.

(5) The sudden drop of resistance of some metals at low temperatures has already been touched on. It means, according to this view, a clustering of the amplitudes closely about the mean.

(6) The negative pressure coefficient has already been explained at some length. It is primarily due to a decrease of atomic amplitude brought about by an increasing frequency at higher pressures. Added to this effect must be another effect, in most cases considerably smaller, due to the crowding of atoms from the second condition into the first. We have already seen that under ordinary conditions the number of atoms in the second condition is small, but in the case of liquid metals the number in the second condition is relatively larger, and we should

¹ C. Barus, Bull. U. S. Geol. Sur. No. 92, 1892, p. 74.

expect the decrease of resistance due to this cause to be higher. This is verified by the only liquid metal for which we have data, mercury.

(7) The decrease of relative pressure coefficient under increasing pressure is to be accounted for by the disappearance of atoms in the second class. This number is not large, and so the effect would be small, as it actually is. The rate of decrease of coefficient at rising pressures is a rough measure of the rate at which the atoms of the second group are being exhausted. If the rate of decrease of atoms of this group were constant at all pressures, we would, of course, have no decrease of the coefficient at higher pressures.

(8) The surprising fact that the decrease of coefficient with pressure is greater at low than at high temperatures is to be explained [by the fact that at low temperatures the atoms of the second group are being exhausted more rapidly, since there are fewer of them, and therefore they contribute a more rapidly decreasing term to the negative pressure coefficient. The more rapid decrease at higher temperatures of some of the metals with low melting points is, on the other hand, probably to be explained by the greater decrease of the thermodynamic quantities which enter the formulas for the amplitude. It is for just these metals that one would expect the greatest decrease, although such a decrease has not yet been proved by experiment.

(9) When a metal is stretched by a mechanical tension the atoms are separated along the line of tension, but are brought closer together in a direction at right angles to the tension. We would expect therefore the number of gaps to increase if the path of the electrons is along the tension, but to decrease if it is at right angles. This corresponds with the observed changes of resistance under tension.

(10) The positive pressure coefficient of resistance of Bi and Sb next engages us. A positive pressure coefficient, according to this view, means that the number of gaps increases as volume decreases at constant temperature. Such an increase may be accounted for by an increase in amplitude, or what is the same thing, a decrease in frequency at constant temperature. A decrease in frequency with decreasing volume is not what we would expect, but may be brought into immediate connection with two other abnormal features of the behavior of Bi and Sb. Both Sb and Bi contract instead of expanding when melted, and the thermal expansion of both is unusually small. If a diagram is plotted of thermal expansion against compressibility (or melting point) of all the metals of these experiments, Sb and Bi will be found to occupy a position apart from the others, with abnormally small expansions. Now it is a conclusion most strongly suggested by the analysis of Debye, and also by my

own examination¹ of the properties of a gas composed of only one atom already mentioned, that the thermal expansion is intimately connected with the law of force between atoms. The subject has by no means been worked through as yet to a satisfactory conclusion, but we may probably assume with safety that an abnormally high thermal expansion means a restoring force increasing with unusual rapidity as the atoms are brought together, and that a small expansion means that the restoring force increases less rapidly than normal with decreasing volume. If we admit the conclusion as justified by the results above that the restoring force in normal metals is linear and independent of the volume, then for a substance with small expansion the restoring force decreases as the volume decreases. Such a decrease of restoring force means a decrease of mean frequency with decreasing volume, and hence an increase of amplitude. It is not difficult to imagine a law of force which will lead to such an unusual result.

In Fig. 1 is represented what we may suppose to be the action between three adjacent atoms. We suppose atom *B* to vibrate, and atoms *A* and *C* to remain at rest. The curve (1) shows the force with which *A* and *B* act on each other as a function of their distance apart. At infinite

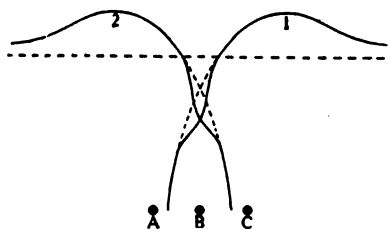


Fig. 1.

separation there is no force (indicated by the dotted horizontal line); as the atoms approach the force is at first an attraction, indicated by (1) lying above the horizontal line, but on closer approach the force rapidly becomes an intense repulsion. The law of force between *B* and *C* is precisely similar to that between *A* and *B* and is shown by

the curve (2). The dotted part of curves (1) and (2) is what we may imagine for normal atoms, while the heavy curve is perhaps like that of Bi and Sb. The anomaly consists merely in a temporary acceleration of the rate of increase of the repulsive force followed by a retardation, as compared with the normal atom. The restoring force on *B* as it oscillates between *A* and *C* is evidently the difference of the forces exerted in opposite directions by *A* and *C*, and will therefore be proportional to the angle between (1) and (2) multiplied by the displacement of *B* from its equilibrium position. It is evident from inspection of the figure that as atoms *A* and *C* approach, that is as volume decreases, the angle of intersection between (1) and (2) decreases, and the restoring force therefore decreases. This decrease of restoring force means a small

¹ P. W. Bridgman, Proc. Amer. Acad., 49, 107, 1913.

thermal expansion, and at the same time a decrease of frequency, and so an increase of resistance.

The slight anomaly in the law of force shown in Fig. 1 is also consistent with the solid having a larger volume than the liquid. To simplify the argument on this point suppose that the solid is at 0° Abs. We will try to show that the atoms are in equilibrium in a position of abnormally large volume. Equilibrium is found in that configuration in which the work done by the attractive forces during a virtual decrease of volume is exactly equal to the work done on the repulsive forces. We may suppose that the repulsive forces act only between immediately adjacent atoms, whereas all the more distant atoms exert attractive forces. Equilibrium will therefore be found where the few repulsive forces balance the many attractive forces. If there is a region in which the repulsive forces are unusually intense at an unusually large distance from the center of the atoms, as we have supposed the case for Bi and Sb, the repulsive forces will balance the attractive forces when the atoms are separated more than usual, and we will have a solid of abnormally large specific volume. If we suppose that this particular law of force is valid only when the atoms have the regular orientation of the crystal, but that the average law of force for haphazard orientations is more nearly normal, we have immediately a suggestion as to why the metal contracts on melting.

The abnormality we have imagined to account for the pressure effect has little effect on the temperature coefficient. The reason for this is that the temperature axis is very nearly normal to the line of constant amplitude in any event, so that a relatively large change in the direction of the line of constant frequency will produce a relatively small change in the temperature derivative of frequency. It is true that the temperature coefficients of resistance of Bi and Sb are not at all abnormal.

(11) The fact that the resistance of an alloy is always greater than that of its components is simply explained by the failure of the atoms of the two metals to fit closely together. In an alloy there are, therefore, a greater number of gaps permanently present than in the pure metal, and consequently the resistance is greater. As far as I know the conductivity of every alloy is less than that of its best conducting component, and in many cases is less than that of either component. The positive pressure coefficient of some alloys may be explained by supposing some such effect as that imagined for Bi, which is plausible enough if we are dealing with molecules instead of atoms, or by supposing that under the constraint of increasing pressure the atoms assume less natural positions, in which the number of gaps is greater. This latter alternative would be suggested by all my experiments on polymorphic transitions, in which the applica-

tion of pressure at constant temperature drives the molecules ultimately into such a configuration that some other configuration is more stable.

In conclusion a few critical remarks are not out of place. I do not offer the view of conductivity here explained as the only *possible* one consistent with all the facts, but merely as the one which seems to me at present most probable in the light of the facts now at our disposal. In one essential particular it does seem to me likely, however, that this view is like that which will be finally adopted. In the mechanism of conduction the properties of the atomic framework play a preponderating part; the atoms are not merely the trees of a forest among which a breeze of electrons plays, as in the classical free electron theory. The new experimental justification for this broad requirement in any new theory is the observation that the change of resistance is intimately connected with the amplitude of atomic vibration. Even if one is still so unconvinced of the truth of quantum theory as to be unwilling to admit the validity of the fundamental equations (1), (2) and especially (3), the fact cannot be escaped that we have found an approximate relation between the change of resistance and certain quantities of a purely thermodynamic character, which are certainly intimately connected with atomic properties.

This theory of conduction is not complete in the sense of the old free electron theory because it does not at present explain Ohm's law. We merely postulate, without searching for a detailed mechanism, that the electrons encounter resistance when they jump a gap between atoms, and that the resistance of the gap is on the average a resistance which obeys Ohm's law. This point is, however, probably one that it will not be difficult to meet. If, for instance, the force encountered by an electron in jumping a gap is like an ordinary fluid frictional force, proportional to the velocity, the requirements are met. The assumption of such a force is already familiar in various branches of electron theory.

The picture which I have presented of conduction in a metal is of a chain of electrons, which are normally at rest, getting under way under an applied electromotive force down a coherent chain of atoms, and gaining in speed until suddenly the continuity of the chain is interrupted by a break which appears somewhere between two neighboring atoms of the chain. The chain of electrons is not entirely stopped by the formation of the gap, but drives itself across by its own electromagnetic momentum, using up some or all of its store of energy in so doing. The gap absorbs the energy by a mechanism not specified, just as a spark absorbs the energy of an interrupted circuit,¹ and from the gap the energy is ultimately absorbed as heat energy.

An alternative picture is that the gaps are not of sporadic, isolated occurrence as above, but that between every pair of neighboring atoms there is a resistance which on the average is proportional to the square of the mean amplitude. An objection to this view is that we must assume the square law, without at present even attempting to make it plausible, as we have above.

Still another point of view is possible, connecting so closely with the old free-electron theory as to account for Ohm's law. When the atoms are at rest at 0° Abs. we may suppose that the electrons travel freely through the substance of the atoms, as the atoms of a gas freely traverse interatomic space, but as the amplitude of atomic vibration is increased, the electrons collide with the *gaps* between the atoms, precisely as they collided with the atoms on the old view. Ohm's law follows immediately, as on the old view. An objection to this viewpoint is that it brings up again the specific heat difficulty. We will have to suppose the electronic velocity independent of temperature, as does Wien. This does not seem to me a formidable difficulty; in fact such a constant velocity seems more plausible under the supposed circumstances than under the conditions imagined by Wien. For there must be some electrons always traversing the substance of the atoms as β -particles, continually emitted by the break-up of the outer ring of electrons in the atoms, and continually reabsorbed. Velocities from such a source as this must be independent of the temperature.

Wien's original picture of electrons passing freely through alley-ways between atoms is not a remote possibility consistent with a close connection between change of resistance and amplitude. The picture may contain much that is true, but probably the details of working out, as given by Wien, must be modified in any event. In particular Wien's assumption must be given up that at very low temperatures the quanta of energy are all located in a few isolated atoms, the rest of the atoms being at rest. It seems more probable now that the quanta are located in modes of elastic vibration. To explain the absence of any specific effect of a change of volume as such, we must suppose according to this view also that the atoms are distorted by pressure.

A combination of Wien's point of view with that advocated by this paper offers many attractive features: We may suppose that the electrons pass freely through the substance of the outer part of the atoms, encountering no resistance even when passing from atom to atom. The electron may, however, collide with the positive nucleus, which is so small that it acts like a point charge. The number of collisions will evidently be directly connected with the amplitude of oscillation of the nuclei.

According to this point of view the absence of any specific volume effect is not so serious a difficulty, because we would not expect it to be nearly as large. I do not at present, however, see the possibility of bringing into line with this view the wide range of facts dealt with qualitatively above.

The chief claim to consideration of the old free electron theory was that it offered an explanation for the Wiedemann-Franz ratio. Wien regretted as one of the weak points of his theory that he had to give up hope of explaining the Wiedemann-Franz ratio when he assumed electronic velocity independent of temperature. The view of conduction offered above does not attempt as yet to explain the Wiedemann-Franz ratio, but such an explanation does not seem to me beyond the bound of ultimate possibilities. In the first place, any mechanically consistent theory, irrespective of whether it assumes thermal conduction performed by electrons or not, must give a ratio involving the electronic charge in the correct way, simply by dimensional reasoning, but need not give the correct form for the coefficient of the charge. This is all the old theory does, the coefficient it predicts is not of the correct form. It is commonly held that because electrical conductors are so much better thermal conductors than electrical insulators the electron must play a large part in the thermal conduction of metals, and that therefore no theory can be correct which does not assign some part of the energy of temperature agitation to the electrons. This conclusion seems to me not necessary; all that is indicated is that the same mechanism which makes easy a transfer of electrons from atom to atom should also make possible an easy heat transfer. Now this is immediately indicated by the view above. Electrons pass freely from atom to atom without break of continuity when the outer parts of the fields of force of the atoms so merge that the effective individuality of the atom is lost. But when the atoms are so merged as to lose their individuality they will function as one large atom, and the energy of heat vibration will jump over whole ranks of atoms, as we know it must if the thermal conductivity of metals is to be accounted for by an atomic mechanism only.

SUMMARY.

The most important result of this paper is the observation that the variations of resistance of a normal solid metal are preeminently concerned with one factor only, the average amplitude of vibration of the atoms, irrespective of whether the change of amplitude is brought about by a change of pressure or of temperature. The proportional change of resistance is approximately twice the proportional change of amplitude.

This suggests that a successful theory of metallic conduction must discard the old viewpoint, which explained resistance in terms of the properties of an assemblage of electrons little affected by the inert framework of atoms, and substitute an explanation in terms of the properties of the atomic framework. There are several possible ways of giving such an explanation, of different physical complexions, all of them suggesting an intimate connection between resistance and amplitude. The view which I have adopted above seems to me that best adapted to bring into line the entire range of facts. This view is that the electrons normally pass freely from atom to atom, but if the atoms are separated beyond a critical distance, as by temperature agitation, gaps appear between the atoms which offer resistance. The resistance at ordinary temperatures is proportional to the number of gaps, and it is not unplausible to expect the change in the number of gaps to be twice the proportional change in the amplitude. At higher temperatures, however, that is large volumes, as in liquid metals, the passage of electrons from atom to atom is on the average difficult, but is facilitated by a collision of unusual violence. An increasing amplitude functions in opposite ways, therefore, at large and at small volumes. Under actual conditions there must be a combination of these two effects in varying proportions. I have shown in detail how the play against each other of these two different effects offers an explanation of the most important features of the behavior of both normal and abnormal metals.

Another point of extreme suggestiveness is that a pressure high enough to compress the metal to less than its volume at 0° Abs. is powerless to change its temperature coefficient of resistance. Consistency with the view advanced here seems to me to demand a distortion of the atom under pressure. Such a distortion has been suggested by many other aspects of my work on high pressures.

The experimental work on which this paper is based was assisted in large part by generous grants from the Bache Fund of the National Academy of Sciences and from the Rumford Fund of the American Academy of Arts and Sciences.

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¹ J. H. Jeans, *Mathematical Theory of Electricity and Magnetism*, Cambridge University Press, 1908, p. 133.

TOLMAN'S TRANSFORMATION EQUATIONS, THE PHOTO-ELECTRIC EFFECT AND RADIATION PRESSURE.

BY S. KARRER.

R. C. TOLMAN uses the following transformation equations when applying his principle of similitude,

$$l' = xl; \quad t' = xt; \quad e' = e; \quad m' = m/x; \quad S' = S.$$

P. W. Bridgman has pointed out that these equations are a particular case out of a large number of possible transformations which follow from the principle of dimensional homogeneity. Tolman has shown that, in many instances, the application of the above equations leads to results in accord with experiment. We wish to point out two important cases where the transformation equations give results which are in accord with present knowledge.

In the first place, let us assume we know from experiment that the kinetic energy E of the electrons emitted from a metal under the influence of light, minus a constant E_0 is only a function $f(n)$ of the frequency n of the incident light, and is independent of the intensity of the light and of the temperature and special properties of the metal. That is, assume

$$E - E_0 = f(n).$$

Then from the equations above it follows that

$$E' - E_0' = \frac{E - E_0}{x} \quad \text{and} \quad n' = \frac{n}{x},$$

$$E' - E_0' = f(n'),$$

$$\frac{E - E_0}{x} = f\left(\frac{n}{x}\right),$$

$$E - E_0 = xf\left(\frac{n}{x}\right) = f(n),$$

x being arbitrary, this functional equation must hold for every value of x . Its solution will be of the form,

$$f(n) = hn,$$

where h is a constant, hence

$$E - E_0 = hn,$$

which is Einstein's photoelectric equation. h must be determined by experiment; the measurements of Millikan give strong support to the correctness of this equation. And further, his results show that h is identical with Planck's radiation constant. From Tolman's point of view this is a very interesting result, but from the standpoint of Bridgman it is a necessary condition that the application of Tolman's equations give a correct result.

It is important to notice that Tolman's equations do not give any definite result in the case of the relation between the photoelectric current and the intensity of the light incident on the metal.

In the second place, we will apply the transformation equations to get the relation between the pressure exerted upon a body by the radiation incident upon it and the density of the radiation. Assume, experiment shows that the pressure p depends only on the energy density E of the radiation, then $p = f(E)$, and since

$$p' = \frac{p}{x^4} \quad \text{and} \quad E' = \frac{E}{x^4},$$

$f(E)$ must satisfy the following functional equation,

$$p = x^4 f\left(\frac{E}{x^4}\right) = f(E).$$

That is

$$p = f(E) = kE,$$

where k is a constant. As is well known, experiment shows that the radiation pressure is proportional to the energy density of the radiation.

SUMMARY.

Tolman's transformation equations lead to Einstein's law of the photoelectric effect if it is assumed that the kinetic energy of the electrons emitted from a metal, minus a certain constant, is only dependent upon the frequency of the incident light.

The equations also lead to the correct relation between the pressure of radiation and the density of the radiation.

LABORATORY OF PHYSICS,
UNIVERSITY OF ILLINOIS,
January, 1917.

ON THE PHOSPHORESCENCE OF THE URANYL SALTS.

BY EDWARD L. NICHOLS AND H. L. HOWES.

IN a recent paper¹ on the phosphorescence of the double salt uranyl ammonium sulphate, the following facts were established:

1. There is no appreciable change of color during decay.
2. The decay of phosphorescence is exceedingly rapid, the intensity falling to one thousandth of its initial value within .0035 second.
3. The very complex fluorescence spectrum at -180° is identical in structure and relative distribution of intensities with that observed during the earlier and later stages of phosphorescence.
4. The curve of decay of phosphorescence differs from the prevailing type in that although as usual two successive processes are distinguishable, the second process is more rapid instead of being slower than the first.

The study of these phenomena has since been extended to several other typical uranyl salts, the curves of decay of which were determined by the method described in that paper and under conditions of excitation, etc., as nearly constant as possible. These curves of decay are of the same new type originally found in the uranyl ammonium sulphate. The two processes, as determined by the customary method of plotting $I^{-1/2}$ as a function of the time are indicated by straight lines differing from one another in slope and the second process has in all cases the steeper gradient. Later experiments in which the intensity of excitation was increased, revealed the presence of a third process not included within the interval of time covered by our earlier experiment.

STUDIES INVOLVING THE FIRST AND SECOND PROCESSES.

The curves shown in Figs. 1 and 2 are typical of the results obtained with all the salts under observation. They represent the decay of the phosphorescence of the following compounds:

Curve.	Substance.
1.....	Uranyl ammonium sulphate
2.....	Uranyl potassium sulphate
3.....	Uranyl nitrate + 6H ₂ O
4.....	Uranyl sulphate
5.....	Uranyl ammonium chloride

¹ Nichols, Proceedings of National Academy of Sciences, Vol. II., p. 328.

The initial intensity, under like excitation, varies greatly in the different salts as also, to some extent, does the rate of decay. It will be noted that the initial intensities of the ammonium and potassium sulphates

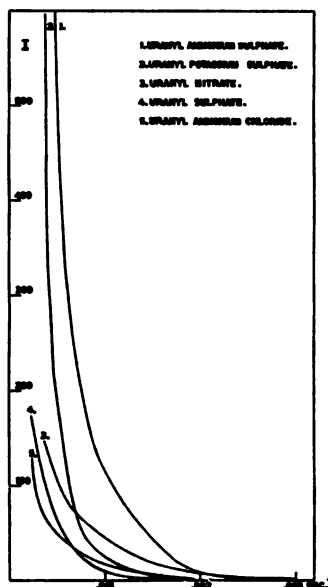


Fig. 1.

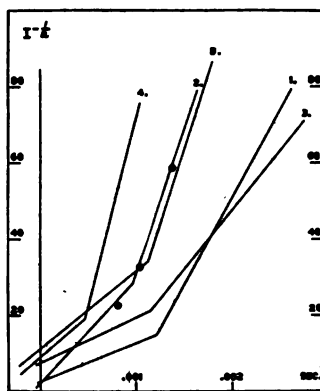


Fig. 2.

for example are several times greater than those of the nitrate, the sulphate and the ammonium chloride. This is however a question of previous history as well as of chemical and physical constitution as was determined in the following manner.

Uranyl potassium sulphate was dissolved in hot water and a mass of the minute crystals which were thrown down on cooling the solution were immediately sealed up in a glass tube. Care was taken throughout these manipulations to protect the precipitate from the action of light.

This sample, still in darkness, was mounted in the synchro-phosphoscope¹ and a curve of decay was taken, the first exposure to exciting light being that at the beginning of the run. The substance then showed temporarily, a brilliancy of phosphorescence much above that to be obtained under ordinary circumstances but was soon reduced to its normal and semi-permanent condition after which the usual curve of decay was obtained.

¹ For a description of this instrument, which was used in all the experiments recorded in this paper, except as otherwise stated, see the *PHYSICAL REVIEW* (2), VII., p. 586; also *Science*, V., XLIII., p. 937.

EXCITATION IN THE PRESENCE OF RED AND INFRA-RED RAYS.

To determine whether red or infra-red rays have an effect on these substances similar to that observed in the case of the phosphorescent sulphides a modification of the apparatus was made such that the surface under examination could be subjected to the intense illumination obtained by focusing the crater of an electric arc upon it. A screen of excellent ruby glass was interposed to cut off all but the longer waves and observations were made through a screen quite impervious to red.

Exposure to this source was found to affect measurably neither the brightness of fluorescence nor of phosphorescence nor the rate of decay. Curves taken after exposure to this source, those taken with the substance subjected to it interruptedly throughout the run, and curves in the determination of which readings were taken alternately with and without red light were all identical with those taken in entire absence from such exposures. The striking contrast between this negative result and the well-known effects of infra-red radiation upon the phosphorescence of the sulphides is notable.

The observations described in the note¹ already cited, showing the complete identity of the spectrum of fluorescence with that of phosphorescence seemed to indicate that the intensity would go over from that of fluorescence to that of phosphorescence without discontinuity. This conclusion was confirmed, within the errors of observation, by measurements just before and after the close of excitation. The only previous instances where this relation has been experimentally established so far as we know are to be found in Waggoner's² studies of phosphorescence of short duration and in our recent paper on the luminescence of kunzite.³

Where, as in these measurements, determinations have to be made through a very wide range and at exceedingly low intensities, doubts naturally arise as to the accuracy of the data for the lowest values. That in the present case the foot of the decay curves, where the brightness is reduced to a few thousandths of the initial intensity, is determined with a precision of the same order as that for higher values may be seen from Fig. 3 (Curve *BB*), in which the portion of the decay curve applying to the second process, *i. e.*, from .001 second onward, is plotted on a scale in which the ordinates for intensity are multiplied by ten. It will be noted that the observations fit the curve as closely as can be expected in work of this character. This is a fact quite in accord with previous

¹ Nichols, *l. c.*

² Waggoner, *PHYSICAL REVIEW*, XXVII., p. 209.

³ Nichols and Howes, *PHYSICAL REVIEW* (2), IV., p. 26.

experiences in the domain of luminescence where observations made at the very lowest intensities which would permit of distinguishing the pattern in the contrast field of a photometer or spectrophotometer have been found quite as reliable as those in which the illumination was much more intense.

In view of the unexpected character of the decay curves for the phosphorescence of the uranyl compounds, the question arises whether the rather unusual mode of excitation employed; *i. e.*, periodically repeated exposures, 120 times a second to groups of sparks of high frequency, might produce such a result or whether the decay curves are characteristic of this class of compounds whatever the mode of excitation. It is true that both Waggoner¹ and Zeller² using a Merritt phosphoroscope found in their studies of phosphorescence of short duration that excitation by means of a spark discharge very similar to our own gave decay curves of the usual type.

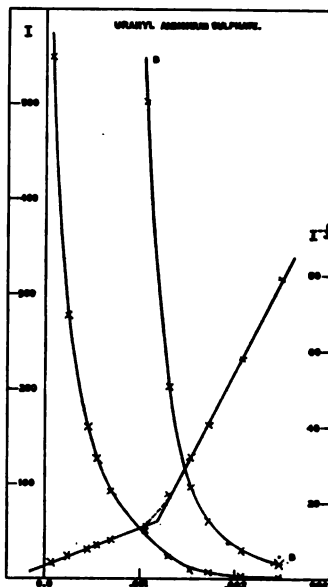


Fig. 3.

It is also obvious from the measurements already described that the interval between excitation in our experiments, *i. e.*, 1/120 second is sufficient for the complete discharge of the phosphorescent glow and since the absence of any effect of red and infra-red indicates that there is no storage of undeveloped energy to be carried over, such as occurs in the phosphorescent sulphides, it seems probable that the decay curves do not vary greatly from that which might be obtained, were it possible to make the experiment, from a single exposure.

To test this a run was made upon the sample of uranyl ammonium sulphate previously used but with the Merritt phosphoroscope.

By driving the disk of this instrument 3,000 revolutions a minute, much the same range of time intervals was available as with the synchro-phosphoroscope.

To further vary the conditions a quartz mercury arc was substituted for the spark gap of Waggoner and Zeller. The arrangement of apparatus was as shown in Fig. 4 in which *DD* is the revolving disk, *H* the mercury lamp, *P* the phosphorescent substance, *LB* the Lummer-Brodhun cube of

¹ Waggoner, PHYSICAL REVIEW, XXVII., p. 209.

² Zeller, PHYSICAL REVIEW, XXXI., p. 367.

the photometer, *SS* a color filter and milk glass screen. The device for shifting the oblique mirror *M* with reference to the aperture *A* in the disk is not shown.

Although the decay was somewhat more rapid in this determination on account of the less intense excitation; the curve was of precisely the type obtained by the previous method.

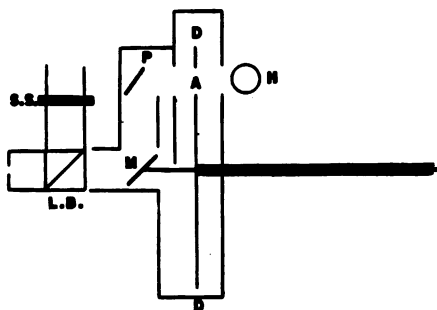


Fig. 4.

Measurements upon some of the bands of brief duration in the spectrum of the phosphorescent sulphides, recently made with the synchrono-phosphoroscope under experimental conditions identical with those described in this paper¹ yield curves of the usual type as-

associated with these sulphides; so that the question of the change of form being due to the method employed is effectually eliminated. It is proposed, however, to further test the possible influence of the mode of excitation and method of measurement. For this experiment a phosphoroscope of the drum type, to be driven at very high speeds, is under construction.

SOLID SOLUTIONS AND SEMI-FLUIDS.

The uranyl salts differ from nearly all if not all phosphorescent substances hitherto studied. We do not have, as in the phosphorescent sulphides, the preparations of Waggoner, the ruby, etc., to deal with a trace of active material in solid solution but with compounds that are in themselves brilliantly phosphorescent. If the peculiar character of the curve of decay is due to that fact it might be expected that uranium glass, in which the active material is considered to be in a state of solid solution, would have a law of decay corresponding to the prevailing type for such solutions, *i. e.*, with the first process as indicated by the curve for $I^{-1/2}$ and time, represented by a line of steeper slope than the line for the second process. A piece of uranium glass gave, however, a decay curve similar to those of the uranyl salts (see Fig. 5). Another preparation which differs from most of the uranyl salts is the uranyl sodium phosphate, a sample of which was made by D. T. Wilber for certain studies in fluorescence recently published.² This substance is a very viscous liquid with the characteristic green fluorescence of the uranyl compounds.

¹ Nichols, Proc. Nat. Acad. Sciences, (1917).

² Howes and Wilber, PHYSICAL REVIEW (2), Vol. 7, p. 394, Mar., 1916.

Being a liquid one might expect, in accordance with the findings of Becquerel for liquids in general,¹ that there would be no observable after-glow. It is true that Becquerel expressed the belief that with a phosphoroscope of sufficient speed, phosphorescence would probably be detected in fluorescent liquids but no one, so far as we know, save Dewar in an unconfirmed statement concerning a supposed phosphorescence of liquid air, has since that time (1859) recorded an instance of phosphorescence excepting in solids and gases.

When a tube containing the phosphate was tested with the synchrono-phosphoroscope no phosphorescence was found of duration sufficient to be detected. Another sample so prepared as to reduce the amount of water to a minimum did however exhibit phosphorescence of measurable duration. This preparation, so slow was its rate of flow, might be regarded as a plastic solid rather than a viscous liquid. A bead of microcosmic salt, colored in the usual manner with uranium oxide, was comparable in its phosphorescence with canary glass.

It appears that the persistence of luminescence is due to the consistency of the substance and disappears as the fluidity increases; also that the peculiar type of decay here described is common, not only to the crystalline uranyl salts in general but also to the gelatinous forms, as in this double salt and to substances in which uranium appears in solid solution as in the case of the canary glass.

THE THIRD PROCESS.

E. Becquerel² in the course of his great pioneer work on phosphorescence made a number of observations on the uranyl salts and on uranium glass. He noted the brilliant initial intensity and very rapid decay and to test the independence of the constant in his equation of decay when the illumination varied he made many measurements. If from his data we compute $I^{-1/2}$ as a function of the time we obtain curves of the same general form as those in Fig. 2.

Becquerel's observations are not numerous enough, taken by them-

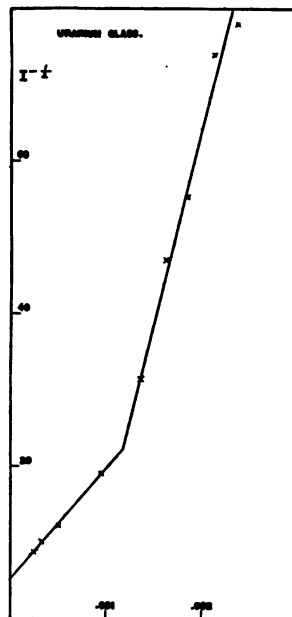


Fig. 5.

¹ See E. Becquerel, *La Lumière*, Vol. I., chapter on Phosphorescence.

² E. Becquerel, *Annales de Chimie et de Physique* (3), LXII., p. 1, 1861.

selves, to determine completely the type of curve. His measurements, however, cover a larger time interval than ours and the values for the longest times indicate an even more rapid decay following the second process. We had indeed found some indications of a similar tendency which had been omitted from our curves as lying almost beyond the range of definite determination.

To investigate the further trend of the curves of decay the intensity of excitation was increased by readjustment of the sparking circuit by which means it was found possible to extend the time interval for more than .006 sec. beyond the cessation of excitation.

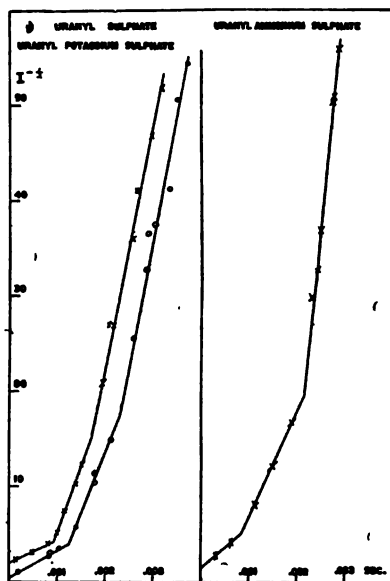


Fig. 6.

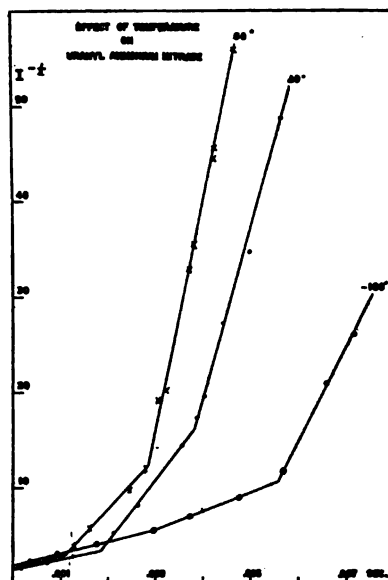


Fig. 7.

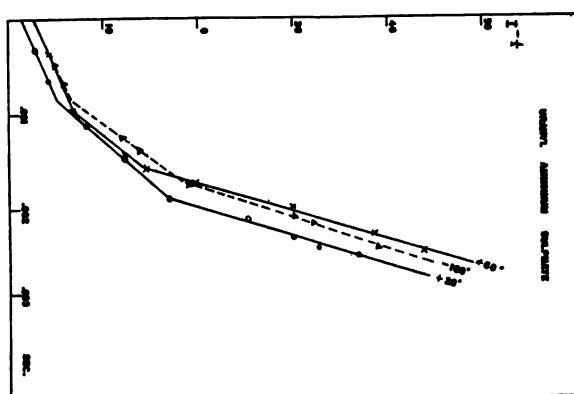


Fig. 8.

Careful, often repeated measurements, of the various salts showed in fact a third linear process beginning where our previous determinations had ceased and having a steeper slope, indicative of still more rapid decay. Typical results are indicated in Figs. 6, 7, 8, etc.

These processes may be numbered for convenience 1, 2 and 3 in the order in which they occur. Processes 1 and 2 are in general of about equal duration for a given salt. The abruptness of transition, however, varies greatly, and in some instances the change of slope is so gradual as to encroach seriously on process 2 at both ends.

THE INFLUENCE OF TEMPERATURE.

The only previous instances of decay of phosphorescence in which the later stages are more rapid than those preceding are noted by Ives and Luckiesh¹ in their study of the influence of temperature on phosphorescence, and by E. H. Kennard² in a more recent paper.

Ives and Luckiesh measured the phosphorescence of one of Lenard and Klatt's sulphides (BaBiK from Leppin and Masche). This substance was found to be very sensitive to change of temperature and the results at 0° C., 22° and 35°, when plotted for $I^{-1/2}$ any time in the usual manner, gave curves varying greatly in slope. The curve for 0 is concave towards the time axis, that for 22° linear and that for 35° *strongly convex*. They show that a linear relation may be obtained for each of these curves by varying the exponent of I .

The effect of temperature in the case of the phosphorescent sulphides, where one has to do with a composite of many overlapping bands of varying duration is undoubtedly different from that to be expected with the uranyl salts where the spectrum in spite of its complexity of structure is a unit. It was deemed of interest however to determine the effect of temperature upon the latter.

For this purpose a specimen of the uranyl ammonium nitrate was mounted within a cylindrical Dewar flask with unsilvered walls and its decay of phosphorescence was determined with the synchro-phosphoroscope at a temperature a few degrees above that of liquid air (about -180°) at +20° and at +60°. The last-named temperature was maintained during the run by means of an electrical heating coil.

The principal change consists in a marked retardation of decay with lowering temperature (see Fig. 7), but this is not a universal characteristic of the uranyl compounds. Uranyl ammonium sulphate, for example (Fig. 8), is but slightly influenced in its rate of decay by change of tem-

¹ Ives and Luckiesh, *Astrophysical Journal*, XXXVI., p. 330 (1912).

² Kennard, *PHYSICAL REVIEW* (2), IV., p. 278 (1914).

perature and the curve for -180° is intermediate between those for $+20^\circ$ and $+60^\circ$.

THE EFFECT OF VARYING THE INTENSITY OF EXCITATION.

To determine the effect of the intensity of excitation a series of measurements were made with the spark gap at various distances from the phosphorescent surface. The substance observed in these experiments was uranyl rubidium nitrate. It was found possible to make observations of the decay of phosphorescence with the excitation reduced to a two hundredth of that usually employed.

From the curves obtained of which four are given in Fig. 9, it will be

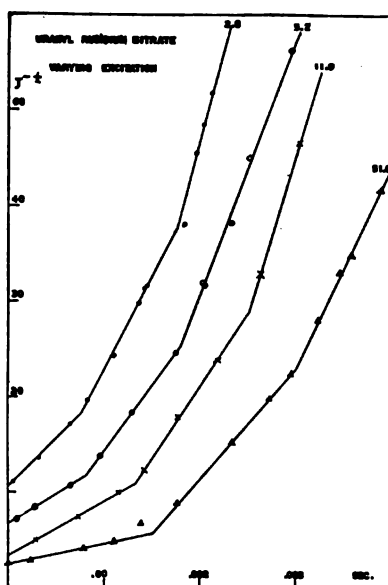


Fig. 9.

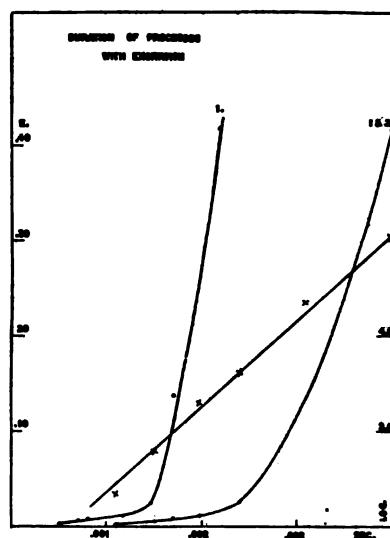


Fig. 10.

noted that all three processes are present whatever be the intensity of the exciting light; also that, taken roughly, processes 1 and 2 are of nearly equal duration and that with decreasing intensity of excitation the duration of each of these processes diminishes.

These relations are better shown in Fig. 10, in which the duration of process 1 and the sum of the durations of processes 1 and 2 counting from the close of excitation are plotted with the intensity of the exciting light as ordinates. Approximately in both cases the duration is proportional to the natural logarithm of the excitation. (See Table I.)

TABLE I.

Variation of Length of Processes with Excitation (Phosphorescence of Uranyl Rubidium Nitrate).

Intensity of Excitation (E).	Nat. Log E .	Duration.		
		Process 1.	Process 2.	Process 1 + 2.
41.70	6.033	.0022 sec.	.0018	.00400
13.70	4.919	.00170	.0014	.00310
2.52	3.220	.00147	.000993	.00240
1.35	2.590	.00118	.00080	.00198
0.900	2.190	.00080	.00090	.00170
0.476	1.560	.00070	.00076	.00146
0.201	0.698	.00050	.00060	.00110

This decrease in the duration of the two processes with falling excitation affords an obvious explanation of the varying character of the curves of decay of phosphorescent substances. Where the excitation is chiefly superficial, as in the case of some powders, the excitation may be nearly of one intensity and the curve made up of well-defined linear processes with sharp inflection points. We have found this to be the case in many instances. Where on the other hand fluorescence is excited within the crystalline mass by rays that have suffered considerable loss by absorption, etc., there will be a wide range of intensities of excitation and a curve results with distributed knees and linear processes shortened and sometimes almost obliterated. We observed this particularly where a clear crystal was mounted with faces perpendicular to the photometer and was excited from behind so that the light emitted by the surface nearest the exciting source passed through the crystal and was partially absorbed. Excitation occurred within the crystal in diminishing amount with increasing depth and the composite phosphorescence reaching the eye under such conditions showed this blending effect to a marked degree. The same crystal when excited from in front gave a curve in which the angles between processes were made more sharply defined. The effect in question is probably a general one and may well account for the perplexing differences in the curves of decay obtained under slightly varying circumstances. Thus one observer will obtain an angular curve where another, studying the same material, can detect no linear processes. The same observer, indeed, in attempting to repeat his measurements will often find the above-mentioned change of type under conditions which seem to be identical but in which the same relations as regards superficial and internal excitation are not preserved.

We found in the study of this effect a crystal one smooth face of which gave the blended curve while the opposite face, which was rough, gave

the angular curve, a change produced and reproducible by merely rotating the crystal through 180° .

THE PHOSPHORESCENCE OF VARIOUS NITRATES.

Observations were made on a series of nitrates previously prepared for the detailed comparison of the fluorescence spectra of that salt.¹ These consist of crystals with 6 H_2O (rhombic); 3 H_2O (triclinic) and 2 H_2O (system undetermined) as water of crystallization and a specimen sealed in glass which had been rendered as nearly anhydrous as was possible without decomposing the nitrate.

The curves of decay indicate a much slower rate of decay for the crystalline forms than for the anhydrous nitrate. Whatever effect the amount of water of crystallization may have is doubtless masked by the far greater influence of the crystalline form. This is perhaps to be expected since as has been shown in the paper just cited, these specimens exhibit as great differences in the structure and appearance of their fluorescence and absorption spectra as commonly exist between entirely distinct uranyl salts. That similar differences are found in the case of salts similar in composition but differing in crystalline form will be brought out in a forthcoming paper.

OBSERVATIONS ON POLARIZED PHOSPHORESCENCE.

Certain crystals of the double chlorides of uranyl exhibit fluorescence spectra consisting of sets of bands polarized at right angles to one another.

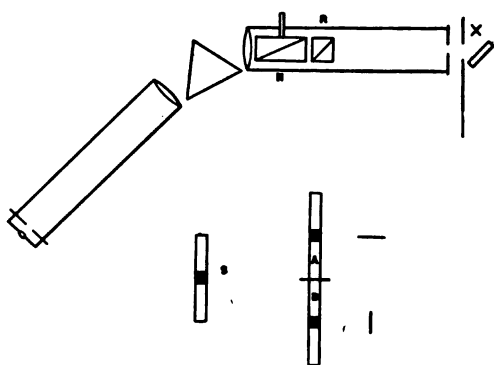


Fig. 11

To determine whether these components after the close of excitation decay independently or without change in their relative intensities, the following experiment was made.

A crystal of the rubidium uranyl chloride that exhibited the phenomenon of polarized fluorescence was mounted behind the disk of the synchrono-phos-

phoroscope and was observed with a spectroscope. The slit of the latter instrument was divided into two parts by means of an opaque strip across the middle (S, Fig. 11).

¹ Nichols and Merritt, *PHYSICAL REVIEW* (2), Vol. IX, p. 113 (1917).

Within the collimator a doubly refracting rhomb *R* and Nicol prism *N* were mounted. The rhomb gave two slit images vertically displaced and the adjustment was such that the lower part (*A*) of one image was contiguous with the upper part (*B*) of the other.

Thus two spectra of the phosphorescent field were obtained corresponding to the two polarized components. These presented the usual distinctive structures at whatever stage of the phosphorescent decay they were observed. By rotation of the Nicol prism the two fields could be brought to equal brightness for any given part of the spectrum and this balance, if made with the sector of the phosphoroscope set so as to give observations at .0005 second after extinction, was found equally correct up to .005 second or as long as phosphorescence was observable. The two components therefore decay at the same rate.

SUMMARY.

1. All uranyl salts thus far examined possess the same type of phosphorescence; *i. e.*, with increasing instead of diminishing rates of decay.

2. This is true not only of the crystalline forms but also of uranyl compounds in solid solution or in the plastic state characteristic of the double phosphates.

3. The initial brightness of phosphorescence under like excitation varies greatly with the different salts; as does also to some extent the rate of decay.

4. The brightness of a salt newly prepared in darkness is greater when first excited than subsequently but it soon reaches a nearly stable condition.

5. Exposure to red and infra-red rays is without effect as regards the rate of decay.

6. The phosphorescence, like the fluorescence of the uranyl salts appears to be independent of the mode of excitation and the structure of the intricate spectrum is the same during excitation and through the observable phosphorescent interval.

7. Changes in the rate of decay are not continuous but occur in definite steps there being at least three successive processes within the interval covered by observations, *i. e.*, about .006 seconds. These processes follow a law such that $I^{-1/2}$ is in linear relation to the time.

8. The first and second processes counting from the close of excitation are of nearly equal duration, increasing in duration with the intensity of excitation in such a manner that the duration of the process is approximately proportional to the natural logarithm of the excitation.

9. In certain salts, such as uranyl ammonium nitrate decay is retarded by cooling, in other cases the temperature effect is slight.

10. Uranyl nitrates with 2, 3 and 6 molecules of water of crystallization vary greatly in the rate of decay but the changes in crystalline form appear to be more important in this respect than the amount of water.

11. In the case of the polarized spectra of the double chlorides, both components decay at the same rate and no change in relative brightness could be detected throughout the range covered by observation.

PHYSICAL LABORATORY OF CORNELL UNIVERSITY,

January, 1917.

NOTES ON THE CHANGE OF RESISTANCE OF CERTAIN SUBSTANCES IN LIGHT.¹

BY T. W. CASE.

THE author has been making a systematic search for substances which show a change of resistance when exposed to light. The results of this search may prove interesting to those who are working in the same field, as several new light-active substances have been found.

Considerable work has already been done upon substances which change their resistance under the influence of light. The best known examples of this effect are selenium, stibnite and cuprous oxide, of which the latter has recently been announced by Professor A. H. Pfund, at Johns Hopkins, *PHYSICAL REVIEW*, 7 (1916), 298.

As Professor Brown, of Iowa University, obtained much better results with a large crystal of selenium than had been obtained originally with the finely divided form, crystallized mineral specimens were used in this investigation where possible.

It may not be out of place here to give a description of the apparatus used, as it was simple and highly effective. It consisted essentially of an arc lamp and a variable speed water motor mounted upon a concrete base. The water motor drove a disk which had 15 sectorial openings; and so served to rapidly interrupt the beam of light produced by the arc, and brought to focus upon the substance under observation. Lead wires ran from this substance to an Ayrton shunt, which was connected to a three-step De Forrest Audion Amplifier. (The third step of this instrument was capable of increasing the energy of the original impulse 12,000 per cent.) High resistance receivers could be inserted in any one of these steps; and if any photoelectric action was produced by the interrupted beam of light, a distinct note would be heard in the receivers. The pitch of this note varied directly with the number of light interruptions per unit of time.

The crystal specimen under examination was held in a small clamp in the path of the interrupted beam of light. Contact was made at the ends of the crystal through the agency of thin platinum foil, which was pressed firmly against the crystal by a spring of variable tension in the

¹ The conditions of this experiment do not cover the minerals in all possible structures. Hence the negative actions are not necessarily conclusive.

Mineral.	Composition. ¹	Conductivity.	Action.
Sulphur.....	Se	N.C. ²	None
Selenium.....		G.C. ³	Fair, easily audible
Tellurium.....		G.C.	None
Antimony.....	Contains sometimes silver, iron or arsenic	G.C.	None
Bismuth.....		G.C.	None
Realgar.....		N.C.	None
Orpiment.....	As ₂ S ₃	N.C.	None
Stibnite.....	Sb ₂ S ₃	P.C. ⁴	Small
Bismuthinite.....	Bi ₂ S ₃	P.C.	Very good
Tetradymite.....	Bi ₂ (TeS) ₃	N.C.	None
Molybdenite.....	MoS ₂	G.C.	Fair
Domeykite.....	Cu ₃ As	G.C.	None
Algodonite.....	Cu ₃ As	G.C.	None
Whitneyite.....	Cu ₃ As	G.C.	None
Argentite.....	Ag ₂ S	G.C.	Good, large fatigue
Hessite.....	Ag ₂ Te	G.C.	None
Galenite.....	PbS	G.C.	Fair
Altaite.....	PbTe		None
Clausthalite.....	PbSe	G.C.	None
Berzelianite.....	Cu ₂ Se	N.C.	None
Crookesite.....	(Cu, Tl, Ag) ₂ Se	N.C.	None
Chalcocite.....	Cu ₂ S	N.C.	None
Stromeyerite.....	(Ag, Cu) ₂ S	N.C.	None
Sternbergite.....	Ag, Fe ₂ S ₃	G.C.	None
Acanthite.....	Ag ₂ S	G.C.	Good, large fatigue
Sphalerite.....	ZnS	N.C.	None
Metacinnabarite.....	HgS	G.C.	None
Guadalcazarite.....	Found near above, contains a little zinc	N.C.	None
Cinnabar.....		N.C.	None
Covellite.....		P.C.	None
Greenockite.....	CdS	N.C.	None
Millerite.....	NiS	P.C.	None
Niccolite.....	NiAs	N.C.	None
Breithauptite.....	NiSb	P.C.	None
Pyrrhotite.....	Fe ₁₁ S ₁₃	G.C.	None
Polydymite.....	Ni ₂ S ₃	G.C.	None
Bornite.....	Cu ₃ FeS ₄	G.C.	None
Linnaeite.....	Co ₃ S ₄	G.C.	None
Chalcopyrite.....	CuFeS ₂	G.C.	None
Stannite.....	Cu ₂ .S.FeS.SnS ₂	G.C.	None

¹ The formulæ given for the minerals are those obtained from the mineral dealers; the order is that of Dana's System of Mineralogy.

² N.C. is used in this work as meaning that the substance is practically a non-conductor in the dark.

³ G.C. indicates that a crystal of the substance one mm. on a side, in the holder used, at an applied potential of 110 volts has a resistance of less than 1 megohm.

⁴ P.C. indicates that a crystal of the substance one mm. on a side, in the holder used, at an applied potential of 110 volts has a resistance greater than 1 megohm.

Mineral.	Composition.	Conductivity.	Action.
Hauerite.....	MnS ₂	G.C.	None
Cobaltite.....	CoS ₂ CoAs ₂	N.C.	None
Gersdorffite.....	NiAsS	G.C.	None
Corynite.....	Ni(AsSb)S	N.C.	None
Ullmannite.....	NiSbS	G.C.	None
Marcasite.....	FeS ₂	G.C.	None
Arsenopyrite.....	FeAsS	G.C.	None
Pearceite.....	Ag ₃ AsS ₄	G.C.	Fair
Sylvannite.....	(Au, Ag)Te ₂	N.C.	None
Calaverite.....	(Au, Ag)Te ₂	N.C.	None
Kermesite.....	Sb ₂ S ₂ O	N.C.	None
Zinkenite.....	PbSb ₂ S ₄	G.C.	None
Sartorite.....	PbAs ₂ S ₄	N.C.	None
Rathite.....	3PbS ₂ As ₂ S ₄	P.C.	None
Emplectite.....	CuBiS ₂		None
Chalcostibite.....	CuSbS ₂	P.C.	None
Galenobismutite..	PbBi ₂ S ₄	G.C.	None
Miargyrite.....	AgSbS ₂	G.C.	Fair
Plagionite.....	5PbS.4Sb ₂ S ₂	N.C.	None
Schirmerite.....	3(Ag ₂ Pb)S.2Bi ₂ S ₂	N.C.	None
Cosalite.....	Pb ₂ Bi ₂ S ₄	G.C.	None
Jamesonite.....	Pb ₂ Sb ₂ S ₄ ¹	P.C.	Fair
Kobellite.....	Pb ₂ (BiSb) ₂ S ₄	P.C.	None
Semseyite.....	Pb ₇ Sb ₂ S ₁₄		None
Freieslebenite.....	(Pb, Ag) ₂ Sb ₄ S ₁₁	P.C.	None
Bournonite.....	3(Cu ₂ , Pb)S.Sb ₂ S ₂	P.C.	Good
Boulangerite.....	Pb ₂ Sb ₂ S ₄	N.C.	Good
Fizelyite.....	Ag ₃ Pb ₂ Sb ₂ S ₁₁	N.C.	None
Guitermanite.....	10PbS.3As ₂ S ₂	N.C.	None
Pyrargyrite.....	Ag ₃ SbS ₃	P.C.	Fair
Proustite.....	Ag ₃ AsS ₃	G.C.	Good
Tetrahedrite.....	Cu ₂ Sb ₂ S ₇	G.C.	None
Stephannite.....	Ag ₂ SbS ₄	G.C.	Fair
Polybasite.....	Ag ₂ SbS ₄	G.C.	Fair
Enargite.....	3Cu ₂ S.As ₂ S ₂	N.C.	None
Epiboulangerite..	Pb ₂ Sb ₂ S ₄	N.C.	None
Cerargyrite.....	Silver Chloride	N.C.	None
Iodyrite.....	AgI	P.C.	Fair
Fluorite.....	CaF ₂	N.C.	None
Laurionite.....	PbCl ₂ .Pb(OH) ₂	N.C.	None
Cumengeite.....	PbCuCl ₂ (OH) ₂	N.C.	None
Quartz (tiger eye)	SiO ₂	N.C.	None
Valentinite.....	Sb ₂ O ₃	N.C.	None
Bismite.....	Bi ₂ O ₃	N.C.	None
Molybdite.....	MoO ₃	N.C.	None
Cervantite.....	Sb ₂ O ₄	N.C.	None
Cuprite ¹	Cu ₂ O	P.C.	Fair
Chalcotrichite.....	Cu ₂ O	N.C.	None

¹ Cuprite was predicted to show this action by Professor A. H. Pfund (Phys. Rev., 7 (1916), 298).

Mineral.	Composition.	Conductivity.	Action.
Silver oxide	Ag ₂ O (pastiles freshly prepared)	P.C.	Fair
Periclasite	MgO	N.C.	None
Manganosite	MnO	P.C.	None
Zincite	ZnO	N.C.	None
Massicot	PbO	N.C.	None
Tenorite	CuO	P.C.	None
Corundum	Al ₂ O ₃	N.C.	None
Hematite	Fe ₂ O ₃	G.C.	None
Ruby spinel	MgAl ₂ O ₄	N.C.	None
Magnetite	FeO.Fe ₃ O ₄	G.C.	None
Franklinite	(FeZnMn)O(FeMn) ₂ O ₃	P.C.	None
Chromite	FeCr ₂ O ₄	P.C.	None
Chrysoberyl	BeAl ₂ O ₄	N.C.	None
Hausmannite	Mn ₂ O ₄	N.C.	None
Minium	Pb ₂ O ₄	N.C.	None
Cassiterite	SnO ₂	G.C.	None
Rutile	TiO ₂	N.C.	None
Brookite	TiO ₂	N.C.	None
Pyrolusite	MnO ₂	G.C.	None
Turgite	Fe ₄ H ₂ O ₇	G.C.	None
Diaspore	Al ₂ O ₃ .H ₂ O	N.C.	None
Göthite	FeO(OH)	N.C.	None
Limonite	2Fe ₂ O ₃ .3H ₂ O	N.C.	None
Brucite	MgO.H ₂ O	N.C.	None
Gibbsite	Al(OH) ₃	N.C.	None
Chalcophanite	(Mn, Zn)O.2MnO ₃ .2H ₂ O	N.C.	None
Psilomelane	H ₄ MnO ₆	N.C.	None
Lampadite	MnO ₂	P.C.	None
Calcite	CaCO ₃	N.C.	None
Siderite	FeCO ₃	N.C.	None
Smithsonite	ZnCO ₃	N.C.	None
Phosgenite	PbCO ₃ .PbCl ₂	N.C.	None
Malachite	CuCO ₃ .Cu(OH) ₂	N.C.	None
Azurite	2CuCO ₃ .Cu(OH) ₂	N.C.	None
Aurichalcite	2(Zn, Cu)CO ₃ .3(Zn, (Cu)(OH) ₂	N.C.	None
Microcline	KAlSi ₃ O ₈	N.C.	None
Albite (soda fels- spar)	NaAlSi ₃ O ₈	N.C.	None
Actinolite	(CaMgFe)SiO ₃	N.C.	None
Beryl	Be ₃ Al ₂ Si ₆ O ₁₈	N.C.	None
Garnet	Ca ₃ Al ₂ Si ₂ O ₁₂	N.C.	None
Diopside	H ₂ CuSiO ₄	N.C.	None
Zircon	ZrSiO ₄	N.C.	None
Cyrtolite	(Altered zircon)	N.C.	None
Gadolinite	Be ₂ FeY ₂ Si ₂ O ₁₀	N.C.	None
Zoisite	Ca ₂ (AlOH)Al ₂ (SiO ₄) ₂	N.C.	None
Epidote	HCa ₂ (Al, Fe) ₂ Si ₂ O ₁₁	N.C.	None
Prehnite	H ₂ Ca ₂ Al ₂ Si ₂ O ₁₁	N.C.	None
Chondrodite	H ₂ Mg ₁₀ Si ₈ O ₂₄ F ₄	N.C.	None
Tourmaline	R ₄ SiO ₆	N.C.	None

Mineral.	Composition.	Conductivity.	Action.
Staurolite	$H_4(FeMg)_4(AlFe)_{24}Si_{11}O_{68}$	N.C.	None
Apophyllite	$H_7KCa_4(SiO_3)_8$ and $4\frac{1}{2}H_2O$	N.C.	None
Stilbite	$H_4(Na_3Ca)Al_3Si_6O_{18}$ and $4H_2O$	N.C.	None
Laumontite	$H_4CaAl_3Si_4O_{14}$ and $2H_2O$	N.C.	None
Penninite	$Hg(Mg, Fe)_4Al_3Si_5O_{18}$	N.C.	None
Talc	$H_2Mg_3Si_4O_{12}$	N.C.	None
Sepiolite	$H_4Mg_3Si_5O_{10}$	N.C.	None
Pyrophyllite	$Al_3Si_2O_8$ and H_2O	N.C.	None
Thaumasite	$CaSiO_3.CaCO_3.CaSO_4.15H_2O$	N.C.	None
Perovskite	$CaTiO_3$	N.C.	None
Columbite	$FeCb_2(Ta)_2O_6$	N.C.	None
Samaraskite	$R_3R_4(Nb, Ta)_6O_{21}$	P.C.	None
Euxenite (columbite yttria)	$R(NbO_3)_3 R_2(TiO_3)_2$ and $3/2 H_2O$	N.C.	None
Vanadinite	$Pb_3V 1/3 PbCl$	N.C.	None
Olivenite	$Cu_3As_2O_8.Cu(OH)_2$	N.C.	None
Descloizite	$Pb_2O_3VO_3$	N.C.	None
Vivianite	$Fe_3P_2O_8$ and $8H_2O$	N.C.	None
Torbernite	$Cu(UO)_2P_2O_8$ and $8H_2O$	N.C.	None
Barite	$BaSO_4$	N.C.	None
Crocoite	$PbCrO_4$	P.C.	None
Wulfenite	$PbMoO_4$	N.C.	None
Agularite	$AgSAs_2Se$	N.C.	None
Chalcedony	SiO_2	N.C.	None
Copalite	(Fossil Copal)	P.C.	None
Lorandite	$TlAsS_3$	N.C.	None
Smoky quartz	SiO_2	N.C.	None

clamp. An E.M.F. of 110 volts was placed in series circuit with the crystal, the audion apparatus and a protective resistance. In the work with very fragile crystals the clamp was replaced by a mounting of the crystal upon a small cork, current being sent through the specimen through the agency of gold leaf contacts at the ends of the crystal.

In some instances, the resistance of the crystal under examination was such that the impressed voltage allowed either too much or too little current to flow through the apparatus. With low resistance crystals this was taken care of by the shunt. With crystals of very high resistance the voltage and size of the crystal were adjusted until the desired current was obtained. A list of the minerals examined, an indication of their photo activity as determined by the note on the receivers, and their relative conductivity in the dark as shown on pages 305-309.

The active substances are being further studied in regard to their resistance changes, fatigue, deduction and induction periods. Two specimens which show remarkable action are acicular crystals of bismuth sulphide (bismuthinite), and a granular lead antimony sulphide. It

suffices to say here that the resistance of a piece of the latter 1 mm. by 10 mm., reduces about 5,000 per cent. from the value in the dark in dull sunlight at an applied potential of 110 volts. If the substance be actually heated up, either by exposure to heat or passing a current which slowly heats the substance, then the resistance is gradually lowered, but this is a slow process. All of the photo active sulphides show the quick light reaction in the red part of the spectrum. Both the bismuthinite and the lead antimony sulphide give fine results when used as a transmitter in a photo phone system, using a manometric flame for transforming the voice variations into light variations. The voice reproduction is very clear and loud. Specimens of lead antimony sulphide at present show no evidence of an induction period or fatigue, outside of the slow heating effect due to the current which reaches an equilibrium after several hours, after which the light reaction then becomes constant. This, as well as its large change of resistance with small variations of light intensity allow it to be used as a very delicate actinometer.

SUMMARY.

1. An apparatus has been described for the detection of certain photo electric phenomena.
2. The change of resistance of certain substances in light are being studied; and a number of new compounds have been found which show a change of conductivity in light.

AUBURN, N. Y.

AN IMPROVED HIGH VACUUM MERCURY VAPOR PUMP.

BY CHAS. T. KNIPP.

THE diffusion pump of Gaede¹ has stimulated a number of investigators in this country to enter the field of pump design with the result that several improvements involving new principles have been published. In a recent number of the *PHYSICAL REVIEW* Langmuir² describes an improved mercury vapor pump "characterized by its extreme speed and the high degree of vacuum attainable." The writer of this note being interested for a number of years in the production of high vacua also seized upon this opportunity to aid, if possible, in simplifying the means by which vacua are produced in the research laboratory and submits the following design made wholly of glass as an improved high-vacuum high-speed mercury vapor pump.

The pump complete, except the usual mercury vapor trap, is shown in the figure, which is one third full size. The bulb to be exhausted and trap are fused to *B*, while the tube *E* is attached to the supporting or rough pump. The mercury vapor rising from the lower bulb, which is heated in a sand or heavy oil bath, streams up through the short tubes *P* and *O* and is deflected downward through an annular throat by the umbrella *N*. The issuing mercury vapor at once condenses on the water-cooled surface of the enveloping tube and, as Langmuir³ pointed out, the gas that comes from *B* is forced mechanically downward from the lower edge of *N* along the cooled surface of the condensing chamber. This accumulated gas is removed through the lateral tubes *b b*, which unite at the top and form the exhaust tube *E*, all being enveloped by the water jacket *XY*, as shown in the figure. This construction keeps the

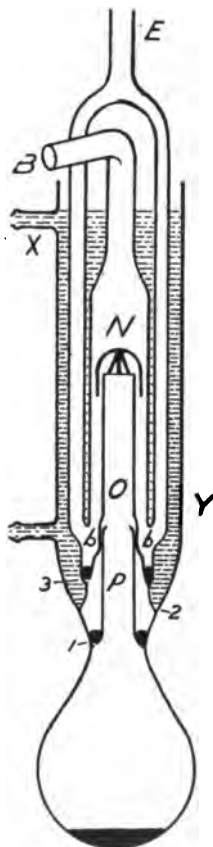


Fig. 1.

An improved high
vacuum mercury vapor
pump.¹ Ann. Physik, 46, 357, 1915.² Phys. Rev., 8, 48, July, 1916.³ Gen. Elect. Rev., 19, 1060, Dec., 1916.

mercury, which collects at the ring-seal 3, cool, and thus removes the objection that mercury vapor having an *upward* velocity would enter the annular condensing chamber. A small opening shown at 3 serves as a valve which allows the accumulated mercury to pass, yet due to surface tension maintains a perfect seal. The short tube *P* is inserted to shield the hot mercury vapor streaming up from the boiler from condensing on the surfaces at 3. The upper end of *P* telescopes loosely into the lower end of *O*, while the lower end is secured by the ring-seal 1, having also a small valve opening in it through which the mercury passes back into the boiler. By making the upper end of *P* conical condensed mercury vapor is caught in the annular space thus formed and automatically seals the space *PO* from the cavity just outside of *P*. The cold mercury collected at the ring-seal 3, and the adjacent water-jacket, thus have but little opportunity of cooling the hot stream of mercury vapor passing up through *PO*, and, furthermore, the temperature gradient between the ring-seals 1 and 2, and 3 and *b b* are not abrupt, hence there is no danger of the glass cracking. This construction very much simplifies the glass-blowing, since the tube throughout the process is kept perfectly symmetrical.

In the absence of a convenient means of measuring the pressure within the discharge vessel quickly, the writer has chosen to express the degree of exhaustion in terms of the cathode dark space and the sparking distance at a parallel gap in air. The induction coil used was a 10 cm. spark Kohl coil, and the parallel gap was between balls 1.75 cm. in diameter. The supporting pump was a Gaede rotary mercury pump backed by a Gaede oil box pump. The mercury was heated in an oil bath by one bunsen burner. The trap beyond *B* was immersed in liquid air. Mercury vapor pumps are not sensibly operative until the vacuum drawn by the supporting pump is of the order of 1 cm. Crookes dark space, depending upon the construction of the pump as regards annular throat, etc. Hence the test for speed should not be from atmospheric pressure. The following tests serve to give an indication of the speed attainable.

Test 1.—With the discharge vessel fused to *B* (through a centimeter tube 30 cm. long) having a volume of 270 c.c. it required, by repeated trial, 29 minutes to lengthen the dark space from 1 cm. to 5 cm. equivalent spark in air; while with the *mercury vapor pump operative it required but 43 seconds.*

Test 2.—With a 6 liter vessel attached by a short large-diameter tube, it required 51 minutes to range from 1 cm. dark space to an equivalent parallel gap in air of 4.65 cm.; which time interval with *this pump*

operative was reduced to $2\frac{1}{2}$ minutes. At the end of another 2 minutes the vacuum was so hard that the 10 cm. Kohl coil was not able to force a discharge.

The advantages of this form of hot blast mercury vapor condensation glass pump may be briefly stated as

1. The symmetrical design simplifies the glass-blowing.
2. Full effectiveness of the hot blast of mercury vapor without sensible loss of heat through a long delivery tube.
3. Effective cooling by a proper placing of water-jacket, ring-seals, and of an internal-shielding tube.
4. The use of simple mercury valves for the direct return of the condensed mercury vapor to the boiler.

LABORATORY OF PHYSICS,
UNIVERSITY OF ILLINOIS,
January 15, 1917.

ELECTRODE POLARIZATION IN GASES.

BY C. A. SKINNER.

IN a recent article¹ I have described experiments which show beyond doubt that there exists a large polarization P. D. between the cathode and the conducting gas of a glow current; and which, at the same time, prove the reliability of wire sound measurements of potential in the "Crooke's dark space," where the error, if any, should be the greatest.

These experiments force the conclusion that, as indicated, by wire sound measurements, there exists in general a polarization P. D., at the anode as well as at the cathode, wherever the current is transmitted from the gas to the electrode by ionic carriers. If the anode be placed in the negative glow its polarization P. D. practically vanishes² presumably because there the current is transmitted to it by free electrons, which probably do not produce much of a polarization. Outside the negative glow the anode polarization is about 20 volts. These facts together with Chrisler's experiments on anode absorption³—which show that in general the negative carriers in the glow current are atoms—leads to the interesting conclusion that the electrons must become largely attached to the atoms shortly after leaving the negative glow.

Lacking any free positive electrons, there is naturally no part of the ordinary glow current where the cathode polarization vanishes. It is found to be of the same magnitude in the negative glow as at the main cathode. Only when the current is made up entirely of free electrons—and possibly ions freed from the electrodes instead of discharging to them—can one assume that no polarization exists at the electrodes. Such a condition obtains approximately at the cathode in the ordinary arc, at both electrodes in the mercury arc, and at a Wehnelt cathode.

Important errors have undoubtedly been made in various experimental determinations where the existence of a polarization, especially at the cathode, has been ignored. An interesting optical corroboration of this fact has been recently observed by Tate, in which he vainly attempted to shorten the single line radiation from mercury by applying, as he supposed, a back field parallel to his spectroscope slit to slow down the

¹ C. A. Skinner, *Phys. Rev.*, February, 1917.

² C. A. Skinner, *Phil. Mag.*, 6, 2, p. 616, 1901.

³ V. L. Chrisler, *Physik. Zeitschr.*, 10, p. 745, 1909.

electrons. He finally found that his supposed back field did not exist, except as a polarization potential difference at the cathode. All those experiments are subject to such an error, where a theoretical potential has been supposedly impressed on a conducting gas by two metal electrodes charged to a definite potential difference.

In the previous article (l.c.) the following law was found with an aluminum cathode placed in the negative glow of a separate ionizing current through hydrogen:

$$V_b = g \left(\frac{j}{n} \right)^2, \quad (1)$$

where V_b is the polarization P. D. and j the current density at the cathode, n the number of positive ions per c.c. in the gas surrounding the cathode and g a constant. This law fits satisfactorily for values of V_b above about 10 volts,¹ and also holds at the main cathode.

In this article is given a simple mathematical theory, with experimental applications, of this electrode polarization as arising from an elastic rebound of the discharging ions from the electrode. An earlier attempt was made to solve the same problem from the same viewpoint. The greater simplicity and applicability of the present theory, as compared with the former, is largely due to the fact that we have not sought to express here the distribution of potential, but have related other magnitudes.

In the earlier theory the polarization region was *assumed* to extend one mean free path of the discharging ion from the electrode. Our empirical equation (1) is however independent of the gas pressure. This fact, together with other general considerations, has led to the conviction that the extent of the polarization region from the electrode is but a small part of a mean free path of the discharging ions. This view simplifies any theoretical treatment of the subject to the extent that the chance of collision between the discharging ions and the gas molecules need not be considered.

THEORY.

The simple law expressed by equation (1) will first be derived. This has been found to hold with aluminum in hydrogen for values of V_b above 10 volts. It will then be shown that the deviation from the law observed below 10 volts may be satisfactorily ascribed to the temperature velocity of the discharging ions.

Case in which the Temperature Velocity of the Discharging Ions is

¹ The apparent deviation for values of V_b above about 80 volts is probably due to the increase in n arising from the ionization produced by electrons freed from the transverse cathode.

Negligible.—On the average the discharging ions enter the polarization region with the kinetic energy which the electric field imparts to them on their last mean free path λ outside that region. Let the drop of potential over this last mean free path be E_λ and the polarization P. D. be V_b . The ion strikes the electrode with the kinetic energy equal to

$$(E_\lambda + V_b)e,$$

where e is the ionic charge. By impact on the electrode it loses on the average a fraction k' of this energy, rebounding with a kinetic energy equal to

$$(1 - k')(E_\lambda + V_b)e.$$

The polarization P. D. arises from the consequent accumulation of the discharging ions, hence its outer boundary is reached by those ions on their first rebound which are reflected in a direction normal to the equipotential surfaces. Their turning point is where all their kinetic energy of rebound is converted into the potential energy $V_b e$. We have then

$$V_b e = (1 - k')(E_\lambda + V_b)e; \quad (2)$$

hence

$$V_b = \frac{1 - k'}{k'} E_\lambda. \quad (3)$$

But we have similarly the relation

$$E_\lambda e = \frac{1}{2} m \bar{u}^2, \quad (4)$$

where $\frac{1}{2} m \bar{u}^2$ is the mean kinetic energy of the ions on their entrance to the polarization region. Substituting from (4) in (3)

$$V_b = \frac{1}{2} \frac{m}{e} \cdot \frac{1 - k'}{k'} \cdot \bar{u}^2. \quad (5)$$

To compare this expression with the empirical equation (1) we introduce the condition that the electric current to the polarization region is carried entirely by the discharging ions. This holds very accurately under ordinary glow current conditions. Then

$$j = neu, \quad (6)$$

in which j is the current density, n the number of discharging ions per c.c. just outside the polarization region, and u their average velocity at that point. Obviously u is proportional to $\sqrt{\bar{u}^2}$, so that, from (5) and (6)

$$V_b = \frac{1}{2} \cdot \frac{m}{e^2} \cdot \frac{1 - k'}{k^1} \cdot g' \cdot \left(\frac{j}{n}\right)^2, \quad (7)$$

where

$$g' = \frac{\bar{u}^2}{u^2}.$$

Equation (7) is identical in form with the empirical equation (1).

Inasmuch as it has been shown in the previous article that (1) fits not only the conditions at the transverse cathode, but also expresses the previously discovered law for the main cathode, we will turn our attention here to the determination of k' . From (2)

$$k' = \frac{E_{\lambda}}{E_{\lambda} + V_b}. \quad (8)$$

From this equation the values of k' have been calculated for cathodes of several metals in hydrogen, nitrogen, and oxygen—the values of E_{λ} and V_b having been obtained from measurements made by Cheny¹ Neuswanger² and myself³—the value of λ being taken as the m.f.p. of the molecule. These are incorporated in Table I. The values of k'

TABLE I.
Values of k' .

Cathode.	Current Density.	Hydrogen.			Nitrogen.			Oxygen.		
		V_b .	E_{λ} .	k' .	V_b .	E_{λ} .	k' .	V_b .	E_{λ} .	k' .
Aluminum.....	"Normal"	78	2.3	.030	94	3.2	.033	167	4.7	.027
	2×"Normal"	101	2.6	.025	112	3.8	.033	207	5.1	.024
	4×"Normal"	130	3.6	.027	128	4.4	.033	248	6.1	.024
Platinum.....	"Normal"	143	2.2	.015						
Nickel.....	"Normal"	174	2.4	.0135						
Steel.....	"Normal"	155	2.2	.014						

are found to vary but little with considerable variation in current density, polarization P. D. and gas pressure—though the effect of the latter is not brought out in the table. In hydrogen k' is about twice as large with aluminum as with platinum, nickel or steel—for which metals it is nearly the same. For aluminum in the different gases an appreciable difference in k' is found, but it is not large.

It should be remarked that k' is derived from average conditions, while the border of the polarization region is really marked by those ions which rebound farther than the average. In that case k' should not necessarily give the actual fractional loss of energy on impact, but merely be proportional to it. The factor of proportionality should be the same however for the different gases and metals, since the relation is in terms of the m.f.p. of the ions.

Under the discussion of the temperature effect a method is given for determining the real value of loss by impact on the electrode.

¹ W. L. Cheny, *PHYS. REV.*, VII., p. 241, 1916.

² Wm. Neuswanger, *PHYS. REV.*, VII., p. 253, 1916.

³ C. A. Skinner, *PHYS. REV.*, VI, p. 158, 1915.

Effect of a Temperature Velocity of the Discharging Ions.—A distinct deviation from the simple theoretical equation (7) or (1) is found when polarization potentials below 10 volts are used. This is best seen in the curve of apparent cathode resistance R_b as in Fig. 1.

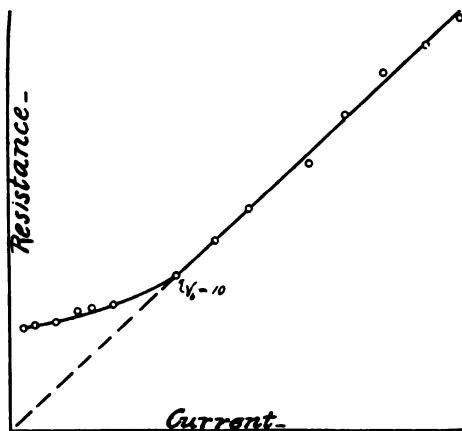


Fig. 1.

From (1)

$$R_b = \frac{V_b}{j}$$

$$= \frac{gj}{n^2}.$$

For constant n the apparent resistance should be a straight line from the origin. For values of V_b below 10 volts the resistance is observed to deviate rapidly toward a constant magnitude as the current is reduced. That is to say, for low values of V_b the number of ions discharging to the cathode, in a given time, is less than the derived law requires.

We assume that with polarizing potentials below 10 volts the law of distribution of potential between the transverse electrodes is the same as above 10 volts. Then the number of discharging ions which *enter* the polarization region in a given time will be that determined by the simple law. If now, the current be smaller than the law requires, it is because a part of the ions rebound from the electrode with a sufficient component velocity to carry them out of the polarization region again. The origin of this excess velocity is conceived to be in the temperature of the ions.

Let n'/n be the fractional part of the ions entering the polarization region, which escape from it again. Then equation 6 becomes

$$j = (n - n')eu \quad (9)$$

and consequently (7) becomes

$$V_b = \frac{1}{2} \cdot \frac{m}{e^2} \cdot \frac{1 - k'}{k'} \cdot g' \left(\frac{n - n'}{j} \right)^2$$

or in terms of (1)

$$V_b = g \left(\frac{j}{n - n'} \right)^2. \quad (10)$$

From this

$$\frac{n'}{n} = 1 - \left(\frac{g}{V_b} \right)^{\frac{1}{2}} \frac{j}{n}. \quad (11)$$

This quantity n'/n represents the fractional part of the entrant ions which, after impact on the electrode, have a sufficient component velocity normal to it to carry them out of the polarization region again. From the law of velocity distribution and the value of n'/n we can calculate the lowest component temperature velocity with which this fraction of the ions enter the polarization region. Those having the lowest velocity possess sufficient component velocity outward, after impact, to just take them out of the polarization region again. Having the entrant velocity and the emergent velocity we can determine the loss on impact.

It is first necessary however to assume the kind of reflection—whether it be diffuse, regular, or of special form. To avoid lengthening the discussion it may be simply stated that, on the basis of a single impact and no collision between the ions within the polarization region, the magnitudes involved render diffuse reflection impossible. As a special form of reflection we will consider, along with regular reflection, the case in which the polarization region fits the surface of the electrode like a skin, approximating a spherical form about each point of impact. Then any direction of rebound tends to carry the ion out of the polarization region. In that case only the actual speed of the molecules, after impact and before, needs to be considered. This is admittedly a rather artificial assumption, but its treatment will be found of value in bringing out clearly the comparative applicability of the idea of regular reflection.

Case of Regular Reflection of the Ions.—For this case we need to consider only the component velocity of the ions normal to the equipotential surfaces, including the electrode. These we will assume to be parallel planes. Without making an appreciable error we may assume—for the highly conductive negative glow—that the entire applied P.D. is condensed in the polarization region of the transverse cathode, and that the flow into this region is by diffusion. This would at least be true for a vanishing value of j .

Let u_0 be the component temperature velocity with which those ions enter the polarization region which rebound with just sufficient velocity to carry them out again. Then similarly to (5)

$$V_b = \frac{1}{2} \cdot \frac{m}{e} \cdot \frac{1-k}{k} \cdot u_0^2, \quad (12)$$

where k represents now, as contrasted with k' , the *real* fractional loss of energy by impact on the electrode. From this we obtain

$$k = \frac{\frac{1}{2} \cdot \frac{m}{e} \cdot u_0^2}{\frac{1}{2} \cdot \frac{m}{e} \cdot u_0^2 + V_b}. \quad (13)$$

The value of u_0 may be calculated from the temperature—assuming the entrant ions have the same mean kinetic energy as the gas molecules. Maxwell's law of velocity distribution gives

$$\frac{dn}{n} = \frac{4}{\pi^{3/2}} \cdot \frac{c^3}{\alpha^3} \cdot e^{-(c^2/\alpha^2)} \cdot dc, \quad (14)$$

where dn/n is the fractional number of the molecules whose speed is between c and $c + dc$; and α is the most probable speed of the n molecules. To obtain the corresponding fraction whose component velocity lies between u and $u + du$ we have from (14)

$$\frac{dn}{n} = \frac{1}{\pi^{3/2} \alpha^3} e^{-(u^2/\alpha^2)} du \int_{-\infty}^{\infty} e^{-(v^2/\alpha^2)} dv \int_{-\infty}^{\infty} e^{-(w^2/\alpha^2)} dw = \frac{1}{\sqrt{\pi} \alpha} \cdot e^{-(u^2/\alpha^2)} du. \quad (15)$$

Let now n'/n be the fractional number of those *entering* unit area of the polarization region, having a component velocity greater than u_0 as defined in (13). For this case dn/n is double the value given in (15), since there n would express those passing through unit area in both directions. We obtain by integration

$$\frac{n'}{n} = \frac{2}{\sqrt{\pi}} \int_{u_0/\alpha}^{\infty} e^{-(u/\alpha)^2} d\left(\frac{u}{\alpha}\right) = 1 - \frac{2}{\sqrt{\pi}} \int_0^{u_0/\alpha} e^{-(u/\alpha)^2} d\left(\frac{u}{\alpha}\right). \quad (16)$$

We have calculated the values of u_0/α from (16), for various values of n'/n , by the aid of a table of definite integrals. These are given in Table II., which shows for example, that only about 15 per cent. of the molecules of a gas have a component velocity greater than α .

TABLE II.

n'/n .	u_0/α .	n'/n .	u_0/α .	n'/n .	u_0/α .	n'/n .	u_0/α .
.10	1.16	.35	.66	.60	.37	.85	.13
.15	1.02	.40	.60	.65	.32	.90	.09
.20	.91	.45	.53	.70	.27	.95	.04
.25	.81	.50	.48	.75	.23		
.30	.73	.55	.42	.80	.18		

TABLE III.

$n'/n.$	$c_0/a.$	$n'/n.$	$c_0/a.$	$n'/n.$	$c_0/a.$	$n'/n.$	$c_0/a.$
1.000	.10	.90	.55	.66	.90	.34	1.30
.995	.20	.87	.60	.62	.95	.27	1.40
.99	.25	.84	.65	.57	1.00	.21	1.50
.97	.35	.81	.70	.54	1.05	.163	1.60
.96	.40	.77	.75	.49	1.10	.12	1.70
.94	.45	.735	.80	.45	1.15	.09	1.80
.92	.50	.695	.85	.41	1.20	.045	2.00

In Table III. are given a series of corresponding values for c_0/α calculated from (14). It shows for example that 57 per cent. of the molecules have a speed greater than α , while only 4.5 per cent. have a speed more than twice its value.

EXPERIMENTAL.

To test the above theory of the temperature effect, measurements have been made with transverse electrodes of aluminum, steel and platinum placed in the negative glow of a separate ionizing current through hydrogen. The apparatus was described in detail in the previous article.

A series of measurements consisted in observing the transverse current produced by various applied potentials between the transverse electrodes—with check readings made in all cases to insure against any progressive change. Table IV. gives a characteristic series from aluminum as cathode. The first two columns contain the observed magnitude of polarization P. D. and corresponding current density at the transverse cathode. The third column gives the calculated values of n'/n as obtained from (11) by assuming that n' vanishes at the higher potentials. This table shows that with a polarization P. D. of .46 volt 67 per cent. of the entrant ions according to the theory, escape without giving up their charge; while with a P. D. of 4.6 volts, 13.5 per cent. escape.

To calculate k —from (13)—it is necessary to obtain the value of $\frac{1}{2} \cdot (m/e) \cdot u_0^2$ in terms of the potential difference in volts required to produce it. We have

$$\frac{1}{2} \frac{m}{e} u_0^2 = \frac{1}{2} \frac{m}{e} \alpha^2 \left(\frac{u_0}{\alpha} \right)^2.$$

For a hydrogen molecule at 20° C.—the assumed temperature of the negative glow¹—

$$\alpha = 1.55 \times 10^5 \text{ cm./sec.}$$

If it carries a univalent charge

¹ Wood—Wied. Ann., 59, p. 238—did not find it much above room temperature.

$$\frac{m}{e} = .7 \times 10^{-14} \text{ c.g.s.e.s.}$$

Then

$$\begin{aligned} \frac{1}{2} \frac{m}{e} u_0^2 &= .84 \times 10^{-4} \left(\frac{u_0}{\alpha} \right)^2 \text{ c.g.s.e.s.} \\ &= 2.5 \times 10^{-2} \left(\frac{u_0}{\alpha} \right)^2 \text{ volts.} \end{aligned} \quad (17)$$

From (17) Table II., and (13) the values of k given in the fourth column of Table IV. were calculated. These show about one half per cent. loss of energy on impact, which is nearly the same for all polarizations between .46 and 4.6 volts. In the next column is given the calculated

TABLE IV.

Aluminum Cathode in Hydrogen.

Gas pressure: 1.48 mm.

Ionizing Current: 1.10 m.a.

V_0 (Volts).	j (M. A.).	π/n .	k .	k_0 .	V_0 .	j .	π/n .
.46	$.80 \times 10^{-2}$	.67	.0049	.042	14.3	13.7×10^{-2}	-.004
.92	1.56	.55	.0048	.029	19.2	15.8	.000
1.85	3.04	.38	.0052	.021	29.0	19.8	-.020
2.80	4.40	.27	.0054	.017	38.9	22.2	.010
3.75	5.44	.22	.0051	.015	48.8	24.8	.015
4.65	6.72	.135	.0060	.015	58.6	27.6	.000
9.45	11.0	.007	—	—	68.5	30.0	-.006

values of the loss by impact for the special form of reflection in which the speed c_0 is used in (13) instead of the component velocity u_0 . This magnitude is found to vary nearly three times within the same range in which the value obtained from u_0 varies only about 20 per cent. Thus the results are found distinctly in favor of the theory of regular reflection. In the succeeding tables the constant for regular reflection only has been calculated.

Table V. gives the mean values obtained from four series of observations on aluminum cathodes—freshly cleaned, by filing, for each series—taken

TABLE V.

Aluminum Cathode in Hydrogen.

V_0 .	π/n .	k .	V_0 .	π/n .
.45	.675	.0048	14.1	-.025
.90	.555	.0048	19.0	-.03
1.82	.395	.0050	28.7	-.015
2.74	.28	.0053	38.6	.015
3.65	.205	.0055	48.4	.007
4.60	.135	.0061	58.2	.004
9.30	-.005	—	68.1	.002

with gas pressures from 1.5 to 2.0 mm., and with ionizing currents varying from 1.1 to 2.0 m.a. These different sets agreed very satisfactorily. This table brings out a slight increase in k with increasing polarization P. D., which appears consistent with the fact that k' in Table I. is about 2.5 per cent., while k in this table is only .5 per cent. If there were no change with polarization P. D. k' should be somewhat less than k instead of greater.

Table VI. gives likewise the means of three similar series of results with steel cathodes of the same dimensions as the aluminum—area: .25 cm².

TABLE VI.
Steel Cathode in Hydrogen.

V_h	π'/n	k	V_h	π'/n
.46	.70	.0040	14.2	.085
.91	.58	.0042	19.1	.035
1.83	.43	.0043	38.8	.02
2.75	.32	.0044	38.6	.003
3.70	.245	.0045	48.5	-.002
4.60	.185	.0048	58.3	-.003
9.4	.09	—	68.2	-.01

For steel, as expected, k is somewhat smaller than for aluminum. It should be, however, in the same ratio to aluminum that is found from Table I. for k' but this is not the case. The ratio for k' is 1 : 2 while for k it is about 4 : 5. An explanation for this discrepancy is suggested by the results from platinum—given in Table VII.

TABLE VII.
Platinum Cathode in Hydrogen. Showing Gradual Change with Time in Service.

V_h	A.		B.		C.		D.	
	π'/n	k	π'/n	k	π'/n	k	π'/n	k
.45	.735	.0033	.675	.0048	.665	.0052	.63	.0064
.90	.61	.0036	.58	.0042	.565	.0046	.485	.0068
1.80	.445	.0040	.40	.0050	.365	.0056	.31	.0071
2.70	.335	.0043	.29	.0051	.27	.0056	.20	.0077
3.65	.245	.0046	.20	.0057	.18	.0062	.125	.0082
4.55	.19	.0047	.14	.0060	.12	.0067	.07	
9.3	.06		.045		.015		-.005	
14.1	.018		.02		-.007		-.023	
19.0	.005		.005		.00		-.017	
28.7	.000		.01		-.005		.00	
38.5	.002		.00		.00		-.003	
48.4	-.005		.00		.00		.005	
58.2	.013		-.01		.003		.01	
68.1	-.015		.01		.00		.025	

The sequence of tests made with platinum are given under the columns *A, B, C, D*. These tests extended over three days' time, during which the platinum was kept in an atmosphere of hydrogen at low pressure. A progressive increase of k is clearly evident—supposedly due to a change in the metal surface arising from the absorption of hydrogen. Under ordinary cathode polarizations of say 140 volts or more, the hydrogen is probably kept expelled from the cathode surface by the resulting bombardment, while with the potentials of five volts and under this is probably not the case. The result is a somewhat different surface condition of the same metal under different polarizations.

Discussion.—We have found that, under the assumption of no appreciable collisions between the discharging ions in the polarization region, they must suffer regular reflection at the cathode. Considering the relative irregularities of the electrode surface, regular reflection seems impossible.

If, however, the reflection be diffuse, then the collisions between the ions within the polarization region must be sufficiently numerous to produce a resultant velocity distribution approximately satisfying Maxwell's law. The "temperature" of the ions at a given equipotential surface would then be less than at the electrode by an amount determined by their stored up potential energy, relative to that at the electrode surface. Thus we should have a velocity distribution at the outer boundary of the polarization region, for the rebounding ions, similar to that for the entrant ions—except that the average velocity of the latter would be higher, and therefore a greater number entering than escaping.

This view probably accounts for the apparent applicability of the theory of a regular reflection of the ions impinging on the cathode. If it be correct then k' derived from the mean conditions should, like k , represent the average loss of energy of the ions which get back to the outer boundary of the polarization region.

THE BRACE LABORATORY,
UNIVERSITY OF NEBRASKA,
December, 1916.

CRITICAL EXAMINATION OF THE LAW OF X-RAY LINE SPECTRA.

BY HORACE SCUDDER UHLER.

THE present paper differs from my earlier one¹ on the same subject in three respects: (i) It includes the wave-lengths of all the accurately measured lines of the *K*-series (only a few of which were formerly accessible to me), (ii) it contains the results of more rigorous calculations, and (iii) it presents the material in greater detail. The object of the paper is to subject to critical examination the functional dependence of the *X*-ray lines of any one sub-series upon the corresponding atomic numbers so that answers may be given to the following questions, which are: How accurately does Moseley's law reproduce the observed wave-lengths? What other empirical formula will represent the numerical data within the given limits of experimental error? And what is the probable order of magnitude of the high frequency radiations of elements having small atomic numbers and of which the spectra have not yet been obtained? All the wave-lengths used (save two) may be found in a paper² by M. Siegbahn entitled "Bericht über die Röntgenspektren der chemischen Elemente (Experimentelle Methoden und Ergebnisse)." These data were chosen because they are the most recent, they were all obtained in the same laboratory with the same spectrometers, and they constitute the most extensive, accurate, and consistent set available.

In Moseley's second paper³ the analytical result of his investigations is concisely stated in the sentence: "From the approximate linear relation between $\nu^{1/2}$ and N for each line we obtain the general equation $\nu = A(N - b)^2$, where A and b are constants characteristic of each line." That b may not equal zero is pointed out very clearly in Moseley's reply⁴ to certain objections raised by F. A. Lindemann. Accordingly, in the present paper, the equation $\sqrt{\nu_N} = a + bN$ will be referred to as Moseley's law. The degree of accuracy with which this relation represents the experimental wave-lengths will now be considered.

¹ Proc. Natl. Acad., Vol. 3, p. 88, Feb., 1917.² Jahrbuch der Radioaktivität und Elektronik, Vol. 13, Part 3, Sept., 1916.³ Phil. Mag., Vol. 27, p. 712, April, 1914.⁴ Nature, Vol. 92, p. 554, Jan. 15, 1914.

The values of the parameters a and b were calculated, by the method of least squares, from the square-roots of the reciprocals of the known wave-lengths (expressed in angstroms) and the corresponding atomic numbers, N . Then, from a , b , and N , the wave-lengths were recalculated.

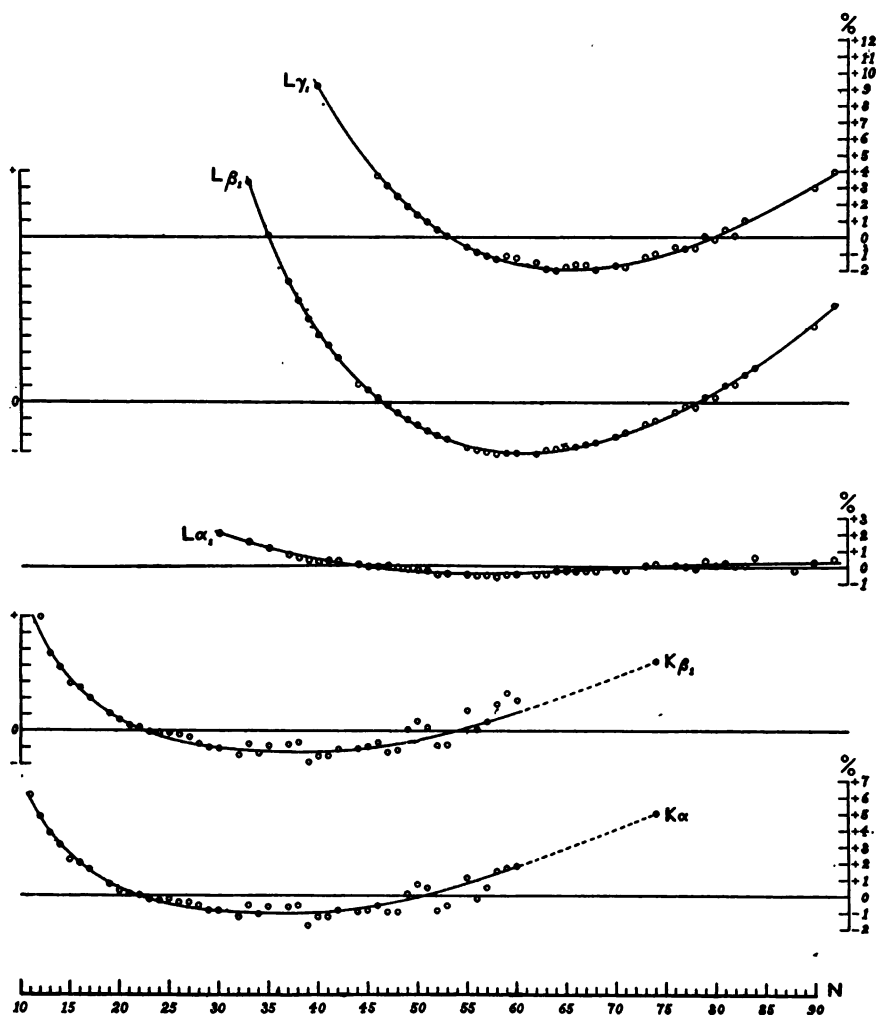


Fig. 1.

Finally the percentages by which the mathematical wave-lengths are algebraically greater than the experimental data were computed. This procedure was followed for the entire experimental range of each of the series $K\alpha_m$, $K\beta_1$, $L\alpha_1$, $L\beta_1$, and $L\gamma_1$. Since some of the lines of the $K\alpha$ series have not been measurably resolved into the two components

α_1 and α_2 , mean values were used for all of the resolved pairs. For this reason the subscript m is written below the first α . The results obtained are shown graphically for five series in Fig. 1, and numerically for the $K\text{-}\alpha_m$ and $L\text{-}\beta_1$ series in the columns headed %, M of Tables I and II. The ordinates (with respect to the corresponding horizontal axes) of the centers of the little circles in the figure are the percentage deviations defined above, the common scale of the diagrams being indicated on the associated vertical axes. The abscissæ of these centers, which represent the atomic numbers, have their scale indicated at the bottom of the figure. The curves drawn through the five sets of points were not obtained by calculation since they are only intended to afford general guides to the eye. The two points at the extreme right ends of the curves for the K series were plotted from wavelengths for tungsten, kindly furnished by A. W. Hull in a private communication. These data were not used in computing the values of the parameters a and b of Moseley's formula. For this reason the curves are indicated by short dashes from $N = 60$ to $N = 74$. Notwithstanding the fact that there is much room for improvement in the portions of the curves pertaining to values of N greater than 50 (especially for $K\text{-}\beta_1$), nevertheless the two points for tungsten seem to accord very well with the general drift of the nearer points which were derived from Siegbahn's tables. Incidentally, the circles for the $K\text{-}\alpha_m$ and $K\text{-}\beta_1$ series show (with the single exception for $N = 59$) precisely the same irregularities of distribution or "accidental" errors. This may indicate that homologous wave-lengths were derived from the same spectrograms and were dependent upon some poorly determined reference line, such as the direct image, for example.

Even a casual inspection of the five graphs brings out the fact that, in general, Moseley's law does not represent at all well the experimental data throughout the entire ranges now at our disposal. The percentage deviations at the extreme points and at the algebraic minima being respectively about + 6, + 5, - 1; + 7, + 4, - 1.2; + 2, + 0.2, - 0.6; + 13.3, + 5.8, - 3; and + 9.2, + 4, - 2; for the $K\text{-}\alpha_m$, $K\text{-}\beta_1$, $L\text{-}\alpha_1$, $L\text{-}\beta_1$, and $L\text{-}\gamma_1$ series. Most of these deviations greatly exceed the experimental errors estimated by the original investigators. For the K series, from germanium to chromium, it is stated¹ that: "The agreement of the lines measured on different plates is very good; the deviations are of the order of magnitude of 0.1 to 0.2 per cent." For the L series, from tantalum to uranium, it is remarked² that: "The accuracy of the

¹ Physikal. Ztschr., p. 48, Feb. 15, 1916.

² Phil. Mag., Vol. 32, p. 41, July, 1916.

measurements is estimated to about 0.3 per cent." With regard to the *L* series, from zinc to lutetium, the author says:¹ "The reflexion angles calculated have an uncertainty of about 0.3 per cent." Unfortunately I have not been able to find data of this kind for the remaining wave-lengths in Siegbahn's list. Hull's values for tungsten, which were obtained by the ionization method, are probably not in error by more than one per cent. Another fact brought out by the graphs is that Moseley's law represents the *L*- α_1 series better than any of the remaining four. This is quite contrary to all the accessible statements in the literature of the subject since they consistently convey the impression that Moseley's law holds very well for the *K* series but that it needs correction in the region of shortest wave-lengths of the *L* series. In fact, O. W. Richardson exceeds the original qualifying remarks of Moseley when he says:² "For the *K* series the agreement is quite exact, but for the *L* series there is a slight deviation from linearity with the elements of very high atomic weight." Since the preceding examination shows conclusively that Moseley's law (as distinguished from his epoch-making association of the chemical elements with atomic numbers and broad point of view) does not satisfactorily represent the experimental wave-lengths over the entire ranges of the five longest sub-series, the next problem is to find a function which will reproduce the numerical data within the estimated limits of experimental error.

The only interpolation formula which I have been able to find in the literature is the one proposed by A. W. Hull.³ For the *K* series he gives $\nu_a = 1.64 \times 10^{15} N^{2.10}$ and $\nu_b = 1.56 \times 10^{15} N^{2.15}$, true frequencies being represented. Since the exponents of *N* do not seem to be promising from the theoretical standpoint, as their values were obtained by the graphical method alone, and since the formula was not given the more crucial test of the *L* series, I have not subjected it to examination by the method of least squares. On the other hand, Hull's function has at least one desirable property, that is, simplicity.

Since last fall I have tested the equation¹

$$\sqrt{\nu_N} = A + BN + \frac{C}{D - N} \quad (1)$$

fairly exhaustively without succeeding either in replacing it by a function involving less than four parameters or in finding any systematic deviation between the wave-lengths computed from it and the experimental data. Any true but slight discordance may, of course, be masked by the

¹ Phil. Mag., Vol. 32, p. 497, Nov., 1916.

² "The Electron Theory of Matter," p. 511, 1914.

³ Proc. Natl. Acad., Vol. 2, p. 267, May, 1916.

inaccuracies in the observed quantities, which is a question that future investigation alone can decide. The particular form of equation (1) was found in the following empirical manner. The residuals of $\sqrt{\nu_N}$ (not percentage deviations of wave-lengths), obtained by applying the method of least squares to Moseley's law, were plotted on a large scale in the hope of obtaining a correction "term" to the linear relation. Inspection of the five sets of points thus obtained suggested that the hyperbola $c_0N^2 + Nr + c_1N + c_2r + c_3 = 0$ ($r \equiv$ residual) would fulfil the requirements of the problem. As stated above, subsequent rigorous calculations have confirmed this belief. The correction expression may be transformed into

$$-r = a' + c_0N + \frac{C}{D - N},$$

where $a' = c_1 - c_0c_3$, $C = c_2(c_1 - c_0c_3) - c_3$, and $D = -c_2$. Since $\sqrt{\nu_N} = a + bN - r$ the final form is equation (1), where $A = a + a'$, and $B = b + c_0$. Since equation (1) may be written $BN^2 - N\sqrt{\nu_N} + (A - BD)N + D\sqrt{\nu_N} - (C + AD) = 0$ the geometric meaning of the new formula is that the square-roots of the frequencies of the X-ray lines are related to the atomic numbers in the same way as the ordinates and abscissæ of the points on an hyperbola. In short, these radicals form a family of hyperbolæ for the five series tested. This result was brought out by the magnification of which the residuals were susceptible, but it could not have been seen so readily by plotting $\sqrt{\nu_N}$ against N directly. See, for example, the approximate straight lines given by Moseley, Siegbahn, and others.

The wave-lengths calculated by the aid of formula (1) and the percentage deviations of the numbers thus obtained are given in the columns headed λ_c , U and $\%$, U , respectively, in Tables I and II. The series $K\text{-}\alpha_m$ and $K\text{-}\beta_1$ were selected as typical since the shorter wave-lengths of the former seem to involve relatively large experimental errors while those of the latter appear to constitute the most accurate set of all. For the same reasons the correction coefficients c_0 , c_1 , c_2 , c_3 were obtained graphically for the $K\text{-}\alpha_m$ series, and by the method of least squares for the $L\text{-}\beta_1$ lines. In fact, as might be expected, in the cases where both methods were applied to one and the same series the gain in accuracy attained by the more rigorous computation was slight and altogether incommensurate to the extra labor entailed. The probable errors of a single residual, expressed in per cent., are ± 0.28 , ± 0.25 , ± 0.11 , ± 0.10 , and ± 0.16 for the $K\text{-}\alpha_m$, $K\text{-}\beta_1$, $L\text{-}\alpha_1$, $L\text{-}\beta_1$, and $L\text{-}\gamma_1$ series, in the order named. It is therefore clear that equation (1) represents the

TABLE I.

Series K- α_m .

El.	<i>N.</i>	λ_0	$\lambda_0, M.$	$\lambda_0, U.$	$\%, M.$	$\%, U.$
W.....	74	0.210	0.220	0.210	+5.0	0.0
Nd.....	60	0.333	0.338	0.334	+1.8	+0.3
Pr.....	59	0.345	0.350	0.346	+1.7	+0.4
Ce.....	58	0.358	0.363	0.359	+1.5	+0.4
La.....	57	0.374	0.376	0.373	+0.5	-0.4
Ba.....	56	0.391	0.390	0.387	-0.2	-0.9
Cs.....	55	0.400	0.405	0.402	+1.1	+0.5
I.....	53	0.439	0.437	0.435	-0.6	-0.9
Te.....	52	0.458	0.454	0.453	-0.9	-1.0
Sb.....	51	0.470	0.472	0.472	+0.5	+0.5
Sn.....	50	0.489	0.492	0.493	+0.7	+0.8
In.....	49	0.513	0.513	0.514	+0.1	+0.3
Cd.....	48	0.541	0.535	0.537	-1.0	-0.7
Ag.....	47	0.565	0.559	0.561	-1.0	-0.6
Pd.....	46	0.588	0.584	0.587	-0.6	-0.1
Rh.....	45	0.617	0.612	0.615	-0.9	-0.3
Ru.....	44	0.647	0.641	0.645	-1.0	-0.3
Mo.....	42	0.712	0.705	0.711	-0.9	-0.1
Nb.....	41	0.752	0.741	0.748	-1.3	-0.5
Zr.....	40	0.791	0.780	0.788	-1.3	-0.3
Y.....	39	0.838	0.822	0.831	-1.8	-0.8
Sr.....	38	0.874	0.868	0.877	-0.6	+0.4
Rb.....	37	0.924	0.918	0.927	-0.7	+0.4
Br.....	35	1.038	1.030	1.042	-0.7	+0.4
Se.....	34	1.107	1.094	1.107	-1.1	0.0
As.....	33	1.172	1.165	1.178	-0.6	+0.5
Ge.....	32	1.259	1.242	1.256	-1.3	-0.2
Zn.....	30	1.435	1.422	1.437	-0.9	+0.1
Cu.....	29	1.541	1.527	1.542	-0.9	+0.1
Ni.....	28	1.655	1.645	1.660	-0.6	+0.3
Co.....	27	1.783	1.776	1.791	-0.4	+0.4
Fe.....	26	1.930	1.923	1.937	-0.4	+0.4
Mn.....	25	2.095	2.090	2.103	-0.2	+0.4
Cr.....	24	2.286	2.279	2.290	-0.3	+0.2
V.....	23	2.500	2.495	2.503	-0.2	+0.1
Ti.....	22	2.744	2.744	2.747	0.0	+0.1
Sc.....	21	3.030	3.032	3.029	+0.1	0.0
Ca.....	20	3.357	3.367	3.355	+0.3	-0.1
K.....	19	3.737	3.761	3.737	+0.7	0.0
Cl.....	17	4.712	4.790	4.723	+1.6	+0.2
S.....	16	5.360	5.470	5.369	+2.0	+0.2
P.....	15	6.168	6.306	6.155	+2.2	-0.2
Si.....	14	7.131	7.349	7.127	+3.1	-0.1
Al.....	13	8.360	8.675	8.347	+3.8	-0.2
Mg.....	12	9.915	10.394	9.909	+4.8	-0.1
Na.....	11	11.951	12.681	11.951	+6.1	0.0

TABLE II.

Series L- β_1 .

El.	<i>N.</i>	λ_{∞}	$\lambda_{\infty} M.$	$\lambda_{\infty} U.$	% <i>M.</i>	% <i>U.</i>
U.....	92	0.720	0.762	0.719	+ 5.84	-0.11
Th.....	90	0.766	0.801	0.764	+ 4.62	-0.22
Po.....	84	0.920	0.940	0.921	+ 2.13	+0.11
Bi.....	83	0.950	0.966	0.951	+ 1.69	+0.08
Pb.....	82	0.983	0.994	0.982	+ 1.09	-0.13
Tl.....	81	1.012	1.023	1.014	+ 1.04	+0.19
Hg.....	80	1.049	1.053	1.047	+ 0.35	-0.15
Au.....	79	1.080	1.084	1.082	+ 0.38	+0.21
Pt.....	78	1.120	1.117	1.119	- 0.28	-0.12
Ir.....	77	1.154	1.151	1.157	- 0.23	+0.23
Os.....	76	1.194	1.187	1.196	- 0.56	+0.19
W.....	74	1.278	1.265	1.281	- 1.05	+0.22
Ta.....	73	1.323	1.306	1.326	- 1.28	+0.23
Lu.....	71	1.421	1.395	1.423	- 1.80	+0.12
Yb.....	70	1.474	1.444	1.475	- 2.07	+0.04
Er.....	68	1.586	1.548	1.586	- 2.42	0.00
Ho.....	67	1.646	1.604	1.646	- 2.56	-0.01
Dy.....	66	1.709	1.663	1.709	- 2.67	-0.01
Tb.....	65	1.775	1.726	1.775	- 2.75	0.00
Gd.....	64	1.844	1.793	1.845	- 2.79	+0.04
Eu.....	63	1.918	1.863	1.918	- 2.87	+0.01
Sa.....	62	2.000	1.937	1.996	- 3.13	-0.21
Nd.....	60	2.167	2.101	2.164	- 3.06	-0.14
Pr.....	59	2.259	2.190	2.255	- 3.05	-0.16
Ce.....	58	2.359	2.285	2.352	- 3.12	-0.28
La.....	57	2.461	2.387	2.455	- 3.00	-0.25
Ba.....	56	2.569	2.496	2.564	- 2.86	-0.20
Cs.....	55	2.684	2.612	2.680	- 2.69	-0.16
I.....	53	2.934	2.870	2.934	- 2.19	0.00
Te.....	52	3.074	3.013	3.074	- 1.97	0.00
Sb.....	51	3.222	3.168	3.223	- 1.67	+0.04
Sn.....	50	3.381	3.335	3.383	- 1.36	+0.06
In.....	49	3.550	3.515	3.554	- 0.98	+0.13
Cd.....	48	3.733	3.711	3.738	- 0.59	+0.14
Ag.....	47	3.929	3.923	3.936	- 0.15	+0.17
Pd.....	46	4.142	4.154	4.148	+ 0.29	+0.15
Rh.....	45	4.372	4.406	4.378	+ 0.78	+0.13
Ru.....	44	4.630	4.682	4.625	+ 1.12	-0.10
Mo.....	42	5.175	5.317	5.185	+ 2.74	+0.19
Nb.....	41	5.493	5.684	5.502	+ 3.47	+0.16
Zr.....	40	5.851	6.090	5.847	+ 4.08	-0.07
Y.....	39	6.227	6.542	6.224	+ 5.05	-0.04
Sr.....	38	6.639	7.045	6.638	+ 6.12	-0.02
Rb.....	37	7.091	7.610	7.092	+ 7.31	+0.02
Br.....	35	8.141	8.962	8.146	+10.09	+0.07
As.....	33	9.449	10.710	9.445	+13.35	-0.04

TABLE I.

Series K- α_m .

El.	<i>N.</i>	λ_m	$\lambda_m, M.$	$\lambda_m, U.$	$\%, M.$	$\%, U.$
W.....	74	0.210	0.220	0.210	+5.0	0.0
Nd.....	60	0.333	0.338	0.334	+1.8	+0.3
Pr.....	59	0.345	0.350	0.346	+1.7	+0.4
Ce.....	58	0.358	0.363	0.359	+1.5	+0.4
La.....	57	0.374	0.376	0.373	+0.5	-0.4
Ba.....	56	0.391	0.390	0.387	-0.2	-0.9
Cs.....	55	0.400	0.405	0.402	+1.1	+0.5
I.....	53	0.439	0.437	0.435	-0.6	-0.9
Te.....	52	0.458	0.454	0.453	-0.9	-1.0
Sb.....	51	0.470	0.472	0.472	+0.5	+0.5
Sn.....	50	0.489	0.492	0.493	+0.7	+0.8
In.....	49	0.513	0.513	0.514	+0.1	+0.3
Cd.....	48	0.541	0.535	0.537	-1.0	-0.7
Ag.....	47	0.565	0.559	0.561	-1.0	-0.6
Pd.....	46	0.588	0.584	0.587	-0.6	-0.1
Rh.....	45	0.617	0.612	0.615	-0.9	-0.3
Ru.....	44	0.647	0.641	0.645	-1.0	-0.3
Mo.....	42	0.712	0.705	0.711	-0.9	-0.1
Nb.....	41	0.752	0.741	0.748	-1.3	-0.5
Zr.....	40	0.791	0.780	0.788	-1.3	-0.3
Y.....	39	0.838	0.822	0.831	-1.8	-0.8
Sr.....	38	0.874	0.868	0.877	-0.6	+0.4
Rb.....	37	0.924	0.918	0.927	-0.7	+0.4
Br.....	35	1.038	1.030	1.042	-0.7	+0.4
Se.....	34	1.107	1.094	1.107	-1.1	0.0
As.....	33	1.172	1.165	1.178	-0.6	+0.5
Ge.....	32	1.259	1.242	1.256	-1.3	-0.2
Zn.....	30	1.435	1.422	1.437	-0.9	+0.1
Cu.....	29	1.541	1.527	1.542	-0.9	+0.1
Ni.....	28	1.655	1.645	1.660	-0.6	+0.3
Co.....	27	1.783	1.776	1.791	-0.4	+0.4
Fe.....	26	1.930	1.923	1.937	-0.4	+0.4
Mn.....	25	2.095	2.090	2.103	-0.2	+0.4
Cr.....	24	2.286	2.279	2.290	-0.3	+0.2
V.....	23	2.500	2.495	2.503	-0.2	+0.1
Ti.....	22	2.744	2.744	2.747	0.0	+0.1
Sc.....	21	3.030	3.032	3.029	+0.1	0.0
Ca.....	20	3.357	3.367	3.355	+0.3	-0.1
K.....	19	3.737	3.761	3.737	+0.7	0.0
Cl.....	17	4.712	4.790	4.723	+1.6	+0.2
S.....	16	5.360	5.470	5.369	+2.0	+0.2
P.....	15	6.168	6.306	6.155	+2.2	-0.2
Si.....	14	7.131	7.349	7.127	+3.1	-0.1
Al.....	13	8.360	8.675	8.347	+3.8	-0.2
Mg.....	12	9.915	10.394	9.909	+4.8	-0.1
Na.....	11	11.951	12.681	11.951	+6.1	0.0

TABLE II.

Series L-β₁.

EL.	<i>N.</i>	λ_{α}	$\lambda_{\alpha}, M.$	$\lambda_{\alpha}, U.$	$\%, M.$	$\%, U.$
U.....	92	0.720	0.762	0.719	+ 5.84	-0.11
Th.....	90	0.766	0.801	0.764	+ 4.62	-0.22
Po.....	84	0.920	0.940	0.921	+ 2.13	+0.11
Bi.....	83	0.950	0.966	0.951	+ 1.69	+0.08
Pb.....	82	0.983	0.994	0.982	+ 1.09	-0.13
Tl.....	81	1.012	1.023	1.014	+ 1.04	+0.19
Hg.....	80	1.049	1.053	1.047	+ 0.35	-0.15
Au.....	79	1.080	1.084	1.082	+ 0.38	+0.21
Pt.....	78	1.120	1.117	1.119	- 0.28	-0.12
Ir.....	77	1.154	1.151	1.157	- 0.23	+0.23
Os.....	76	1.194	1.187	1.196	- 0.56	+0.19
W.....	74	1.278	1.265	1.281	- 1.05	+0.22
Ta.....	73	1.323	1.306	1.326	- 1.28	+0.23
Lu.....	71	1.421	1.395	1.423	- 1.80	+0.12
Yb.....	70	1.474	1.444	1.475	- 2.07	+0.04
Er.....	68	1.586	1.548	1.586	- 2.42	0.00
Ho.....	67	1.646	1.604	1.646	- 2.56	-0.01
Dy.....	66	1.709	1.663	1.709	- 2.67	-0.01
Tb.....	65	1.775	1.726	1.775	- 2.75	0.00
Gd.....	64	1.844	1.793	1.845	- 2.79	+0.04
Eu.....	63	1.918	1.863	1.918	- 2.87	+0.01
Sa.....	62	2.000	1.937	1.996	- 3.13	-0.21
Nd.....	60	2.167	2.101	2.164	- 3.06	-0.14
Pr.....	59	2.259	2.190	2.255	- 3.05	-0.16
Ce.....	58	2.359	2.285	2.352	- 3.12	-0.28
La.....	57	2.461	2.387	2.455	- 3.00	-0.25
Ba.....	56	2.569	2.496	2.564	- 2.86	-0.20
Cs.....	55	2.684	2.612	2.680	- 2.69	-0.16
I.....	53	2.934	2.870	2.934	- 2.19	0.00
Te.....	52	3.074	3.013	3.074	- 1.97	0.00
Sb.....	51	3.222	3.168	3.223	- 1.67	+0.04
Sn.....	50	3.381	3.335	3.383	- 1.36	+0.06
In.....	49	3.550	3.515	3.554	- 0.98	+0.13
Cd.....	48	3.733	3.711	3.738	- 0.59	+0.14
Ag.....	47	3.929	3.923	3.936	- 0.15	+0.17
Pd.....	46	4.142	4.154	4.148	+ 0.29	+0.15
Rh.....	45	4.372	4.406	4.378	+ 0.78	+0.13
Ru.....	44	4.630	4.682	4.625	+ 1.12	-0.10
Mo.....	42	5.175	5.317	5.185	+ 2.74	+0.19
Nb.....	41	5.493	5.684	5.502	+ 3.47	+0.16
Zr.....	40	5.851	6.090	5.847	+ 4.08	-0.07
Y.....	39	6.227	6.542	6.224	+ 5.05	-0.04
Sr.....	38	6.639	7.045	6.638	+ 6.12	-0.02
Rb.....	37	7.091	7.610	7.092	+ 7.31	+0.02
Br.....	35	8.141	8.962	8.146	+10.09	+0.07
As.....	33	9.449	10.710	9.445	+13.35	-0.04

chosen data very satisfactorily and that it will be found very reliable for interpolation, and for extrapolation over a short range of atomic numbers. To facilitate such calculations the numerical values of A , B , C , and D are given in Table III. The column headed "Range" contains the extreme values of N used in calculating these four coefficients.

In using this table two facts should be noted: (i) Siegbahn used the grating space $d = 2.814 \times 10^{-8}$ cm. for rock salt, and $d = 7.621 \times 10^{-8}$ cm. for gypsum; and (ii) the values of λ_N resulting from the use of equation (1) and the tabulated numerical coefficients will be expressed in angstroms.

TABLE III.

Series.	A .	B .	C .	D .	Range.
$K-\alpha$	-0.18724	+0.026898	+ 21.74629	+131.388*	11-74
$K-\beta_1$	-0.44795	+0.027774	+ 69.60474	+170.987	12-74
$L-\alpha_1$	-0.55066	+0.013658	-108.60609	-225.000*	30-92
$L-\beta_1$	-0.43675	+0.009386	+ 66.94285	+180.973	33-92
$L-\gamma_1$	-0.26791	+0.012837	+ 19.38426	+145.197	40-92

* Obtained by graphical treatment of *residuals*.

Before passing to the question of extrapolating to $N = 1$ it may not be superfluous to call attention to the following points. In the first place, with the exception of the top number, the values of D in Table III are approximately integers. Undoubtedly these constants may be replaced by the nearest integers without sensibly decreasing the present accuracy of fit of equation (1), it being at most necessary to apply small corrections to the associated values of A , B , and C . Since slight changes in the experimental data exert an appreciable influence on D , and as the value of this constant is very close to an integer for the series $L-\beta_1$ (-0.015 per cent. error), the wave-lengths of which seem to be affected by the smallest accidental errors, I am still of the opinion that, if the correct formula for X -ray line series, when found, contains a binomial of the type $(D - N)^{-n}$, D will be exactly a whole number; signifying, perhaps, a physical change in origin for N . If this be true, it might be desirable to express ν_N as a function of $D - N$ instead of N alone. In the second place, realizing that equation (1) possesses the undesirable feature of involving four parameters I tried the formulæ $\nu_N = a + b(c + N)^{-1}$ and $\nu_N = a' + b'(c' + N)^{-2}$ which were suggested by the expression obtained by squaring equation (1). It was found that both of the functions just given are altogether unsatisfactory.

An attempt will now be made to arrive at a necessarily tentative answer to the third question proposed in the first paragraph of this paper.

The problem is to determine the order of magnitude of the X-ray lines of hydrogen by making use only of wave-lengths of the same series and purposely avoiding the introduction of any auxiliary theoretical assumptions. By plotting, on a large scale, atomic numbers and wave-lengths as abscissæ and ordinates respectively it becomes apparent at once that the loci of the five longest series approach the axis of ordinates very gradually as the atomic numbers decrease. Accordingly the wave-lengths deduced for hydrogen may be expected to depend to an undesirable degree upon the particular form of function employed for extrapolation. Especial care must therefore be taken to choose an algebraic expression which will implicitly influence the results to as slight an extent as possible. In spite of the fact that the $K\alpha_m$ series only has to be extrapolated from $N = 11$ to $N = 1$, neither the linear law nor equation (1) will suffice. Moseley³ assigns to b [$\nu_N = A(N - b)^2$] the values 1 and 7.4 for the " $K\alpha$ line" and the " $K\beta$ line," respectively. Hence, his formula leads to an infinite wave-length for the $K\alpha_m$ line of hydrogen. For the $K\beta_1$ series the same equation causes λ_N to increase as N decreases from 11 to 8, to become infinite for $N = 7.4$, and to decrease when N continues from 7 to 1. Siegbahn⁶ writes the law in question as $\sqrt{\nu} = a(N - N_0)$, and finds that the values of N_0 are 6.97, 6.09, and 9.92 for the interval Ta to U of the α_1 , α_2 , and β_2 sub-series of the L group. Equation (1) exhibits the same peculiarity, the right member vanishing when N equals 1.35, 5.88, 11.54, and 9.72 for the series $K\beta_1$, $L\alpha_1$, $L\beta_1$, and $L\gamma_1$, respectively. For the $K\alpha_m$ series and $N = 1$ it leads to the relatively enormous wave-length 2.42μ . A variety of explanations of these apparent anomalies suggest themselves at once. (a) The elements of small atomic number may not radiate the K and L series. This idea is consistent with the fact that the M series has only been found for the elements of high atomic number (79 to 92) from gold to uranium. (b) The X-ray wave-lengths may pass through arithmetical maxima and then decrease. This would be somewhat analogous to the correlation of the lines of certain principal series in the visible and infra-red regions of the spectrum which correspond to negative values of the frequency in the formula of Ritz. (c) The concordance of formula (1) with the experimental data may be illusory. Although the third suggestion may be the correct one it seems fair to call attention to the fact that the errors involved in the given wave-lengths (especially for the elements of high atomic number) exert an appreciable influence on the values of the parameters A , B , C , and D . Thus, for the $K\alpha_m$ series, substitution of $N = 1$ in equation (1) leads to $\sqrt{\nu_1} = -0.1872 + 0.0269 + 0.1668 = 0.0065$ or $\nu_1 = 0.000042$.

Slight changes in the coefficients would therefore produce a very great alteration in the wave-length obtained from $\sqrt{\nu_1} = A + B + C(D - 1)^{-1}$.

The least objectionable way of extrapolating is to apply the method of least squares to the computation of the coefficients of a sufficient number of terms of the power polynomial $\nu_N = a_0 + a_1N + a_2N^2 + \dots + a_kN^k$, care being taken to use about fifteen of the largest wave-lengths. An inspection of the left ends of the five curves of Fig. 1, or reference to the original papers, will show that the data in these regions are fortunately the most accurate. Using only the first three terms of the polynomial and then putting $N = 1$ leads to 366 Å., 95 Å., and 22 Å. respectively, for the series $K\text{-}\alpha_m$, $L\text{-}\alpha_1$, and $L\text{-}\beta_1$. Four terms give 130 Å. and 49 Å. for $K\text{-}\alpha_m$ and $L\text{-}\beta_1$. The corresponding values of a_3 were -0.000000698 and $+0.000000320$, and hence further calculations were omitted as superfluous. Accordingly, the evidence obtained solely from the experimental wave-lengths leads to the tentative conclusion that the X -ray lines for all the elements will have wave-lengths inferior to 510 Å., which is the limit last attained by Lyman. Incidentally it may be noted that, for the $L\text{-}\beta_1$ series, extrapolation to $N = 92$ (uranium) by aid of the cubic (a_0, a_1, a_2, a_3) gives an error of $+9.6$ per cent. in the calculated wave-length. Therefore, the close fit of formula (1) cannot be due entirely to the fact that it involves four independent parameters.

In conclusion four sentences in Moseley's second paper³ will be quoted for reasons which will appear later. "On this principle the integer N for Al , the thirteenth element, has been taken to be 13, and the values of N then assumed by the other elements are given on the left-hand side of Fig. 3. This proceeding is justified by the fact that it introduces perfect regularity into the X -ray spectra." "Now if either the elements were not characterized by these integers, or any mistake had been made in the order chosen or in the number of places left for unknown elements, these regularities would at once disappear. We can therefore conclude from the evidence of the X -ray spectra alone, without using any theory of atomic structure, that these integers are really characteristic of the elements." The point to be brought out is that the last sentence must not be taken so literally as to mean that the X -ray data alone necessitate associating the integers 1, 2, $\dots$, 13, $\dots$, 74, $\dots$, 92 with H, He, $\dots$, Al, $\dots$, W, $\dots$, U, respectively; for, the "regularities" will not disappear if the coördinate plane be expanded or contracted parallel to the axis of N . More specifically, as far as the X -ray data are concerned, the atomic numbers need not be integral and they do not have to form an arithmetical progression explicitly.¹ In fact, in making the computa-

¹ The discrete values of $\log N$, say, may be taken as a new sequence of atomic numbers, thus involving the accepted series implicitly.

tions I shifted the origin to the "center of gravity" of the N sequence, divided by 10, etc. These comments would be highly pedantic and irrelevant if removed from the following remarks. The facts that equation (1) leads to infinite wave-lengths for values of N appreciably greater than unity, that the M series has only been excited in the heaviest elements, and that—as Siegbahn points out—the intensities of the lines of any one sub-series decrease, on the whole, and finally vanish (experimentally) as the atomic number becomes smaller, suggest that the mechanisms which give rise to these radiations may cease to function or even to exist (loss of outer rings of electrons) for definite values of N . In short, if it should ever become desirable, for theoretical or other reasons, to use different sets of atomic numbers for the K , L , and M series the transformation of coördinates would not disturb the fundamentally important regularities discovered by Moseley.

SUMMARY.

1. It is shown that Moseley's law, $\nu = A(N - b)^2$, does not satisfactorily represent the X -ray series over the entire ranges of wave-lengths now known.

2. The formula $\nu^{1/2} = A + BN + C(D - N)^{-1}$ —where A , B , C , and D are constants for any one sub-series—is proposed and shown to reproduce the observed data within the given limits of experimental error.

3. The uncertainty pertaining to extrapolation of the series as far as hydrogen is discussed. The tentative conclusion is reached that the X -ray lines of hydrogen, if they exist, probably have wave-lengths less than about 400 angstroms.

4. Attention is called to the fact that the series of natural numbers commencing with 1 is not the only arithmetical progression which may be used as the sequence of atomic numbers consistent with Moseley's "regularities."

SLOANE PHYSICAL LABORATORY,
YALE UNIVERSITY,
February 1, 1917.

PROCEEDINGS
OF THE
AMERICAN PHYSICAL SOCIETY.

ON SOME VERY LARGE VARIATIONS IN THE ADSORPTION OF CERTAIN
SPECIMENS OF CHARCOAL.¹

BY HARVEY B. LEMON.

A PRELIMINARY report given at the November meeting a year ago indicated that among samples of charcoal great differences in the adsorptive power existed which could be associated with the temperature of carbonization of the cocoanut shell, 825° to 875° C. being given as the range within which samples passed from the condition of being good adsorbers to that of being very poor ones.

Subsequent work has shown further that a single specimen can be made to show all of the variations previously observed among different specimens. All samples consistently improve with use, *i. e.*, with repeated "outgassing-heating" and adsorption, provided the former is not carried on at too high a temperature. A sample which is quite poor, requiring over two hours to do a given task, (25 gm. adsorbing 1 liter air) with frequent testing becomes very good, *i. e.*, will do the same in less than one minute. During this process a certain characteristic cycle of curves [$\log \text{ pressure} = F(\log \text{ time})$] is traversed, representing the behavior of successive tests.

Furthermore a good specimen may be ruined, even although repeatedly used, provided the outgassing-heating temperature is increased on successive runs. At any stage in the process however the specimen may be restored in whole or in part. With certain limitations, such a reversal of operations at any point does not interfere with subsequent progress in the initial direction. In all these experiments the same cycle of curves mentioned in the preceding paragraph is found, the family here being developed in the opposite order. Considerable further investigation will be needed before these phenomena can be interpreted with certainty.

Phenomena of quite a different type have been also observed, but merely in a qualitative manner. When charcoal is evacuated the gas is not given off at a uniformly increasing rate as the temperature is raised but comes off in great quantities at certain critical temperatures. Five minutes pumping at certain

¹ Abstract of a paper presented at the Chicago meeting of the Physical Society, December 2, 1916.

temperatures may result in getting out more gas than would be obtained in five hours, thirty degrees lower. Spectroscopic observations show N_2 comes out freely at temperatures as low as 150° , the rush of O_2 begins at 300° and is greatly augmented again at 600° . Notable is the extremely high temperature 750° to 800° before H_2 makes a conspicuous appearance in the spectrum. At 900° H_2 dominates. For quantitative results experiments on pure gases are in progress.

RYERSON PHYSICAL LABORATORY,
UNIVERSITY OF CHICAGO,
November 10, 1916.

PRELIMINARY NOTES ON THE TORSIONAL ELASTICITY OF DRAWN TUNGSTEN WIRES.¹

BY L. P. SIEG.

PRELIMINARY experiments have been made on drawn tungsten wires, ranging in diameter from 0.099 cm. to 0.510 cm. for the determination of the coefficient of simple rigidity. The following is a summary of results:

1. For amplitudes well within the elastic limit the period varies with the amplitude, the variation being as great as 7.6 per cent.
2. The logarithmic decrement varies with the amplitude.
3. Statistically determined the torsional stress is proportional to the torsional strain within the limits in 1 above.
4. The coefficient of simple rigidity varies from 9.88 to 18.95 times 10^{11} dynes per cm^2 . This is the largest rigidity that can be found in the literature, being in its largest values nearly two and one half times as great as that of steel.
5. The coefficient has been determined statically, and found to range from 9.08 to 17.28×10^{11} dynes per cm^2 . These latter values are found uniformly smaller than the corresponding values kinetically determined.
6. The range in values for the rigidity is nearly 100 per cent: This does not equal the range in values in cadmium, and in lead² which is about 140 per cent.
7. The increase in the coefficient does not change in any regular manner with the change in size of the wire. Therefore new wires are to be used that are all definitely drawn from the same original lug. The present wires were chosen at random, and it is not at all certain that they were drawn from the same original piece.
8. With any given wire the results for the elasticity are remarkably constant, being only slightly affected by fatigue, and by a variation in the period of vibration.

Acknowledgment is made to the Nela Laboratory and to Dr. Worthing for their kindness in supplying the wires.

STATE UNIVERSITY OF IOWA,
November 15, 1916.

¹ Abstract of a paper presented at the Chicago meeting of the Physical Society, December 2, 1916.

² See Winkelmann's *Handbuch der Physik*, Vol. 1, p. 652.

THE INTENSITY FACTOR IN BINAURAL LOCALIZATION AND AN EXTENSION OF WEBER'S LAW.¹

BY G. W. STEWART AND O. HOVDA.

THE object of this experimental study was to isolate and to study the intensity factor when a pure tone is used. The actual ratio of intensities at the ears, using a frequency of 256 d.v., required to give a certain angular displacement in localization is ascertained.

It is found that if the intensity ratio is one to ten, then the displacement is 45 degrees from the median plane and on the side of the greater intensity. It is shown that, with a single source of sound about the head, this ratio of intensities with this frequency can but seldom occur in practice.

With such a single source at a distance of five meters, the intensity ratio at the ears does not exceed 1.2. The conclusion is that the intensity factor, when a pure tone is used, is very small.

The data obtained show clearly that the response (in the form of an angular displacement in localization) is proportional to the logarithm of the actual intensity ratio. This is a very interesting extension of Weber's Law.

STATE UNIVERSITY OF IOWA,
November, 1916.

AN EXPERIMENTAL AND THEORETICAL INVESTIGATION OF BINAURAL BEATS.¹

BY G. W. STEWART.

BINAURAL beats, occurring when two sources varying slightly in frequency are presented one to each ear, can be considered under three heads, primary maxima and minima, secondary maxima and localization. The primary maxima and minima (intensities) occur at phase differences of 0° and 180° respectively, but the minima are not zero. The secondary maxima occur at phase differences of $180^\circ - \delta$ and $180^\circ + \delta$. (See abstract, *PHYS. REV.*, Series 2, 3, p. 146, 1914.) The localization is external and in front for 0° phase difference, it wanders about the head on the side of the greater frequency for phase differences from 0° to $180^\circ - \delta$ at which phase it is in the ear, it is then internal, crossing the head, until the phase difference is $180^\circ + \delta$, when it is in the other ear. From $180^\circ + \delta$ to 360° the localization is on the side of the less frequency, wandering from the right ear toward the front. The author seeks an explanation upon physical grounds.

It is shown that skull conduction will explain the primary maxima and minima with more than one assumption as to what causes the response. The two assumptions tried are that the organ responds to intensity and that it responds to the absolute value of the displacement.

The secondary maxima is explained by the assumption of a second pair of

¹ Abstract of a paper presented at the Chicago meeting of the Physical Society, December 2, 1916.

organs which are independent of the first pair, and the assumption that there is a change of phase of 180° introduced in skull conduction.

The assumption of the two pairs of organs gives an explanation of all the intensity and localization phenomena observed. There appear to be no objections to the assumption from either an anatomical or physiological viewpoint.

STATE UNIVERSITY OF IOWA,
November, 1916.

SOME EFFECTS OF CROSS-MAGNETIZING FIELDS UPON HYSTERESIS.¹

BY N. H. WILLIAMS.

THE effect of an alternating field applied at right angles to the direct field while the hysteresis curve was being traced has been investigated for steel specimens of varying degrees of hardness. The direct field reached a magnitude of 150 gauss while an alternating field of maximum intensity, 80 gauss, was applied in some cases. With soft specimens, all traces of hysteresis disappeared, while with very hard steel it was reduced till the width of the curve was less than a tenth of the width of the original curve.

For soft specimens, the magnetization curve with the alternating field applied lies almost wholly outside and beneath the normal hysteresis curve. For hard steel it lies within the normal curve, except near the top, where it passes outside and above it.

It has been possible to compute the induction that should correspond to any field intensity, and points thus determined are found to lie very near to the experimental curve. Their slight deviation from the experimental curve point to two conclusions: (1) The observed effects are not due to mechanical shock, as has been supposed by some. (2) With hard specimens there is a distinct rotational hysteresis or tendency for a flux to maintain its magnitude as it is forced to change its direction.

UNIVERSITY OF MICHIGAN,
November, 1916.

AN ACOUSTICAL THERMOMETER.¹

BY F. R. WATSON AND H. T. BOOTH.

TEMPERATURE may be measured by the variation of a physical property of a substance with the temperature. As common examples, we have the mercury thermometer, the gas thermometer, the resistance thermometer, thermocouples and radiation pyrometers, which involve the properties of expansion, resistance, thermo-electricity, and radiation. In addition to these, the velocity of sound is shown theoretically to vary with the temperature. Thus

$$V_t = V_0 \sqrt{1 + \alpha t} = V_0 \sqrt{\frac{\theta}{273}},$$

¹ Abstract of a paper presented at the Chicago meeting of the Physical Society, December 2, 1916.

where V_t and V_0 are the velocities of sound at t° C. and 0° C. respectively and θ is the absolute temperature. Combining this with $V = N\lambda$, where N is the pitch and λ the wave-length, we get

$$N^2 = \frac{V_0^2}{\lambda^2 273} \cdot \theta = \text{constant} \times \theta, \text{ if } \lambda \text{ is constant.}$$

Therefore $N^2 \propto \theta$.

This theory was tested experimentally in the present investigation, the pitch being found for a cylindrical resonator of constant length when it was heated to different temperatures. The cylindrical resonator was arranged as a Rayleigh resonator¹ with a flat, circular disc suspended at a loop $1/3$ of the length of the tube from the closed end. This meant that the tube responded to its first overtone and its length, l , was equal to $3/4 \lambda$. When the suspended disc was adjusted to make an angle of 45° to the axis of the tube, it was rotated by sounds whose pitches were the same or nearly so as that of the resonator.

The experimental procedure was as follows: The resonator was maintained at a constant temperature, while a Helmholtz siren generated a tone whose pitch was varied until the resonator showed a maximum deflection. The pitch of this tone was assumed to be the pitch also of the resonator. The temperature was measured by a thermo-couple inserted in the resonator, and the pitch was found from the counting device on the siren. Observations were then repeated at higher temperatures. The results gave a linear relation between N^2 and θ , which departed somewhat from the theoretical linear relation. The measurements were repeated with other instruments and arrangements of apparatus. Up to 400° C., a satisfactory linear relation was found, but above this temperature, convection currents and other disturbances made the readings uncertain. Further work is contemplated using a double resonator so that the suspended system may be removed from the heat.

LABORATORY OF PHYSICS,
UNIVERSITY OF ILLINOIS.

A GENERAL METHOD OF PRODUCING THE STROBOSCOPIC EFFECT, AND ITS APPLICATION IN THE TONODEIK.²

By L. E. DODD.

THE stroboscopic effect requires two experimental arrangements: (1) a stroboscopic screen, (2) periodic illumination, or its equivalent, of this screen. While the effect has been produced heretofore by a variety of special devices, the periodic illumination, or its equivalent, of the screen has been by periodic intensity changes in a beam of light of uniform intensity over the cross-section, where the intensity of the beam periodically changes as a whole, as it does in the special case where the beam varies in intensity at the source.

¹ Rayleigh, *Theory of Sound*, Vol. 2, pp. 44-45.

² Abstract of a paper presented at the Chicago meeting of the Physical Society, December 2, 1916.

This method of periodic illumination of the screen is one of two possible general methods. The other is to impart to a beam of light of non-uniform intensity over the cross-section a back and forth periodic motion perpendicular to the direction of the beam. When such an oscillating beam falls upon a suitable stroboscopic screen the stroboscopic effect is very distinctly observed. The oscillating beam is readily provided by reflection of a stationary beam from a vibrating (phonic) mirror. Any given half-period pause of the beam results in a definite intensity at a given point on the screen, and the next following half-period pause results in a slightly different intensity at the same point, while the next following pause gives the previous intensity. Thus at the point in question there is periodically changing intensity similar to that due to flares of a manometric flame (first general method) of the same frequency as the phonic mirror causing oscillation of the non-uniform beam, and the stroboscopic effect follows. This suggested explanation accounts for observed facts.

The method of a non-uniform oscillating beam is applied in an instrument, the tonodeik, for indicating the absolute pitch of sounded tones. (See *Scientific American*, Nov. 4, 1916.)

STATE UNIVERSITY OF IOWA,
November, 1916.

A PROPOSED METHOD FOR THE PHOTOMETRY OF LIGHTS OF DIFFERENT COLORS.—II.¹

BY IRWIN G. PRIEST.

AT the Washington meeting of the American Physical Society, April, 1915,² the author communicated a paper under the above title. In that communication, it was proposed that the color difference be eliminated by means of the rotatory dispersion of quartz, and the theory of computing the relative candlepowers of two lamps by the proposed method was outlined. It was stated that the method had been tried only qualitatively, the author having had no opportunity to test it quantitatively. During the past few weeks he has done some preliminary quantitative work with this method and desires to submit the essential results of these experiments in the present communication. However, it is first necessary to correct an error in the previous abstract which was not noticed until returning anew to this subject a few weeks ago. If the equation in that paper is to hold, the symbol V^λ (defined there as "relative visibility of light of wave-length λ ") must be understood to stand, not for "visibility"³ but for "luminosity" of the comparison source, *i. e.*, relative "visibility" times relative radiant power in the com-

¹ Abstract of a paper presented (by title) at the New York meeting of the Physical Society, December, 1916.

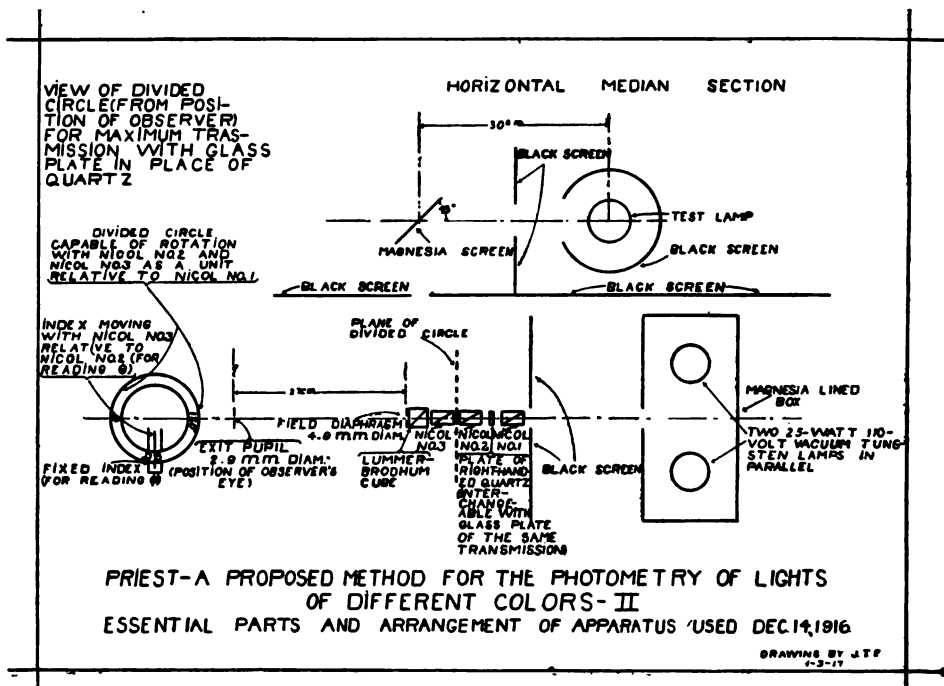
² *PHYS. REV.* (2) 6, 64.

³ Nutting, *Trans. Ill. Eng. Soc.*, 9, 633 (1914).

parison source.¹ Likewise on p. 66, line 4, instead of "'visibility' curve," read "luminosity curve of the comparison source."

The following is a brief statement of the essential features and results of the experiments just made.²

1. The apparatus and method used were essentially as described in the previous communication to the Physical Society. However, some modifications in apparatus and symbols have been introduced, so that it will be clearer to present the subject anew and without reference to the particular symbols and conventions of the previous abstract.



The essential features of the apparatus are shown in the accompanying figure. The following particulars may be noted:

(a) The photometric field was circular, divided along a horizontal diameter. It subtended about $1^{\circ} 2'$ at the eye and was surrounded by a dark field.

(b) The maximum and minimum brightnesses of the screen illuminated by the test lamp were (computed roughly from the distance of the lamp, angle of incidence and reflecting power) 156 meter candles and 30 meter candles.

¹ In future discussions of this subject, however, the author intends to use the symbol V to represent relative "visibility" in the sense used by Nutting in his paper on Visibility of Radiation (Trans. Ill. Eng. Soc., 9, 633). Luminosity for wave-length λ will then be represented by $L_{\lambda} = V_{\lambda} E_{\lambda}$ where E_{λ} represents relative radiant power for wave-length λ .

² It is expected that a fuller statement of more extended work will be published later in the Scientific Papers of the Bureau of Standards.

This screen was viewed through a 2.9 mm. circular pupil by reflection in the Lummer-Brodhun cube (cemented silver strip type).

(c) The quartz plate was 1.000 mm. thick (this being known to within about one micron) and its surfaces were accurately perpendicular to the optic axis.

(d) The glass plate, interchangeable with the quartz had (by test) the same transmission as the quartz.

(e) The spectral energy distribution in the comparison source was established by operating it at a voltage (mean of a number of observations by different observers) giving a color match with an acetylene flame of known energy distribution,¹ in place of the test lamp (the quartz plate being replaced by glass).

2. Symbols used have meanings as follows. (Refer also to accompanying figure.)

λ , wave-length of radiation.

V , relative visibility of radiation.

E , relative radiant power, comparison source.

α , rotation of plane of polarization by 1 mm. plate of quartz perpendicular to optic axis.

ϕ , angle read at fixed index } (see figure).

Θ , " " " movable " }

ϕ_s and Θ_s , values of ϕ and Θ observed for standard matched in color and brightness with comparison source.

ϕ_x and Θ_x , similar values for unknown candlepower.

I_s , standard candlepower.

I_x , unknown candlepower.

3. With the apparatus as shown, and, in terms of the above symbols, the general form of the equation for relative candlepower is:

$$\frac{I_x}{I_s} = \frac{\left[\int_{400}^{740} VE \sin^2 (\phi_s - \alpha) d\lambda \right] \sin^2 \Theta_s}{\left[\int_{400}^{740} VE \sin^2 (\phi_s - \alpha) d\lambda \right] \sin^2 \Theta_s}$$

If ϕ_s and Θ_s are observed with glass in the path instead of quartz, the above becomes:

$$\frac{I_x}{I_s} = \left[\frac{\int_{400}^{740} VE \sin^2 (\phi_s - \alpha) d\lambda}{\int_{400}^{740} VE d\lambda} \right] \frac{\sin^2 \Theta_x}{\sin^2 \phi_s \cdot \sin^2 \Theta_s},$$

which represents the condition and form used in the present experiments.

For convenience we will represent the factor

¹ Coblenz and Emerson, Sci. Paper, Bureau of Standards, No. 279.

$$\frac{\int_{400}^{740} VE \sin^2 (\phi_z - \alpha) d\lambda}{\int_{400}^{740} VE d\lambda}$$

by R .

Assuming the spectral energy distribution data of Coblentz and Emerson for acetylene and their data for relative visibility¹ we have computed R for all values of ϕ . We have also computed R for $\phi = 65^\circ.32$ (the value used in the present experiments) from Nutting's visibility data and the energy distribution used by him in computing visibility from luminosity. (See accompanying table.) Both of these values of R have been repeatedly determined on plots of different sizes and with different planimeters. So far as errors of computation are concerned, the mean values given here are considered accurate to about 0.2 per cent.

4. Under the above conditions, the method has been tested quantitatively to a first approximation in the following way:

One of the Bureau of Standards vacuum tungsten working standards has been used as a test lamp. (B.S. Lamp No. 1292 giving 28.46 m.h.c.p. and having an efficiency of 1.20 w.p.m.h.c. at 100.75 volts.)

The ratio of the candlepower of this lamp at 64.91 volts (3.14 w.p.m.h.c. giving a color match with a carbon lamp at 4.0 w.p.m.h.c.) to its candlepower at 99.86 volts (1.22 w.p.m.h.c.) was determined by four observers by the proposed method and also by equality of brightness in the same photometer, with the color difference present. The results obtained have been compared with the value computed from the tables of Middlekauff and Skogland.² These results are presented in the accompanying table. Our ratios involve the application of somewhat uncertain small apparatus corrections (for polarization) of the order of a few tenths of one per cent., and in the author's opinion they *may* be in error by as much as one or two per cent. of their value. He sees no reason, however, to suspect that the errors can be greater than this. Care has been taken to test for polarization errors in the Lummer-Brodhun cube. The voltages were determined by a calibrated Brooks deflection potentiometer used with a newly calibrated volt box and the small corrections

TABLE OF VALUES OF THE RATIO $\frac{\text{Candlepower at 64.91 Volts}}{\text{Candlepower at 99.86 Volts}}$

applied. The voltages of the comparison and test lamps were carefully kept constant to about 0.01 volt, and the voltage on the test lamp was measured to this accuracy.

5. Qualitative test has shown that this method may be used to eliminate the color difference between the photometric standards (carbon or vacuum

¹ Mean of 125 subjects, 1916, Paper in course of publication, Scientific Paper, Bureau of Standards. Also *PHY. REV.* 9, 88 (Jan. 1917).

² *Sci. Pap., Bur. of Stand., No. 238, Oct. 10, 1914.*

tungsten) and gas filled tungsten lamps, by using a 1 mm. quartz plate just as in the apparatus described above.

Vac. Tung. Lamp B.S. 1292—Dec. 14, 1916.

Observer.	Color Difference Eliminated by Proposed Method $\phi = 65.32$ Degrees.			Color Difference Present.		
	Assuming Visibility and Energy Data of Coblentz and Emerson (Sci. Paper, Bureau of Standards, No. 279 and a Forthcoming Paper, Phys. Rev., p. 88, Jan. 1917). $R = 0.4532$.		Assuming Visibility Data of Nutting ("Mean Visibility," Trans. Ill. Eng. Soc. 9, 638, and Energy Data of Coblentz, Sci. Paper, Bureau of Standards, No. 156, p. 261). $R = 0.4505$.	By Equality of Brightness in Same Photometer as Used in Proposed Method, But Not Using Quartz Plate.		Computed From the Tables of Middlekauff and Skogland (Sci. Paper, Bureau of Standards, No. 238, Based on Mean Results of Other Observers Using Contrast Method).
	Ratio Mean of 10 Settings.	Dev. From Mean of All Observers.		Ratio Mean of 10 Settings.	Dev. From Mean of All Observers.	
I.G.P.....	0.1972	0.0000	Obtained from first column by reducing in proportion to change in R .	0.1994	0.0020	
P.V.W.....	.2006	.0035		.2003	.0029	
C.G.P.....	.1944	.0028		.1925	.0049	
H.J.M.....	.1966	.0006		.1975	.0001	
Mean....	0.1972	0.0017 or 0.86%	0.1960	0.1974	0.0025 or 1.27%	0.1966

Conclusion.—The essential purport of this communication may be summarized as follows:

1. The quantitative experiments here reported demonstrate the practicability of the proposed method and show that (as outlined in this paper) it is not affected by any gross errors. It is thought worthy of consideration as a practical routine method of photometry.

2. The establishment of its consistency with other methods to an accuracy better than 2 per cent. will require much more experimental work. The precision and reliability of the method can no doubt be made better than that attained with the present quickly improvised apparatus.

3. The method factor R has now been accurately computed by graphic integration for 1.000 mm. of quartz for all values of ϕ using the visibility and energy distribution data of Coblentz and Emerson. Persons interested may obtain these factors by applying to the Bureau of Standards.

4. In connection with the integrating sphere, this method may afford a valuable aid in the photometry of the high-efficiency gas-filled lamps.

NATIONAL BUREAU OF STANDARDS,

December 29, 1916.

Revised February 1, 1917.

THE EFFECT OF OXYGEN ON THE PRODUCTION OF BAND AND LINE SPECTRA
IN THE ELECTRIC FURNACE.¹

BY ARTHUR S. KING.

THE several flames employed in the production of luminous vapors for spectroscopic studies involve the ignition of a gas such as hydrogen or acetylene in the presence of oxygen. The use of a plentiful supply of oxygen in a tube furnace promised to give a source in which the process is different from that in the flame, since in the furnace the heat is supplied by the incandescent tube instead of by the chemical action of the flame, while the oxygen is free to combine with the metallic vapor. Experiments were therefore carried out in which a strong stream of oxygen was passed directly through the furnace tube in which titanium or iron was being vaporized. The spectra thus obtained were compared with those given at approximately the same temperature by the furnace in vacuum. Pyrometer measurements did not show an increase of temperature resulting from the use of oxygen, but rather a slight decrease, as if the stream of fresh gas exerted some cooling effect. The bands due to the oxide were given strongly for each metal, this method being an excellent one for studying the spectra of the compounds of metals with oxygen or other gases.

With titanium, the relative intensities of the line and band spectra depended solely on the supply of oxygen. A strong stream of the gas seemed to turn all of the titanium vapor into the oxide, so that the band spectrum alone was visible, even the strongest lines being suppressed. This condition held for as high temperatures as were used, up to about 2600° C., though a temperature of about 2300° was more favorable for the band spectrum. The experiments showed that a rapid current of the gas is necessary to give oxidation of the metallic vapor, since a moderate supply, as when the furnace is operated in oxygen atmosphere, is used up by the graphite tube.

The use of iron vapor with oxygen, on the other hand, gave the line spectrum together with the oxide bands, even when a strong stream of the gas was passed through the tube. This is probably due to a less ready combination of iron vapor with oxygen than is the case with titanium. The iron spectrum was examined from λ 3200 to λ 6500 with regard to changes in the relative intensities of lines caused by the oxygen. No pronounced differences of this sort were observed, the line spectrum being practically the same except as to width of lines whether the furnace is operated in vacuum, in still air, or with a current of oxygen through the tube. A current of oxygen proved a very effective means of suppressing the "cyanogen" bands, which are given strongly by the furnace when only a small residue of air is present.

MOUNT WILSON SOLAR OBSERVATORY,
February 1, 1917.

¹ Abstract of a paper presented at the New York meeting of the American Physical Society and the A. A. A. S., December 26-30, 1916.

EXPERIMENTS WITH THE ELECTRIC FURNACE ON THE ANOMALOUS DISPERSION
OF METALLIC VAPORS.¹

BY ARTHUR S. KING.

THE object of the work was to make a fairly comprehensive study of the anomalous dispersion of metallic vapors, especially with regard to the spectroscopic peculiarities of lines which show the effect in different degrees.

The method of "crossed prisms" was employed, and the chief problem was the production, in a mass of metallic vapor, of a density gradient such that white light when passed through this vapor prism would undergo a change in the index of refraction near those wave-lengths which the vapor strongly absorbs. The electric furnace with a special form of tube was adapted to this purpose, the upper portion of the horizontal graphite tube being filed away from end to end of the length heated. The substance to be vaporized was placed in the trough thus formed, and a considerable density gradient was obtained in the vapor as it rose from the molten mass to the open space above. A beam of white light, narrowed by passage through a slit, traversed this vapor prism and was focused on the slit of a plane-grating spectrograph. The absorption spectrum thus obtained registered the degree of anomalous dispersion produced by each spectrum line through the curvature of the strip of continuous spectrum adjacent to the line in question.

The spectra of iron, titanium and chromium were studied through a long range of wave-length. In addition, certain lines of calcium, strontium, barium, manganese, sodium, potassium and lithium were examined. It was found possible to obtain anomalous dispersion for any line which was emitted strongly in the furnace at the temperature employed, but the conditions for the production of an effective vapor prism differed according to the character of the line concerned. Thus a low temperature of the vapor was required to give the proper distribution in the case of substances of low melting point. At higher temperatures the vapor distribution in the tube becomes more uniform and the critical condition for anomalous dispersion is lost, the spectrum showing merely wide absorption lines; although it often happens that when this stage is reached, the vapor of another substance in the tube, requiring a higher temperature to show its spectrum, gives a strong effect, so that a simultaneous condition of strong anomalous dispersion for one substance and absence of it for another may be observed. This contrast may be carried further by the production of inverted anomalous dispersion when the temperature is raised so high that lines show the opposite direction of change of refractive index to that which they showed at lower temperature. If another element whose lines require the higher temperature for their appearance is present in the tube at the same time, the striking phenomenon is presented of opposite anomalous dispersion effects for the lines of the two elements.

¹ Abstract of a paper presented at the New York meeting of the American Physical Society and the A. A. A. S., December 26-30, 1916.

The calcium line λ 4227 was thus inverted with respect to neighboring lines of chromium, the D lines of sodium with respect to barium lines, iron and chromium lines with respect to those of titanium, and manganese lines with respect to iron lines. This inversion of the effect appears to result from the particles which emit the low-temperature lines being denser in the cooler region above than near the surface of the hot tube, while the prism formed by a vapor requiring higher temperature has its denser portion below.

Such an inversion may be obtained for lines of the same element when these lines require very different temperatures for their production. This was demonstrated by the production of opposite anomalous dispersion at the same time for lines in the calcium spectrum, the effect for λ 4227 being inverted with respect to that for the H and K lines. Among the weaker calcium lines in this region, one group followed the anomalous dispersion of λ 4227 while another was affected similarly to H and K. The evidence thus seems clear that each group of emitting (and absorbing) centers may form an independent vapor prism, even when these particles belong to the same element.

The experiments showed no direct connection between the magnitude of anomalous dispersion effects and the wave-length. While for most elements the lines whose character adapts them to show the effect are in the blue and violet, the strong anomalous dispersion obtained for the D lines, barium lines in the red, and λ 6708 of lithium show that lines of strong absorption may give large effects in any region of the spectrum.

MOUNT WILSON SOLAR OBSERVATORY,
February 1, 1917.

matically in Figs. 1 and 2; Fig. 2 is an enlarged drawing of a portion of Fig. 1. The dimensions are given on the figures in centimeters. The

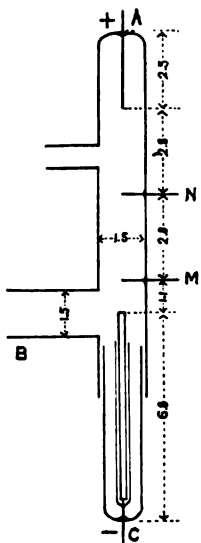


Fig. 2.

tube, in a general way, is similar to the one used by Righi, but is somewhat simpler in design. It consists of a small tube, Fig. 2, having the anode at *A* and the cathode at *C*. Opening into this, at *B*, is a much longer and wider tube, *BD*, Fig. 1. The tube *AC* I will call the *side tube*, and the tube *BD* the *long tube*. The anode is a platinum wire .046 cm. in diameter; and the cathode is an aluminum wire, .328 cm. in diameter, covered to within a centimeter of its tip with a small glass tube to confine the discharge to its end.

The whole tube, which we may call a Righi tube, is connected in series with a high potential battery, *B*, and a non-inductive resistance of from half a million to a million ohms, *R*, as shown in Fig. 3. *kk* is a double switch which connects the battery to the rest of the circuit.

A static induction machine can be used in place of the high-potential battery, but the battery is better as it gives a steadier and larger current. The battery had 2,000 cells, and I usually used about 2,100 or about 2,700 volts.

In all the experiments, I used the air of the room and did not attempt to dry it. To exhaust the tube I used a Gaede rotating mercury pump, or a Gaede oil pump; and to measure the pressure, a MacLeod gauge. The Righi effects were best obtained at pressures varying from .1 to .2 mm. of mercury.

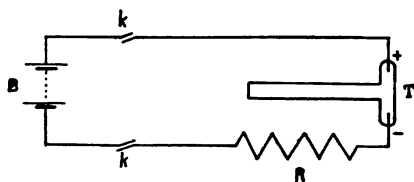


Fig. 3.

To produce the magnetic field, I used a solenoid *S*, Fig. 1, having a soft iron core, which gave an approximately uniform transverse field in the region between the cathode and anode of the tube. Its strength could be varied from zero to 2,600 gauss.

When no magnetic field is present, there is the usual Geissler tube appearance in the side tube *AC*, that is a striated pink glow in the neighborhood of the anode and a bluish-violet glow in the neighborhood of the cathode, but there is no glow anywhere in the long tube. As the strength of the magnetic field is gradually increased, the pink striæ are pressed more and more against the side of the tube, their

number increases, and they move downwards until they fill the whole of the Faraday dark space. When the field reaches a certain strength, depending upon the pressure of the air in the tube, the striæ coalesce, forming a continuous pink column, and the discharge through the tube becomes periodic, emitting a note. At the same time, there appears in the long tube, *BD*, a bluish-violet glow extending from the cathode to *E*, see Fig. 1; a pinkish glow extending from *E* to *F*, and a bluish-white glow extending from *F* to *D*. The short bluish-violet column is either attracted to, or repelled from, the north pole of an auxiliary magnet, and produces a spot of fluorescence at the point where it touches the tube. This bluish-violet column Righi assumes to be made up of his magnetic rays. The pinkish column is bent upwards in an arc by the north pole of an auxiliary magnet, and the bluish-white column is bent downwards in an arc. The directions in which these columns are bent seem to show that a current flows away from the region of *F*, on both sides of it. Righi therefore calls this region *F* the "virtual anode." The column from *E* to *D*, we may call the "Righi column." Righi assumes that it is produced by the disintegration of the magnetic rays. He assumes that the magnetic rays under the action of the magnetic field move out along the lines of force. When they have reached the weaker parts of the field they are supposed to disintegrate, producing free positive and negative ions. The positive ions so produced are assumed to form the virtual anode. The portion of the Righi column from *E* to *F*, I shall call the "positive column" because it resembles in color the positive column in the ordinary Geissler tube, and the portion from *F* to *D*, I shall call, by contrast, the "negative column."

In the region of the virtual anode, for several centimeters, the column is often double, there being both a positive and negative column present at the same time. In this case, it presents much the appearance of a vibrating string. Occasionally the column will be double throughout the whole length of the tube: for instance, on one occasion the negative column was of about equal brightness throughout the whole length of tube, and the positive column decreased in brightness from the fixed to the free end of the tube, being very faint towards the free end.

The virtual anode may exist at any point, *F*, along the tube. As the strength of the magnetic field increases, it moves out along the tube toward *D*. Under certain conditions, as for example when an electrostatic induction machine is used, *F* may be close to *E*, and the negative column may occupy nearly the whole length of the tube.

The "magnetic rays" have been investigated by Thirkell,¹ More and

¹ Proc. Roy. Soc., 83A, 324-334, 1910.

Rieman,¹ More and Mauchly² and others. Both Thirkell and More and Mauchly have come to the conclusion that they are negatively charged particles moving in spirals along the lines of magnetic force. In fact More and Mauchly state that they have found that all of the Righi effects can be obtained without a magnetic field by using a Wehnelt hot lime cathode.³ Recently the Righi effects have been discussed by Holm⁴ and Righi.⁵

The bluish-violet column from *B* to *E*, in agreement with the investigations of Thirkell and More and Mauchly, we may assume to consist of cathode rays, since it begins at the cathode and makes the glass fluorescent where it touches it. These investigations do not explain, however, the existence of the positive and negative columns, the virtual anode, and the periodicity of the discharge.

I first repeated the experiments described by Righi in his publications, and my experiments confirmed his experimental results. Thus, I found that the length of the Righi column depended upon the strength of the magnetic field, increasing as the strength of the field increased; and that the frequency of interruption of the discharge decreased as the strength of the field increased. The frequency could in this way be varied through several octaves.

2. CAPACITY OF THE LONG TUBE.

Early in my experiments, I noticed that when the finger was touched to a metal band holding the long tube in place the note emitted by the tube became very much louder and the frequency of the discharge was lowered, the frequency normally being very high, about 2,000 a second.

I therefore wrapped a brass band, 10 cm. wide, *bb*, Fig. 1, around the tube and connected it to earth. The frequency was then reduced to about one fourth of its original value. When this band was made twice as wide, the frequency was again reduced. By decreasing the width of the band, and also the strength of the magnetic field, the pitch can be made to vary from zero, or say one discharge every ten seconds, to a pitch above the limits of audibility. In other words, with an earthed metal band on the tube, any frequency that is desired can be obtained.

¹ Phil. Mag., 24, 307-316, 1912.

² Phil. Mag., 26, 252-267, 1913.

³ R. Reiger, Sitzungsberichte der Physikalisch-medizinischen Sozietät in Erlangen, 37, 20, 21, 1905, states that when using a Wehnelt cathode it is very difficult to get a continuous discharge. This fact, as will be shown later, is probably the reason why More and Mauchly obtained the Righi effects with this kind of a cathode.

⁴ Phys. Zeitschr., 16, 79, 80, 1915.

⁵ Nuovo Cimento, 9, Gennaio—Febbraio, 1916.

When the band was on the tube, there was a negative column from *bb* to *D*, and, usually, a *double* column from *bb* to *E*. By a "double" column, I mean that the auxiliary magnet divides the luminous column into two parts, one part being bent up and the other down.

These experiments show that the frequency of the discharge depends not only upon the strength of the magnetic field, but also upon the capacity of the long tube. If we assume that the long tube acts as a conducting cylinder, its calculated capacity would be about 20 cm. in electrostatic units. Other experiments, which will be described later, would seem to indicate that this is of the order of the maximum effective capacity of the tube.¹

The connection between the strength of the magnetic field and the frequency of the discharge probably depends upon the fact that the length and strength of the stream of electrons proceeding from the cathode, into the long tube, increases with the strength of the field, making the effective capacity of the tube greater, that part of the tube only being effective as capacity where the gas has become ionized by the presence of the electrons.

3. ACTION OF THE MAGNETIC FIELD.

The component of the magnetic field transverse to the side tube, due to the solenoid *S*, Fig. 1, was approximately uniform between the electrodes; not varying more than 10 per cent. along the line joining them. It was transverse to the line joining the electrodes, and in the direction of the axis of the long tube. Its action is twofold: (1) it produces a periodic interruption of the discharge in the side tube, and (2) it sends a periodic charge of electrons into the long tube.

The first fact is shown by the following experiment. I disconnected the long tube from the side tube at *B*, Fig. 1, and closed the aperture so formed with a glass plug. I then examined the action of the magnetic field upon the discharge in the side tube. Using a rotating mirror to observe it, I found that for a certain strength of the field the discharge through the tube became periodic, the frequency decreasing as the strength of the field increased, and varying through several octaves. When the field reached a certain strength, the discharge in the tube ceased. The *periodicity* of the Rhigi effect is, therefore, due to the action of the field upon the discharge through the *side tube*. Its frequency is modified by the presence of the long tube, *BD*, but the periodicity does not depend for its existence upon it.

¹ For discussions of the capacity of a Geissler tube, see: G. Wiedermann, *Die Lehre von der Elektrizität*, Bd. 4, 505-511, 1885; J. Borgmann, *Phys. Zeits.*, 2, 651-653, 1901; S. N. Taylor, *Phys. Rev.*, 18, 338-344, 1904; R. Reiger, *l.c.*, 21-39.

The periodic discharge of electrons into the long tube is shown by the existence of the cathode stream, already described, from the cathode to *E*.

Repeated experiments showed that the Righi effects are never present in the long tube unless the discharge through the side tube is an interrupted one.

It was also found that even if the discharge is an interrupted one, the Righi effects are not produced unless the cathode is opposite the opening in the long tube at *B*, Fig. 1. If the distance between the electrodes is kept constant, but the cathode is lowered until it is 3 or 4 centimeters below the opening at *B*, the effects are not obtained. This would seem to show that there must be a projection of the electrons directly from the cathode into the long tube. The action of the same field upon the *positive column*, when a portion of it is opposite the opening at *B*, does not produce the same effect.

Table I. shows how the potential difference, between the anode and the

TABLE I.

Pressure in Mm. of Mercury.	Field in Gausses.	Potential Across Tube in Volts.		Differ- ence.	Current in Milliam- peres.		Differ- ence.	
		Without Field.	With Field.		Without Field.	With Field.		
.32	200	750	650	-100	1.00	1.09	+.09	Long tube discon- nected from side tube.
	245	780	680	-100	1.05	1.15	+.10	
	265	800	700	-100	1.02	1.15	+.13	
	290	Discharge became discontinuous.						
.20	265	1,050	800	-250	.75	.95	+.20	
	300	Discharge became discontinuous.						
.15	340	900	750	-150	1.10	1.20	+.10	Long tube connected to side tube.
	410	Discharge became discontinuous.						
.13	170	1,040	850	-190	.84	1.00	+.16	
	220	1,010	790	-220	.78	.98	+.20	
	280	Discharge became discontinuous.						

cathode, and the current through the tube vary as the magnetic field is increased. Two cases are given, first, when the long tube, *BD*, was *disconnected* from the side tube at *B*, and, second, when the long tube was *not disconnected*. The differences of potential were measured with a quadrant electrometer, and the currents with a milliammeter.

To determine the distribution of the potential in the side tube, and the effect of a magnetic field upon it, platinum wires were sealed into the walls of the side tube at the points *M* and *N*, Fig. 2. The wire *M* was in the negative glow, just outside the Crookes dark space, and the wire *N* was just outside the positive striations. The difference of potential

between M and the cathode was approximately equal to the cathode fall. The results, given in Table II., were obtained when the long tube was *disconnected* from the side tube.

It is seen from the two tables that *as long as the discharge through the side tube is continuous*, the effect of the transverse magnetic field is to

TABLE II.

The Long Tube Disconnected from the Side-Tube.

Pressure in Mm. of Mercury.	Field in Gauss.	Potential Difference Between	Potential Across Tube in Volts.		Differ- ence.	Current in Milliamperes.		Differ- ence.
			Without Field.	With Field.		Without Field.	With Field.	
.20	135	C and A	1,080	1,020	-60	.80	.84	+.04
		C and N	940	890	-50	.76	.79	+.03
		C and M	950	860	-90	.78	.80	+.02

considerably *decrease* the difference of potential between the electrodes of the tube, and to correspondingly *increase* the current flowing through it. It is also clearly seen that when the field reaches a certain strength the discharge becomes discontinuous. Readings of the electrometer and milliammeter for field strengths greater than this obviously have no value. Table II. also shows that the cathode fall of potential amounts to about nine tenths of the total fall of potential between the electrodes. Just why the magnetic field for a certain strength should extinguish the discharge, it is difficult to say.¹ The fact that the potential decreases as the magnetic field increases until the discharge becomes interrupted, suggests that the action of the field is to reduce the potential until it becomes less than the "maintenance potential," *i. e.*, the minimum potential necessary to *maintain* the current through the tube. The value of the maintenance potential will depend upon the shape of the tube and its electrodes, the pressure of the air in it, the value of the impressed electromotive force and resistance in the circuit, and probably also upon the strength of the magnetic field.

The *minimum potential* curve, A , and the *maintenance potential* curve, B , for varying pressure were obtained for the tube and are shown in Fig. 4. The minimum potential curve, A , shows the smallest potential that will *start* a discharge through the tube. Curve B was obtained by gradually decreasing the potential applied to the circuit, by a potentiometer arrangement, until the discharge through the tube stopped.

¹ See Thomson, *Conduction of Electricity through Gases*, 2d edition, pages 576-579; Riecke, *Ann. Phys.*, 4, 592-616, 1901; Stark, *Ann. Phys.*, 12, 31-51, 1903; Schwienhorst, *Diss. Gottingen*, 1903; Earhart, *Phys. Rev.*, 3, 103-114, 1914.

There was a non-inductive resistance in the circuit of about half a million ohms, and the current flowing through it varied from .34 to .13 of a milliampere.

4. SIGN OF THE CHARGES INSIDE THE LONG TUBE.

The charges on the *inside* of the long tube were examined both with a brass band touching the walls of the tube, and with a platinum wire placed in its axis. The brass band was 5 cm. wide and was attached to a wire sealed into the end of a long glass tube 8 mm. in diameter which could be introduced into a suitable holder at the free end *D* of the long tube. In this way it could be placed at any desired place in the tube. The platinum wire, .046 cm. in diameter and 5.5 cm. in length, was

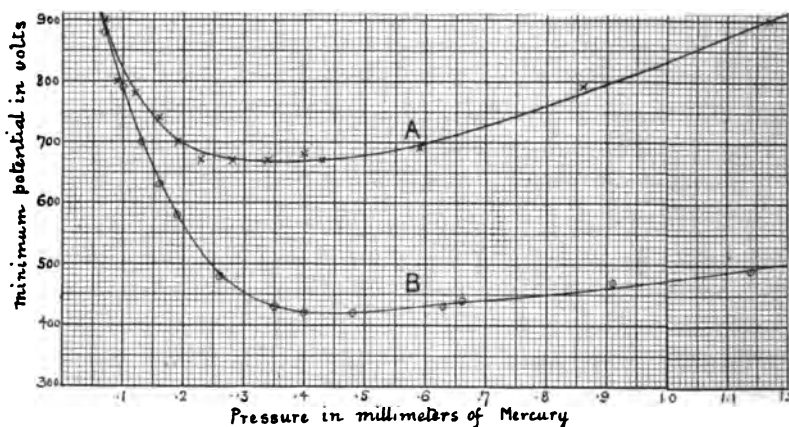


Fig. 4.

similarly sealed into the end of a long glass tube. The band, or wire, was connected to an aluminum leaf electroscope, and the sign of the charge on it was determined by determining the sign of the charge on the electroscope. The results obtained were the same both for the band and the wire, and were briefly the following.

If the magnetic field was not strong enough to make the Righi column long enough to touch the brass band, or wire, the behavior of the electroscope was irregular, but if it was strong enough, on closing the high potential key, *kk*, Fig. 3, with no magnetic field, I got a *positive* charge on the electroscope; then with the magnetic field on I got a deflection of *indeterminate* sign, that is a deflection due to a charge rapidly alternating in sign; and finally opening the high potential key, still keeping the magnetic field on, I got a residual *negative* charge. So then, I had an initial positive charge, then a charge rapidly alternating in sign, and finally a residual negative charge. The farther the band, or wire, is from

the side tube the more slowly does the leaf charge up for the initial positive charge. The behavior of the electroscope, therefore, indicates that when the magnetic field is strong enough to make the discharge interrupted there is a charging of the tube alternately positively and negatively, and that it probably begins with a positive and ends with a negative charge.

The residual negative charge may be the result of an oscillation due to the capacity of the long tube, or it may be due to the magnetic field diverting a stream of electrons into it. The initial positive charge is explained by the fact that the long tube always has a positive charge, of a greater or less value, when there is a steady discharge through the tube. Since nine tenths of the total fall of potential across the electrodes takes place in the neighborhood of the cathode, the whole of the long tube must be very nearly at the potential of the anode. In order to have this potential it must take on a positive charge. Therefore, when the discharge passes through the side tube, the long tube will begin to take on a positive charge. It will continue to do this until the discharge through the side tube becomes interrupted, when its positive charge will flow back towards the cathode, and may or may not produce a complete electrical oscillation in the long tube. That high frequency oscillations are possible in Geissler tubes is shown by an independent experiment which will be described later in this paper.

If an earthed brass band is placed around the outside of the long tube, causing the separate interruptions to succeed each other very slowly, for instance one every four seconds, the discharge builds up gradually, the glow first appearing faintly in the long tube from *bb* to *B*, Fig. 1, growing brighter and brighter and then suddenly flashing up in the side tube. When this flash takes place, the deflection of the electroscope takes place. Immediately after the flash, all the glow both in the long tube and the side tube goes out, and the action repeats itself.

As seen in a rotating mirror, both the positive and negative parts of the Righi column and the discharge in the side tube appear to be synchronous. If the two parts of the Righi column are not perfectly synchronous, and I do not think that they are, the interval of time between their separate occurrences must be very much less than the audible period of the discharge in the tube, probably less than .0001 second.

It is significant that, even when the magnetic field is not on, every time the high potential key, *kk*, is either closed or opened there is an instantaneous flash of light in the long tube.

5. THE PRODUCTION OF THE RIGHI EFFECTS WITH A SPARK GAP.

While experimenting with the arrangement of the circuit shown in Fig. 3, I discovered, accidentally, that all the Righi effects could be obtained without any magnetic field if a suitable spark gap was placed in the circuit. The discovery resulted from the fact that there was a loose contact, and consequently sparking, at one contact of the double switch *kk*. I noticed that when the sparking occurred there was luminosity in the long tube which much resembled the Righi effect. I did not, at first, pay much attention to it, as Righi in his book, and in subsequent papers,¹ has spoken of the importance of not having a loose contact in the circuit, as this would, according to him, produce a spurious effect. I thought, first of all, that I had a spurious effect, but, by putting into the circuit a suitable spark gap on the anode side, and a high resistance on the cathode side of the tube, I found that I could get all the Righi effects without any magnetic field.

At first, not all spark gaps would give the effect. The terminals of the gap must, apparently, be of relatively large area, and be so arranged that the length of the gap remains approximately constant. It is probably necessary that the gap shall be so constructed that the terminals cool off rapidly, preventing arcing. Later I had no difficulty in obtaining the effect with terminals of large surface constructed of zinc, copper or brass. The terminals which I finally adopted were zinc cylinders with

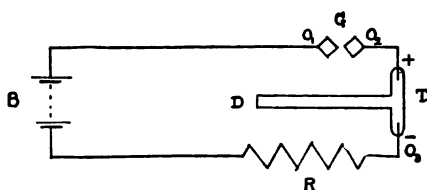


Fig. 5.

flat ends, 9.3 mm. long and 12 mm. in diameter. They were not used end on, that is with their flat faces parallel to each other, but with the flat ends making an angle of 45° with each other, so that the sparking took place between two adjacent edges, as this arrangement

seemed to give more regular results.

The circuit used is shown diagrammatically in Fig. 5. It will be noted that there are four elements in it, the battery *B*, the gap *G*, the tube *T*, and the high resistance *R*. In order that well-developed and characteristic Righi effects shall be produced, I found that the gap must be placed next to the anode of the tube, and the high resistance next to the cathode of the tube. No other arrangement will give the effects completely. For instance, if *G* and *R* are interchanged, the characteristic Righi effects are not obtained, nor are they obtained if *G* and *R* are both put on the same side of the tube. On the other hand, if the

¹ For this and other interesting matters, see Righi, *Phys. Zeits.*, 15, 528-534, 558-563, 1914.

anode and cathode connections to the tube are interchanged, that is, if the aluminum electrode is made the anode and the platinum electrode the cathode, a good Righi column is obtained, showing that the *shape* of the electrodes and their *position* with respect to the long tube have very little to do with the production of the Righi effects, when these effects are produced with a spark gap.

When the gap is closed, there is, of course, no Righi column, and the discharge through the side tube is continuous. If the gap is now opened and gradually widened, the current through the circuit decreases, remaining continuous, until it has fallen to a certain definite value, when, for a certain critical length of the gap, it becomes periodic. When this happens a short Righi column appears in the long tube. As the length of the gap is further increased the length of the Righi column increases until it fills the whole tube. When the gap becomes too long, the discharge goes out altogether. The frequency of interruption of the discharge decreases as the length of the gap increases. Using 2,700 volts and 800,000 ohms in the circuit shown in Fig. 5, the Righi column appeared when the gap was about .1 mm. long, was brightest at about .3 mm., and went out at about .4 mm.

In most of my experiments, with the spark gap, I used about 2,700

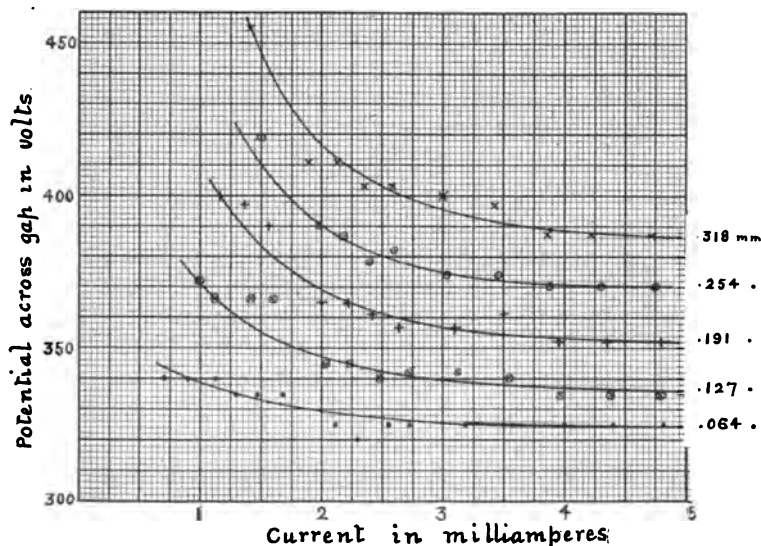


Fig. 6.

volts in the circuit. For this voltage, the Righi effects appear best at a pressure of about .16 mm. of mercury. For 2,100 volts, they begin to appear at about .2 mm., are brightest at about .13 mm., and disappear at about .05 mm.

I examined the charges on the inside of the long tube, when using a spark gap to produce the Righi effects, just as I did when using a magnetic field to produce them, and I got the same results as before. Also, in this case, I found that if the discharge was stopped before the spark gap was opened, there being therefore no luminosity in the long tube, I got a residual positive charge.

From these results, it now appeared that the Righi effects were not dependent directly upon the action of a magnetic field, and were probably produced by the periodic interruption of the current through the tube. It therefore became necessary to study the properties of the spark gap,

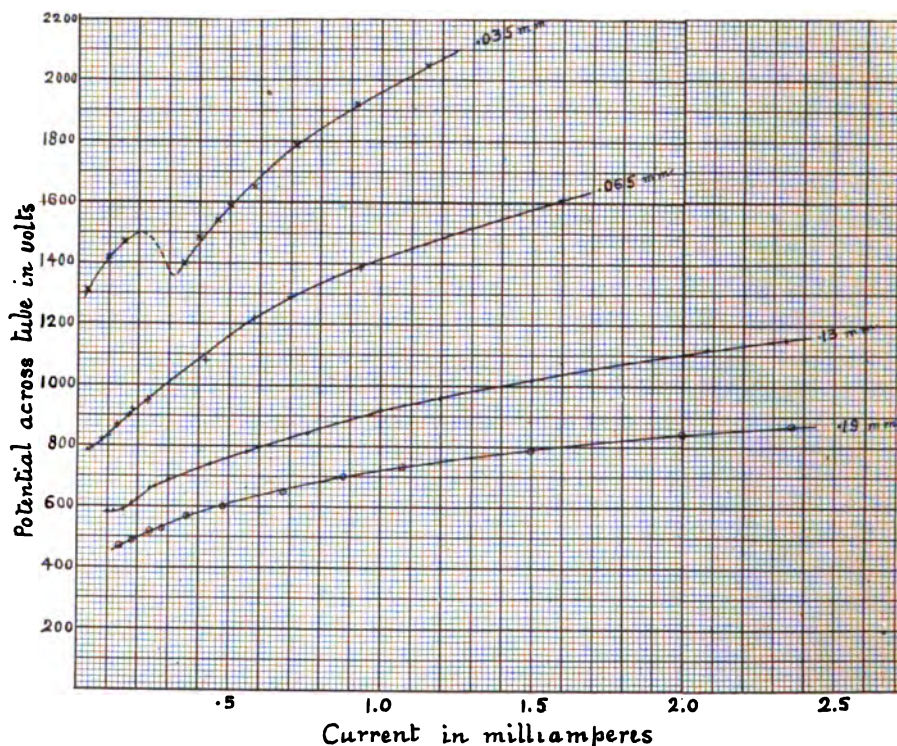


Fig. 7.

and to determine the volt-ampere characteristics both of the spark gap and the Righi tube. The discharge across the gap was examined with both a spectroscope and a microscope and was found to be a true glow discharge, for all lengths, and not an arc discharge. The spectrum observed was, in all cases, the spectrum of nitrogen and not that of the metal of the electrodes. The volt-ampere characteristics for the spark gap, for different lengths of gap, are shown in Fig. 6; and for the tube, for different pressures in the tube, in Fig. 7.

Riecke,¹ Stark² and Earhart³ have investigated the characteristic curves for the *glow discharge* for Geissler tubes of special form; and Stark⁴ and Malcolm and Simon⁵ for spark gap discharges; and my curves agree, in a general way, with those obtained by them. The shape of the characteristic will, of course, depend upon the shape of the tube, and the form of the gap, respectively. The characteristics, as has been shown by Kaufmann,⁶ completely describe the behavior of the gap and the tube in the circuit, and enable us to tell whether the conditions in the circuit are stable or not for a given current by the use of his criterion of stability

$$R > - \frac{\partial E}{\partial i},$$

where E is the potential across the gap, or tube; i , the current through it, and R the external resistance in the circuit.

In the circuit shown in Fig. 5, for 2,100 volts and about 500,000 ohms, the discharge becomes discontinuous, or unstable, when the current falls to about 1.8 milliamperes. Now an inspection of Fig. 7 shows that for this current the characteristic for the tube is a rising one, and therefore the conditions are stable so far as the tube is concerned, and the interruption is not due to the tube. An inspection of Fig. 6, on the other hand, shows that for 1.8 milliamperes the characteristic for the gap is a falling one, and that therefore the instability might be due to the gap. However, calculation shows that this possibility must be dismissed as the resistance in the circuit is very large, varying from half a million to a million ohms, and the slope of the characteristic curve, $\partial E/\partial i$, for 1.8 milliamperes is small compared with it, about 30,000 $\frac{\text{volts}}{\text{amperes}}$. Some other cause must therefore be found for the periodic extinction of the spark, as the falling characteristic of the gap is not *alone* sufficient to cause it.

A long series of experiments seems to show that the interruption of the discharge across the gap is due to two facts, (1) that the Righi tube has an appreciable electrostatic capacity, and (2) that the characteristic of the gap is a falling one. The capacity of the tube, apparently, produces an electrical oscillation in it, and this oscillation combined with

¹ Ann. Phys., 4, 592-616, 1901.

² Phys. Zeits., 3, 274, 275, 1902; Ann. Phys., 12, 1-30, 1903.

³ Phys. Rev., 1, 85-95, 1913.

⁴ Phys. Zeits., 4, 535-537, 1903.

⁵ Phys. Zeits., 8, 471-481, 1907.

⁶ Ann. Phys., 2, 158-179, 1900; see also Barkhausen, Problem der Schwingungserzeugung, Leipzig, 1907.

the falling characteristic of the gap produces the interruption. It is found that if the tube is taken out of the circuit, the discharge across the gap is continuous for all lengths of the gap; and also that if the gap is taken out of the circuit, the discharge through the tube is continuous. The periodic interruption of the current flowing through the circuit is therefore due to the combined action of the gap and tube in series.

Light is thrown upon the problem by the fact that the tube can be replaced, in the circuit, by a *high resistance shunted by a capacity*; thus the Righi tube shown in Fig. 1 may be replaced, approximately, by a high non-inductive resistance, R_1 , of half a million ohms, shunted by an air condenser, C , having a capacity of 93 cm., as shown in Fig. 8. The current flowing through the circuit and the periodicity of the discharge were then about the same as when the tube was present.

Further experiments showed that in order to make the discharge across the gap discontinuous, the capacity must be connected directly to the gap. It will not make the discharge interrupted if a high resistance is placed between it and the gap. For instance, a wire, O_2O_5 , Fig. 9,

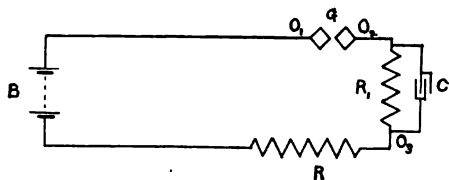


Fig. 8.

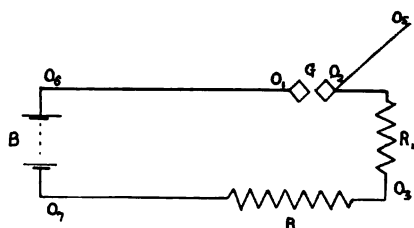


Fig. 9.

800 cm. long and .062 cm. in diameter, and having a capacity free in air of 39 cm., when attached at O_2 made the discharge discontinuous. If it was attached at O_3 , R_1 being a high resistance of half a million ohms, it did not make the discharge discontinuous. Also if attached at O_1 , it did not make the discharge discontinuous. Further it was found that the capacity must be attached between the gap and the high resistance. If it is attached at O_2 and the high resistance is placed between O_1 and O_5 the interruption is not produced. Thus it is evident that they must both be on the same side of the gap. It is however indifferent whether they are attached to the anode or the cathode of the gap.

If a neon tube is touched to any part of the wire O_2O_5 it glows. This fact, I think, indicates that electrical oscillations are set up in the system $O_5O_2O_1$ by the charging of the wire O_2O_5 .

The wire, attached at O_2 , can be replaced by a capacity connected across the gap; the discharge then becoming discontinuous as before.

Elaborate experiments using capacities across the gap and at different points of the circuit seem to show that the periodic extinction of the discharge is produced as follows.

When the key of the circuit is closed, the system formed by the capacity O_2O_5 and the battery wires O_1O_6 charges up until the potential across the gap is high enough to make the discharge pass across it. The time that this takes will depend upon the time constant, CR , of the circuit, where C is the capacity of the wire or condenser, and R is the resistance of the circuit, and also upon the length of the spark gap. Since R is large, this time will not be very small, but will be of the order of the observed period of the discharge, namely a few ten-thousands of a second.

The resistance of the circuit is so great, and its inductance, .000112 henry, so small, that oscillations cannot take place in it as a whole; but oscillations do take place in the free wire and the gap, $O_5O_2O_1$, Fig. 9; the wire going to the battery, O_1O_6 , acting as an earth. In the circuit used, the battery wires going from the double switch, kk , Fig. 3, to the battery two floors above were parallel to each other, about 6 inches apart, and 50 feet long. The oscillations are shielded from the rest of the circuit by the high resistances R and R_1 . The distribution of the oscillations around the circuit was determined roughly by touching a neon tube to different parts of it.

The oscillations resemble the well-known Poulsen and Wien oscillations but differ from them in the fact that in these the supply circuit necessarily has large inductance and small resistance, whereas the Righi circuit has a very small inductance and a very large resistance.

The discharge is put out in the following manner: The condenser, or wire, charges, the spark jumps across, and a damped oscillating current is set up across the gap. At the same time the battery B tends to build up a steady current of a few milliamperes or less. The growing steady current, and the damped oscillating current, will be superposed upon each other. Every time the resultant current approaches zero it will be extinguished on account of the falling characteristic of the gap. If the oscillating potential across the gap, due to the condenser, is great enough, the discharge will light up again across the gap in the opposite direction. When, however, the oscillating potential is no longer great enough to do so, the discharge will stop altogether. The action will then be repeated, and the audible periodicity of the discharge will be produced. The same action has already been described by Barkhausen¹ for the case of Wien oscillations.

¹ Loc. cit., § 46, pp. 94. 95.

The current oscillations were actually observed with a Braun tube and a rotating mirror, for the case in which .095 microfarad and 3.01 henries were connected in series across the gap, the period being, therefore, approximately 3×10^{-3} sec. A strongly damped train of from 1 to 7 half oscillations for every interruption of the spark could easily be observed. The damping as observed in the mirror, or as determined by the position of the maxima on the fluorescent screen of the Braun tube appeared to be *linear* within the errors of observation; the tops of the peaks, in one case, being 2.70, 1.75 and .90 cm. above the point of rest, and 1.75 and .90 cm. below it. I tried to take photographs of the images on the fluorescent screen, but the light was not intense enough.

A calibration of the Braun tube showed that the deflection of 2.70 cm. corresponded to a maximum amplitude of the oscillating current of .33 amperes. The steady current across the gap, when no inductance and capacity were connected across it, was only about .002 ampere. This shows that quite a small capacity would make the discharge interrupted.

I found it to be generally true that if a capacity is put around a spark gap across which a *glow* discharge is passing, the discharge will become discontinuous if the capacity is *great enough*.¹ If the capacity is not great enough, the interruption of the discharge will not take place. For instance, in the case of Fig. 9, the wire O_2O_5 must be about 400 cm. long to produce the interruption. Such a wire has an electrostatic capacity of about 1.25 cm. And for the case of Fig. 8, the air condenser, C , had to have a capacity of at least 2 cm. before it would make the discharge periodic.

There are in these effects two periodicities to be considered, first, the audible periodicity produced by the separate spark discharges and, second, that of the electrical oscillations producing the interruption of the discharge.

The audible periodicity is approximately equal to the time that it takes for the gap to charge up to the sparking potential. This will depend upon the length of the gap. A curve showing the minimum sparking potential across the gap that I used is shown in Fig. 10.

If we neglect the effect of ionization in the gap, the rise of potential across it for the circuit shown in Fig. 9 will be given by the equation

$$(I) \quad V = V_0(1 - e^{-(t/RC)}),$$

where V_0 is the electromotive force of the battery, R is the high resistance in the circuit, and C is the capacity of the wire, or of the condenser around the gap. And the period of the audible periodicity will be given,

¹ See Hittorf, Ann. Phys., 20, 719-726, 1883; Lehmann, Ann. Phys., 56, 333-336, 1895.

approximately, by

$$(2) \quad t = CR \log_e \left(\frac{V_0}{V_0 - V} \right),$$

where V is equal to the minimum sparking potential across the gap. V for a given length of gap can be determined from Fig. 10. If V , V_0 , R and C are known, the audible period can be determined.

I observed that as R , C , or the length of the gap, was increased the audible period was increased. t was calculated from the known values

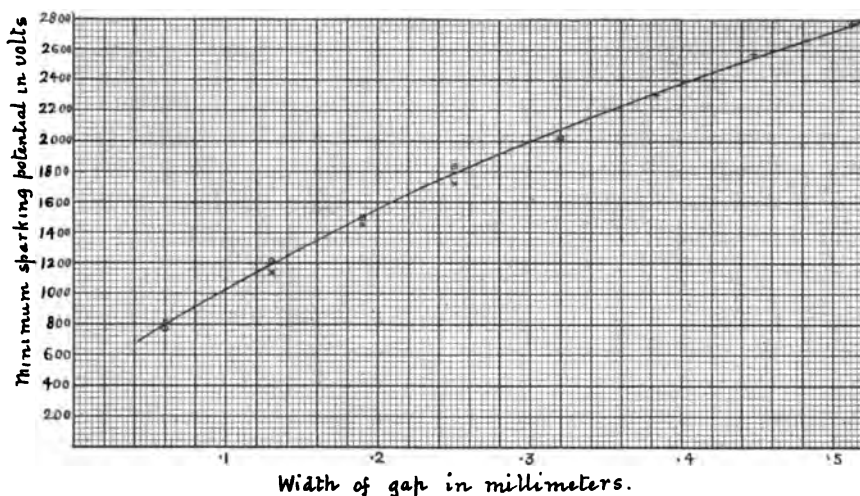


Fig. 10.

of C , R , V_0 and V , and the values obtained were of the order of the audible period.

If we assume that the discharge across the gap goes out when the oscillatory current is equal and opposite to the steady current, since the maximum value of the oscillatory current, neglecting damping, is

$$(3) \quad V \sqrt{\frac{C}{L}},$$

where L is the inductance of the circuit, and the value of the steady current is

$$(4) \quad \frac{V_0}{R},$$

we shall have as the condition for the production of an interrupted discharge

$$V \sqrt{\frac{C}{L}} \equiv \frac{V_0}{R}.$$

Solving for C , we get

$$(5) \quad C \equiv \left(\frac{V_0}{V} \right)^2 \frac{L}{R^2}$$

as the equation for the minimum value of the capacity which must be shunted around the gap, or attached to the cathode side of the gap, to make the discharge an interrupted one.

V , the minimum sparking potential, shown in Fig. 10, is related to the length, l , of the gap approximately by the formula

$$(6) \quad V = k_0 + kl,$$

where k_0 and k are constants.

Experiments confirmed (5) except that the value of the inductance, L , of the oscillating circuit seemed to have little effect upon the value of C . If anything, the experiments indicated that the greater L the less C had to be. This may perhaps be due to the fact that the minimum value of C is always very small, and that any inductance that can conveniently be used has a relatively large distributed capacity. So that, when we add inductance to the circuit we add more or less capacity.

The experiments described show, I think, that when the *Righi effects* are obtained without a magnetic field, with a spark gap in the circuit, they may be explained by assuming that the capacity of the long tube and the instability of the spark gap produce a periodic interruption of the current. There is thus produced an alternate charging and discharging of the long tube, the negative column in the long tube corresponding to the charging, and the positive column to the discharging. An actual electrical oscillation takes place in the long tube, made visible by the light resulting from the ionization of the gas. The virtual anode is the region in which the charging and discharging streams are both visible. These two streams probably occur at the other points along the tube, but on account of lack of luminosity at these points, are not visible.

The action is probably as follows: When the double switch leading to the battery is closed, the potential across the gap rises until there is a discharge across it. When this happens, there is a sudden flow of current through the side tube, and the long tube charges up positively. There is then a reaction of the current in the side tube on account of its capacity and inductance, and a flow of current in the opposite direction. This reduces the total current flowing across the gap, and on account of the instability of the gap, the current through it becomes extinguished. It is possible that there may be more than one oscillation of the current in the long tube, but I do not think that there is. The oscillation can

only take place in the conducting system DTO_2O_1B , Fig. 5. It cannot extend into the portion O_2R on account of the high resistance R . The luminous column, however, in the side tube from A to M , Fig. 2, when under the action of a transverse magnetic field, is seen to be always pressed against one side of the tube only. If there were an oscillating current from A to M , there would be *two* columns, pressed against opposite sides of the tube. Therefore, I think that there is probably only one oscillation in the long tube, a flow in and a flow out.

It is well known that alternations of current of low frequency can exist in Geissler tubes,¹ and also that if the luminous column of the discharge is subjected to a transverse magnetic field it will be separated into two columns bent towards opposite sides of the tube, corresponding to the opposite currents. The question arose as to whether this would be true for alternations of current in the long tube where the alternations, if they exist, must be of very high frequency, probably several millions a second. I found by experiment that such oscillations of current in Geissler tubes *are* divided into two streams by a transverse magnetic field.

A tube 10 cm. long and 1.5 cm. wide had platinum wire electrodes exactly alike and 7 cm. apart sealed into its ends. It was connected in series with a condenser and the secondary of a small induction coil, and subjected to the action of a transverse magnetic field. A double positive column was obtained, the two columns being pressed against opposite sides of the tube. The two columns were, in many cases, equally bright, indicating, I think, that there must have been several oscillations for each break of the primary circuit. The capacity was varied from 2×10^{-6} mf. to 1.25 mf. When the capacity was increased to 2.5 mf., only one column was present. For 1.25 mf. two columns were present, but one was much less bright than the other. When the tube was connected directly to the terminals of the secondary without any condenser in series with it, there was only a single column when acted upon by the magnetic field.

To examine the action for very high frequencies, a wire 70 cm. long was attached to each side of the tube, and a spark several millimeters long was caused to jump from the secondary of the induction coil to the end of one of the wires, the end of the other wire on the other side of the tube being free in air. The tube then glowed, and when subjected to the magnetic field showed a double column. Since the tube with its attached wires must act as a linear oscillator, or a Hertz oscillator,

¹ Thomson, Phil. Mag., 32, 321-336, 445-464, 1891; Lehmann, Verh. d. Naturwissensch. Ver. Karlsruhe, 15, 69-71, 1902.

the frequency must be very high. In this case, since the wires were each 70 cm. long, the frequency was probably of the order of 10^8 a second.

The fact that, to get the best results with the Righi tube, the gap must be placed next to the anode of the tube, and the high resistance next to the cathode, is probably explained by the apparent high resistance of the region around the cathode,¹ shown by the high value of the cathode fall of potential. On the other hand, the region of the positive column behaves as though it had a relatively small resistance. If then the gap is placed next to the cathode of the tube, the region of the cathode will act like a high resistance between it and the long tube, and tend to prevent oscillations from being set up in the long tube.

6. A THIRD WAY OF PRODUCING THE RIGHI EFFECTS.

While experimenting on the pressure most favorable for the Righi effects, I found that they could be produced without either a magnetic field or a spark gap, if the pressure in the tube was made low enough. For instance, using about 2,100 volts and 500,000 ohms in series with the tube, I got a well-marked column with all the Righi effects when the pressure of the air was reduced to .035 mm. The current through the tube at this pressure was about .2 milliamperes. Above this pressure, the discharge through the tube was continuous. Below it, it became discontinuous and the Righi column appeared.

The appearance of periodicity at low pressures is probably explained by the fact that at these pressures the current is very small, and that the volt-ampere characteristic for the tube becomes a falling one for very small currents. The discharge would then become unstable for a certain value of the current according to Kaufmann's criterion. The characteristic for the tube for .035 mm. of pressure, shown in Fig. 7, would suggest that such is the case. The curves given by Riecke² and Earhart³ also show that when the current is very small the characteristic becomes a falling one.

In this case it will be observed that the Righi effects are produced not only without a magnetic field, but even without the presence of a spark gap. So apparently the capacity of the tube and its periodic charging are the only elements that are necessary to produce the Righi effects.

The explanation for this case is probably the same as for the case in which there is a spark gap, except that in this case the instability of the

¹ See Reiger, l.c., p. 35; Lehmann, l.c., pp. 71-73; Wiedemann and Schmidt, *Ann. Phys.*, 62, 462, 1897.

² Loc. cit., p. 596.

³ Loc. cit., pp. 88-91.

discharge through the tube takes the place of the instability of the discharge across the gap.

7. CONCLUSIONS AND SUMMARY.

Since I have shown that the Righi effects can be produced without any magnetic field by inserting a spark gap in the circuit, and, if the pressure is low enough, without either magnetic field or spark gap, it may be assumed that the magnetic field does not play a fundamental part in the production of the effects.

In Righi's case, the magnetic field apparently interrupts the discharge. There then follows a rising of the potential across the gap until the discharge takes place again. At this instant, there is a charging of the long tube, and this results in an electrical oscillation in it, in virtue of its capacity and inductance. It may consist of a single oscillation or of several; probably of a single one.

My conclusions therefore may be summarized as follows. The effects observed by Righi in a special form of Geissler tube, and believed by him to arise from a new kind of ray, called by him the magnetic ray, are not due to any new form of ray, but are due to the periodic charging and discharging of the tube.

In support of this view, I have shown:

1. That the long horizontal extension of the Righi tube has an appreciable electrostatic capacity.
2. That the principal action of the magnetic field is to periodically interrupt the discharge through the tube.
3. That for each interruption, charges appear in the long tube, an initial positive charge and a residual negative charge.
4. That the Righi effects can be produced without any magnetic field, by using a suitable spark gap in the circuit.
5. That the current through a circuit of high resistance and small inductance, containing a battery and a spark gap, can be changed from a steady to an interrupted current by attaching a capacity to one of the terminals of the gap; and that this periodic interruption of the current is due (1) to an oscillation across the gap produced by the attached capacity, and (2) to the negative slope of the volt-ampere characteristic of the gap. A theory is given for this action.
6. That alternations of current of very high frequency can exist in a Geissler tube, and that their presence is made evident by a *double* luminous column when the tube is subjected to a transverse magnetic field.
7. That the Righi effects will be produced without either magnetic field or spark gap if the pressure in the tube is made low enough.

In conclusion, I wish to thank the authorities of Clark University, and Professor A. G. Webster in particular, for placing all the resources of the physical laboratory at my disposal during the course of this investigation.

CLARK UNIVERSITY,
WORCESTER, MASSACHUSETTS,
November 15, 1916.

THE SINGLE-LINED AND THE MANY-LINED SPECTRUM
OF MERCURY.¹

BY T. C. HEBB.

FRANCK and Hertz have shown² that under certain conditions a single-lined spectrum of mercury can be obtained. They found that one of the necessary conditions was that the vapor of mercury be bombarded by electrons having a speed equal to or greater than that produced by falling through 5 volts. As they had already shown that the ionizing potential of mercury was 4.9 volts they were led to an application of the relation $Ve = h\nu$ where $h = 6.56 \times 10^{-27}$ erg. sec. and ν is the frequency of the single-lined spectrum $\lambda = 2536.72$ Å. U. The experiments of Franck and Hertz are exceedingly suggestive and would appear to offer new light on the structure of the atom.

These results have since been repeated and extended by McLennan and Henderson.³ They found that they could get the single-lined spectrum of mercury for any potential difference between 5 and 12.5 volts, but that with a potential difference below 5 volts they obtained no spectrum and that with a potential difference of 12.5 volts or higher they always obtained the many-lined spectrum of mercury. In order to explain the occurrence of the many-lined spectrum at 12.5 volts they suggest that possibly a new type of ionization occurs at this point. They have further shown that they can obtain single-lined spectra of both cadmium and zinc. In a still later article McLennan has shown⁴ that he can also obtain a single-lined spectrum of magnesium. As the ionizing potentials of cadmium, zinc and magnesium have not been determined directly the authors apply the above quantum relation $Ve = h\nu$ in order to determine the value for each of the metals.

The results embodied in this paper are considered worthy of publication even though the work is incomplete. The writer had occasion to spend a limited time at the Ryerson Physical Laboratory during the spring of 1916 and at a suggestion from Professor Millikan attempted

¹ This and the following paper were presented before the American Physical Society at the Chicago meeting, December 1, 1916.

² Deutsch. Phys. Gesell. Verh., 16, 11, pp. 512-517, June 15, 1914.

³ Roy. Soc. Proc., 91, pp. 485-491, Aug. 2, 1915.

⁴ Roy. Soc. Proc., 92, pp. 305-312, Mar. 1, 1916.

to repeat the experiments of Franck and Hertz and of McLennan and Henderson and if possible throw light on the relations of the single-lined and the many-lined spectrum of mercury.

The results to be stated later were obtained with the following apparatus:

A (Fig. 1) was a quartz tube 20 cm. in length, 1.9 cm. internal diameter and 2.1 cm. external diameter. *B* and *C* were iron caps which were

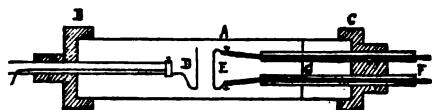


Fig. 1.

joined to the tube with Khotinsky cement. Through *B* passed a glass tube which was connected to the air pump and through which also passed the wire leading to the platinum anode *D*, mounted on the end of

the glass tube. Passing through *C* were two glass tubes through which passed the iron electrodes which held the cathode *E*. The latter was attached to the iron electrodes with clamps. The glass tubes were fastened to the iron caps *B* and *C* with Khotinsky cement. The above arrangement of apparatus made it possible to renew the platinum cathode and also to vary the distance between the anode and cathode. The central part of the tube was placed in an asbestos gas-heated furnace with a sheet-iron bottom. The collimator of the spectrograph passed through a hole in the furnace and the slit was placed directly against the quartz tube. *B* and *C* were surrounded with copper water jackets and the iron electrodes at *F* were cooled with a stream of air in order to prevent the cement from softening. With this arrangement and with the iron electrodes $1/8''$ in diameter it was found possible to use 20 amperes through the cathode and heat the furnace with a gas flame and not have the cement connections soften even after hours of operation. It should be stated that the diagram shows a horizontal section and that the mercury usually stood directly under the anode and cathode. To prevent the heat from passing along the mercury and melting the cement at *C* or *B* the tube was inclined so that the mercury always stayed in the *C* end of the tube and at *G* there was placed an iron stop which prevented it from reaching the end of the tube. The cathode consisted of a piece of platinum .0025 cm. thick and about 1.5 cm. long. The width varied in different cases from 2 mm. to 4 mm. and the current to heat it from 10 to 20 amperes. The platinum was either used bare or coated with barium oxide. The photographs were taken with a small quartz spectrograph. The slit was directed towards some point between the anode and cathode. The lens was of such size that it gathered light from all points between the anode and cathode if they were placed 1.5 mm. apart.

With the apparatus outlined as above one of the first difficulties encountered was due to the occurrence of the many-lined spectrum. It was found almost impossible to make exposures at voltages used by other experimenters for any considerable length of time without having an arc strike and then the plate showed the many-lined spectrum. In order to obviate the striking of the arc the potential difference between the electrodes was lowered step by step, until finally at a little under 5 volts it was found that it was possible to photograph for hours without the appearance of the many-lined spectrum. It thus appears from these experiments that the many-lined spectrum can be obtained for any potential difference above 5 volts. As the other experimenters, as well as the writer, have obtained the single-lined spectrum above 5 volts we have the interesting fact that both the single-lined and the many-lined spectra of mercury may be obtained above 5 volts. McLennan and Henderson (l.c.) obtained the single-lined spectrum for potential differences as high as 12.5 volts. The writer has been able to obtain it for potential differences as high as 21.5 volts. No attempts have been made, as yet, to obtain it at higher voltages, but there seems to be no reason why it could not be obtained.

The fact that the many-lined spectrum of mercury can be produced for the same potential differences for which the single-lined spectrum has been obtained raises the question as to whether there is such a thing as a true single-lined spectrum of mercury above 5 volts. Or to put it another way can the radiation $\lambda = 2536.72 \text{ \AA. U.}$ be produced above 5 volts without the appearance of any of the other radiations of mercury? The writer restricts the question to a potential difference greater than 5 volts for reasons which will appear later. Further, if either the single-lined spectrum or the many-lined spectrum can be produced independently of one another above 5 volts, what are the conditions necessary in order to bring out either the one or the other? The spectrogram labelled I. in the accompanying plate is submitted relative to the above questions. This spectrogram was taken under the following conditions. The maximum and minimum potential differences between anode and cathode were 9.0 and 8.4 volts. The distance between the anode and cathode was between 2 and 3 mm. The temperature immediately outside of the tube was about 160° C. The slit of the spectrograph was arranged so that part of it received light from between the anode and cathode and part from below them. The time of exposure was 3 hours. That part of the plate which received light from between the anode and cathode shows the many-lined spectrum with the line $\lambda = 2536.72 \text{ \AA. U.}$ relatively stronger than the others, whereas, that

part of the plate which received light from below the anode and cathode shows only the line $\lambda = 2536.72 \text{ \AA. U.}$ Further, the lines other than the line $\lambda = 2536.72 \text{ \AA. U.}$ have blunt ends showing no tapering effect. It is hoped that this appears in the reproduction. In as much as the lines other than the line $\lambda = 2536.72 \text{ \AA. U.}$ show no tapering effect, this spectrogram seems to indicate that the single-lined spectrum can be produced without any of the other radiations being present. The spectrogram also seems to show that the occurrence of the single-lined spectrum or the many-lined spectrum depends upon the density of the electronic discharge. From the bluntness of the lines it would also appear as if there were a minimum density of electronic discharge which was necessary in order to produce the many-lined spectrum. It should be pointed out that no variation in the density of the mercury vapor could be present to explain the above spectrogram.

Another spectrogram bearing on the above questions is shown in II. of the plate. The maximum and minimum potential differences in the case of this spectrogram were 21.5 and 20 volts. Ordinarily such a voltage would produce an intense arc, but in this case the cathode consisted of uncoated platinum which had been heated in a good vacuum for 30 or 40 hours. As a consequence the electronic discharge from it was very small. In no other way did the experiment differ materially from others where strong spectra were obtained. The spectrogram shows only the line $\lambda = 2536.72 \text{ \AA. U.}$

Spectrogram III. also bears on the above questions. It was taken under the following conditions. The maximum and minimum potential differences were 5.5 and 4.7 volts. The cathode was coated with barium oxide. The distance between the anode and cathode was about 3 mm. The temperature immediately outside the tube was about 165° C. The time of exposure was 4 hours. The plate shows the line $\lambda = 2536.72 \text{ \AA. U.}$ very plainly and some of the other stronger lines of the many-lined spectrum faintly. The writer is unable to decide whether the many-lined spectrum in this case came in slowly or whether it was produced by some sudden increase in the density of the electronic discharge.

Although the above spectrograms are conflicting in their evidence, the writer is of the opinion that it is possible to produce the radiation $\lambda = 2536.72 \text{ \AA. U.}$ without any of the rest of the radiation of mercury being present. Further, in order to produce the single-lined spectrum for potential differences greater than 5 volts, it seems to be necessary to keep the density of the electronic discharge below a certain minimum value and the greater the potential difference used the smaller must this minimum value be.

In endeavoring to locate the lowest potential difference at which the many-lined spectrum was produced, it was found possible to obtain the single-lined spectrum for potential differences considerably below the minimum as obtained by the other experimenters. Spectrogram IV., for instance, was obtained with maximum and minimum potential differences of 2.5 and 1.9 volts. The plate in this case was exposed 3 hours. Slight traces of the line were seen on a plate exposed 4 hours with maximum and minimum potential differences of 2.0 and 1.4 volts. These results raise a question relative to whether a potential difference across the electrodes is necessary in order to produce the single-lined spectrum. The writer is not able to answer this question. Whatever may cause the single-lined spectrum, it decreases in intensity very rapidly with decreasing voltage. In order therefore to get it at these low voltages every means for increasing the intensity of radiation must be used. But even after the radiation has been made as intense as possible a point is reached where the fogging of the plate masks the effect looked for. Spectrogram V. shows how intense the radiation can be obtained for maximum and minimum potential differences of 4.1 and 3.5 volts. The plate in this case was only exposed 2 hours. A comparison of spectrograms IV. and V. will give an idea relative to the decrease in intensity of the radiation $\lambda = 2536.72 \text{ \AA. U.}$ with the decrease in voltage.

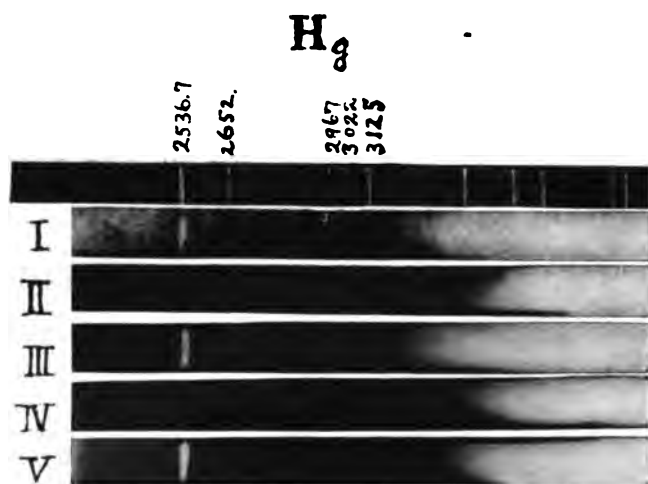
The production of the single-lined spectrum of mercury at such low voltages as recorded above seems to cast doubt on the deductions of Franck and Hertz. These experimenters came to the conclusion that ionization was necessary in order to produce the single-lined spectrum and as ionization occurs at 4.9 volts for mercury they did not believe that the single-lined spectrum could be produced below that voltage and their experimental results seemed to bear out their theory. If ionization is necessary to produce the single-lined spectrum then we must conclude that the electrons leave the hot body with a velocity greater than has been supposed. The velocity of emission would have to be of the order of 2 or 3 volts. Richardson and Brown have determined the velocity of emission from platinum and from platinum coated with an oxide.¹ In the case of platinum they never found the velocity greater than 0.6 volt and in the case of the oxide-coated platinum the greatest velocity obtained was 1.2 volts.

As these results seemed to point to an explanation it was thought worth while to try and measure the velocity of emission right in the tube under observation. For this purpose the anode was connected to the one pair of quadrants of an electrometer with the other pair grounded.

¹ Phil. Mag., Vol. 16, p. 353 (1908).

The negative end of the cathode was joined to the earth through a variable potential difference, so that its potential could be varied with respect to the earth. If the hot cathode gave out only electrons then by raising its potential a value could be reached for which no charge could pass to the electrometer. As is well known both positive and negative particles are given out from a hot body. With the two streams of particles coming from the cathode it was found that by raising the potential of the negative end of the cathode a value was reached which gave no deflection of the electrometer. This means that the positively charged particles forced out by the potential of the cathode just balanced the negatively charged particles which, owing to their initial velocity, overcame this same potential. If the potential of the cathode was raised above this point the electrometer acquired a positive charge and if it was lowered below this point it acquired a negative charge. Were the positive stream to be eliminated then the potential would have to be still higher in order to prevent the electrometer from acquiring a charge. In the case of the writer's apparatus it was found that twenty or thirty hours of heating and pumping were not sufficient to greatly reduce the positive stream. This was probably due to the great amount of matreial from which it was necessary to drive the occluded gases. In only one case was the positive stream reduced to small proportions. This was for the case of pure platinum. As time was limited no effort was made to overcome the difficulties. The general results obtained, however, were these. With pure platinum for both anode and cathode the potential necessary in order to get zero deflection of the electrometer after 20 or 30 hours of heating was about .25 volt. Had there been no positive stream this value would have been raised. This result agrees quite well with that of Richardson and Brown for uncoated platinum. With platinum coated with barium oxide the potential necessary to get zero deflection of the electrometer was very much greater than for pure platinum. After long heating and pumping a value as high as 2.3 volts was obtained. As there was still a large positive stream this value would have to be raised considerably, perhaps to 3 volts, in order to stop the negative stream. To conclude from the above results that the velocity of emission is of the order of 3 volts would, it appears, be unsafe for there are a number of uncertain factors. The results, however, are suggestive and the phenomena need further investigation.

As the velocity obtained for pure platinum could not be much in doubt an attempt was made to find the potential difference for which the single-lined spectrum appeared for uncoated platinum electrodes which had been heated in a vacuum for 30 or 40 hours. It was found, however, that no trace of a line appeared for voltages in the vicinity 4.9.



Even potential differences as high as 20 volts gave only a very weak line after 4 hours' exposure. (See II. of the plate.) This was no doubt due to the smallness of the electronic discharge. It is evident that the question relative to whether ionization produces the single-lined spectrum is still unanswered.

Another result obtained in connection with these experiments which needs explanation is the fact that an arc was made to operate at a potential difference as low as 3.2 volts. The arc was never found to strike at a potential difference lower than 5 volts but after it was once in operation there were times when the maximum voltage across it was as low as 3.2 volts. The spectrum from it then showed the many-lined spectrum of mercury. If a velocity of emission of the order of 2 volts be assumed for the electrons then the arc at these voltages can be explained, otherwise it must be assumed that the arc itself produces the necessary ionization to keep itself in operation. If the velocity of emission of the electrons is of the order of 2 or 3 volts then it appears that the arc should be capable of striking at 2 or 3 volts if ionization is all that is necessary. This result has not been observed.

The experiments outlined in this paper have shown: (1) that the mercury arc can be made to strike at any potential difference greater than 4.7 volts, thus producing the many-lined spectrum; (2) that the arc having been made to strike will operate as low as 3.2 volts, producing the many-lined spectrum. This needs a special explanation for the production of its ions unless it is shown that the velocity of emission is sufficient to account for the discrepancy; (3) that the single-lined spectrum can be obtained for all potential differences between 2.5 and 21.5 volts. In order to explain its production below 5 volts it must be assumed that there is a large velocity of emission or else the idea that ionization is necessary for the emission of light must be abandoned. It may be that ionization is necessary to produce the many-lined spectrum but that a jarring effect is all that is necessary to produce the single-lined spectrum; (4) that whether the many-lined or single-lined spectrum of mercury appears above 5 volts depends upon the density of the electronic discharge.

The writer wishes to express his gratitude to the staff of the Ryerson Physical Laboratory, University of Chicago, for placing at his disposal the facilities for doing these experiments, and especially to Professor Millikan, whose interest in this subject inspired the work.

Since doing the above experiments the writer has become associated with the University of British Columbia and expects to continue this work.

THEORETICAL CONSIDERATIONS RELATING TO THE
SINGLE-LINED AND THE MANY-LINED SPECTRUM
OF MERCURY.

R. A. MILLIKAN.

AT the meeting of the Physical Society of Chicago in November, 1914, I raised a question as to the reality of the so-called one-line spectrum which had seemed to appear in Franck's and Hertz's experiment. I pointed out that if we took the point of view of the pure experimentalist, Wood's demonstration of the enormous absorbing and emitting power of mercury vapor for line of wave-length 2536 Å. along with its transparency for other lines means that with increasing volume excitation the relative intensity of these other lines to that of 2536 must continually increase. Otherwise stated, the opacity of the mercury vapor for line 2536 requires that the light of this wave-length which comes out of a tube come only from the surface layers, while the light of other wave-lengths will come from the whole volume. With weak excitation, therefore, line 2536 might be enormously more intense than the other lines and thus appear on Franck's and Hertz's photographic plates, although the other lines might still be present but merely be too weak to be noticed with the exposures used.

On the other hand, from the point of view of Bohr's theory, a single-line spectrum is not to be expected at least at line 2536. This can be seen from the following considerations: It has long been suspected, and indeed was proved more than a year ago by Tait¹ and again quite recently by Goucher,² that though when negative electrons are driven through mercury vapor there is inelastic impact at 4.9 volts, there is very little ionization at that point or indeed at any potential less than about 10.4 volts when strong ionization sets in. This means from the point of view of Bohr's theory, that at 4.9 volts an electron is merely displaced from one particular orbit to the next outer orbit and when it returns it emits the longest line of the series corresponding to a return to this particular orbit. Line 2536 should therefore be the longest wave-length in a series, as it is in fact, and none of the shorter wave-lengths

¹ PHYS. REV., VII., 686, 1916.² PHYS. REV., VIII., 561, 1916.

of this particular series should appear at 4.9 volts, but the whole series should appear when this electron is knocked entirely out of the atom. This would occur, let us assume, at 10.4 volts, a potential which corresponds, in accordance with the quantum relation, to the convergence wave-length (1188 Å.) of the series of which line 2536 is the long wave-length head. The many-lined spectrum which should suddenly appear at 10.4 volts should then be merely the whole series of which line 2536 is the member of longest wave-length.

But now, since the normal spectrum of mercury actually has many lines of greater wave-length than 2536, these must, according to the Bohr theory, belong to orbits farther out from the nucleus than that producing line 2536 for its member of longest wave-length, and these lines should always be produced when 2536 is produced. For if in the permanent state of mercury vapor some of its atoms have electrons in these outer orbits, they must be knocked entirely out at potentials which should not be greater and may be much less than 5.5 volts. To see how this follows from the Bohr theory let the subscripts 1 and 2 apply to any two adjacent series, such as the K and L series in X-rays. The Bohr theory then states that

$$\nu_{\beta_1} - \nu_{\alpha_1} = \nu_{\alpha_2}.$$

Kossel¹ first published tests of this relation taken from X-ray data. The equation gives also the correct relation between the Lyman ultra-violet series of hydrogen, the Balmer series, and the Paschen infra-red series, since these all fit perfectly the Bohr theory. Even where the simple form of Bohr's theory, which it will be recalled was developed only for a single negative electron rotating about a positive center, does not apply, the foregoing equation should still hold, for it states merely the energy relations which must be satisfied no matter what influences the electrons in the various orbits may exert upon one another.² If then, ν_{∞_1} and ν_{∞_2} represent the frequency limits toward which the series corresponding to two adjacent orbits run, then the energy relations underlying equation 1 may also take the form

$$\nu_{\infty_1} - \nu_{\alpha_1} = \nu_{\alpha_2}. \quad (2)$$

This is merely the well-known Rydberg-Schuster law³ discovered wholly empirically and tested quite generally for metallic spectra, but now seen to be a necessary consequence of the Bohr theory of the atom.

¹ Verh. d. D. Phys. Ges., 16, 953, 1914.

² See Presidential Address American Physical Society on "Radiation and Atomic Structure," *PHYS. REV.*, May, 1917.

³ Bailey's Spectroscopy, p. 488.

Now the series of which the mercury line 2536 is the long wave-length head runs toward the limit $\lambda = 1188 \text{ \AA}$. Applying equation 2, we obtain

$$\frac{1}{1188} - \frac{1}{2536.7} = \frac{1}{x} \therefore x = 2234.$$

This is indeed exactly the short wave-length limit toward which one of the well-known mercury series runs, viz., the series 3125, 2652, 2482, ..., 2234, and according to the Bohr theory it should be the series which corresponds to the orbit just outside that which gives rise to the series of which line 2536 is the member of longest wave-length. The value of $h\nu$ in volts corresponding to this wave-length 2234, is 5.5. If, then, there are electrons in this orbit they should be knocked entirely out at 5.5 volts and the whole series of which line 2234 is the convergence frequency should appear. The longer wave-length members of this series should appear at still lower potentials; if they do not so appear it should be because there are no electrons in these orbits.

But even if in the normal mercury vapor there are no electrons in orbits which are larger than that corresponding to the series whose longest wave-length member is 2536, yet since this line is stimulated it must act photoelectrically, as Van der Bijl¹ has just been urging, on the surrounding atoms, for the photoelectric long wave-length limit of mercury is about 2800 $\text{\AA}$. and *the return of these detached electrons, or of others which take their places, should be accompanied by all lines of series corresponding to outer orbits which, according to our hypothesis, are normally empty.* As a matter of fact the lines which do appear faintly but altogether definitely on both I. and III. of the foregoing plate, though less clearly on the reproduction, are 3125 and 2652 of the above-mentioned series and in addition 2967 and 3022, which, as Prof. F. A. Saunders kindly tells me, are members of the same triplet series; this series being

				Limit.
3650	3022	2804	...	2491
3125	2652	2482	...	2234
2967	2534	2378	...	2149.

In the preceding discussion it has been assumed that the emission of light by mercury is due solely to the partial or complete formation of positive atom-ions. But it is conceivable that the light-emission is in part at least due to the simple stimulation of neutral mercury atoms by corpuscular impact. In this case many lines would be expected to appear simultaneously and at a potential which could scarcely be expected to bear any relation to the value of $h\nu$ for any one of the lines.

¹ Proc. Phys. Soc., Chicago meeting, Dec. 1, 1916.

From whatever point of view, then, I approach the excitation of mercury vapor I can see no reason to expect line 2536 to appear without the lines of longer wave-length. These lines would be expected to be weak if there are few or no electrons in orbits greater than that corresponding to 2536, but they should nevertheless be present. I initiated Dr. Hebb's experiments, therefore, in the hope that they would bring to light these weak lines of longer wave-length than 2536 and accompanying it.

His results seem to me to indicate strongly that these lines are in fact always present along with 2536, their appearance on the photographic plate depending, as is to be expected from Wood's experiments, simply on the volume excitation, or in a given tube, upon the density of the exciting electron stream. Thus in I. of the plate the reason these additional lines appear in the upper half and not in the lower is that, as is well known, the electron stream is a sharply limited bundle outside of which the excitation is too weak to permit the appearance of the weaker lines in the exposure time used. The sharp limits of the cathode beam explain fully the bluntness of the ends, without assuming that a definite density is necessary to stimulate the many-lined spectrum. That these weak lines are present in II. when the P. D. is 20 volts but the electron stream weak, no one, I fancy, would be inclined to doubt.

The experiments on the starting potential of the arc seem to me to demonstrate, first, that this potential depends upon the density of the exciting cathode stream; second, that this potential may be anything above five volts.

Both of these conclusions are in accordance with the foregoing theory. For the striking of an arc merely indicates the rise in the number of positive ions to a certain large value and the consequent concentration of the potential-fall in the neighborhood of the cathode. But with the reciprocal relations which have been postulated between the stimulation of light at 4.9 volts and the photoelectric production of ions by this light, there may exist at any potential in excess of 4.9 volts, the possibility of an indefinite multiplication of positive ions. With a weak cathode stream this possibility will not be realized in the neighborhood of 5 volts, since, in view of the strong absorption by mercury vapor of line 2536 the photoelectrons will be formed close to where the light is stimulated, *i. e.*, close to the anode. The negatives so formed will not then fall through an appreciable potential and hence will not help the original cathode stream in the stimulation of light. The denser the original cathode stream, however, the more positive ions will be formed by the light and consequently the more will the total fall of potential become

concentrated near the cathode, and the sooner will negative photoelectric ions begin to be formed where they can aid the original cathode stream in the stimulation of more light. As soon as this condition ensues there arises the possibility of an indefinite multiplication of positive ions and the formation of an arc.

If the cathode stream is weak then the arc will not appear in the neighborhood of 5 volts. It would, however, be expected to appear at the ionizing potential, namely, at 10.4 volts, since at this point there is in any case a large multiplication of ions. This is where the arc actually did appear in McLennan and Henderson's experiments.¹

It may be mentioned that the fact that no measurable ionization has ever been observed in mercury vapor at potentials lower than 4.9 volts would signify, from the point of view here presented, that when the mercury atom is in its normal, stable condition, the orbit corresponding to line 2536 is the outermost one which contains any appreciable number of its 80 negative electrons.

Whether the foregoing theory is correct or not there can be no doubt that a mercury arc can be maintained at much less than 10.4 volts, for I have myself held one at 3.5 volts, but in this case as in Dr. Hebb's experiments, I suspect that the contact electromotive force was sufficient to raise the actual energy of impact to 4.9 volts. As further evidence it may be mentioned that Rau² observed the appearance of the mercury line 4359 at 8 volts, which is below the value, 10.4 volts, which McLennan found to be the lowest at which any of the lines of the many-lined spectrum could appear.

That the quantum relation holds both at 4.9 volts and at the ionization potential, 10.4, volts is rendered probable by Tait's and by Goucher's experiments, as well as by those of McLennan; for both the observed potential of inelastic impact, and that of ionization seem to agree, within the limits of error of the experiments, with predicted values. The present experiments are not at all at variance with McLennan's but they extend them by showing, first, that an arc may be produced without raising the potential to the ionizing value, and second, that weak lines of longer wave-length than 2536 probably always appear with it. It is only the shorter and shorter wave-lengths *of a given series* which ought to appear successively with successive increases in the applied potential. This prediction is also in line with Rau's results.

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¹ McLennan and Henderson, Proc. Roy. Soc., 91, 485, 1915, and McLennan and Keys, Proc. Roy. Soc., 92, 591, 1916.

² H. Rau, Ber. d. Phys. Med. Ges. Wurzburg, 1914.

ON THE NUCLEAR CHARGES OF ATOMS.¹

BY FERNANDO SANFORD.

IN his second paper on "High Frequency Spectra," published in *Phil. Mag.*, XXVII., 703 (1914), Moseley says: "The frequency of any line in the x -ray spectrum is approximately proportional to $A(N - b)^2$, where A and b are constants and N is the atomic number."

For the α line of the K-radiation Moseley found $b' = 1$, and for the principal line of the L-radiation, $b = 7.4$. Moseley surmised that the significance of N in these equations might be explained on van den Broek's hypothesis that N represents the number of units of positive electricity in the atomic nucleus. He did not show what this unit of positive electricity is nor how the nuclear charge may be computed.

Since the publication of Moseley's work there have been a number of attempts to apply Einstein's photoelectric equation, $\frac{1}{2}mv^2 = eV = \nu h$ to the energy of the cathode rays in a vacuum tube and the frequency of the x -rays which they excite. These attempts have generally been unsuccessful, apparently because the frequency of the x -rays has been measured by their absorption in aluminum. It is apparently for this reason that Barkla decides in his Bakerian Lecture of the present year that the quantum equation does not apply in the case of x -rays.

The recent work of Duane and Hunt,² Albert W. Hull³ and David L. Webster⁴ seems to show that this method of estimating the frequency of x -rays is not reliable, and that the Einstein equation does apply fully to the energy of cathode rays and the frequency of the x -rays which they excite, as well as to the "corpuscular rays" which are emitted by metals when excited by x -rays.

If we accept these conclusions, and if we make the further assumption that x -rays are due to electrons which are revolving in elliptical or circular orbits about a positively charged central nucleus with a rotational frequency equal to the frequency of the x -rays which they emit, we may compute the charge upon the central nucleus and show that it is directly proportional to the square root of the frequency of the x -rays, and hence

¹ Read before the Pacific Physical Society at Berkeley, California, Dec. 2, 1916.

² *PHYS. REV.*, VI, 166 (Aug., 1915).

³ *Ibid.*, VII., 156 (Jan., 1916).

⁴ *Ibid.*, VII., 599 (June, 1916).

that it may also be computed from the Moseley equation by substituting the nuclear charge for the square root of the x -ray frequency.

Thus, from the Einstein equation, $mv^2 = 2h\nu$, and $\nu = mv^2/2h$. If the radiating electrons are moving in circular orbits with revolution frequencies which are equal to the vibration frequencies of the x -rays which they induce, we have for their orbital velocities $v = 2R\pi\nu$, where R is the orbital radius and ν the revolution frequency of the electrons. Accordingly, $\nu = v/2R\pi$. Equating the above values of ν , we have $mv^2/2h = v/2R\pi$ and

$$Rv = h/m\pi. \quad \text{Equation 1.}$$

If we assign the ordinarily accepted values to h and m , viz., $h = 6.55 \cdot 10^{-27}$ and $m = 8.8 \cdot 10^{-28}$, we have $Rv = 2.37$. If we substitute λ , the wave-length of the x -rays, for their frequency in the Einstein equation, we have $mv^2 = 2h \cdot 3 \cdot 10^{10}/\lambda$, and $v^2 = 6h \cdot 10^{10}/m\lambda$. Assigning the values already used to h and m , we have

$$v^2 = 44.7 \cdot 10^{10}/\lambda \quad \text{and} \quad v = 6.69 \cdot 10^5/\sqrt{\lambda}. \quad \text{Equation 2.}$$

If we assume that the revolving electrons are held in their orbits by the attraction of the electro-positive nucleus, we shall have the equation $mv^2/R = Qe/kR^2$, where Q = the positive nuclear charge, e , the negative electronic charge and k the specific inductive capacity within the atom. Since we know nothing of this specific inductive capacity, we can only neglect it in our equation, and we accordingly write $Q = mRv^2/e$.

From equation 1, $R = h/mv\pi$. Substituting this value, $Q = v h/e\pi$. calling $e = 4.77 \cdot 10^{-10}$, $h/e\pi = 4.37 \cdot 10^{-18}$. From Equation 2, $v = 6.69 \cdot 10^5/\sqrt{\lambda}$. Substituting these values in our equation for the value of Q , we have

$$Q = 2.95 \cdot 10^{-12}/\sqrt{\lambda}. \quad \text{Equation 3.}$$

Since Moseley's equations give $\sqrt{\nu} = A(N - 1)$ for the α line of the K-radiation, while for the corresponding line of the L-radiation $\sqrt{\nu} = A(N - 7.4)$, it follows that the same equations, but with different values of A , must apply to the nuclear charges of the atoms which produce these spectral lines.

The nuclear charges for both the α and β lines of the elements whose K-radiation wave-lengths have been measured by Moseley and others are given in Table I. as calculated from the equation $Q = 2.95 \cdot 10^{-12}/\sqrt{\lambda}$ and from the equation $Q = A(N - 1)$, where A has a value of $8.51 \cdot 10^{-10}$ for the α line and $8.9 \cdot 10^{-10}$ for the β line. The wave-lengths used in the computation are taken from Moseley's measurements as given on page 235 of Kaye's "*X-Rays*," from Malmer's paper in *Phil. Mag.*, 28, 787 (1914) and from the abstract of a paper by Siegbahn and Stenström

in Science Abstracts, Vol. 19, No. 1129 (Oct. 25, 1916). The original paper in Phys. Zeits. from which the abstract was taken has not yet reached our library.

TABLE I.
K—Radiation.

Atom.	No.	λ_{α} .	Q_{α} .		λ_{β} .	Q_{β} .	
			$\frac{2.95 \cdot 10^{-18}}{\sqrt{\lambda}}$.	$8.51(N-1)10^{-10}$.		$\frac{2.95 \cdot 10^{-18}}{\sqrt{\lambda}}$.	$8.9(N-1)10^{-10}$.
Na.....	11	11.951	85.3	85.1			
Mg.....	12	9.915	93.7	93.6	9.477	96.5	97.9
Al.....	13	8.362	102	102	7.954	104.7	107
Si.....	14	7.136	110.3	110.6	6.744	113.6	115.7
P.....	15	6.168	119	119.2	5.808	122.5	124.7
S.....	16	5.360	127	127.6	5.018	132	133.5
Cl.....	17	4.730	135.6	136.1	4.394	140.5	142.4
K.....	19	3.747	155	153.3	3.456	159	160
Ca.....	20	3.363	160.6	161.5	3.090	167.5	169
Se.....	21	3.030	169	170	2.778	176.5	178
Ti.....	22	2.751	178	178.6	2.516	186	187
V.....	23	2.509	187	187	2.294	194.5	196
Cr.....	24	2.293	194.5	195.7	2.086	204	204.6
Mn.....	25	2.103	203	204	1.860	216	214
Fe.....	26	1.938	212	212.4	1.756	223	222.5
Co.....	27	1.798	220	221.6	1.629	230.3	231.5
Ni.....	28	1.662	228.4	229.8	1.506	239.7	240
Cu.....	29	1.549	238	238	1.402	250	249
Zn.....	30	1.445	244	246.6	1.306	258.5	258
Y.....	39	0.841	322	323	0.757	339	338
Zr.....	40	0.800	330	332	0.718	348.5	347
Nb.....	41	0.750	340.6	340.5			
Mo.....	42	0.716	349	349	0.631	372	367
Ru.....	44	0.641	368.5	366	0.569	392	383
Pd.....	46	0.587	387.5	383	0.522	408	400
Ag.....	47	0.562	393.5	392	0.499	418	410
Cd.....	48	0.535	403	400	0.471	430	418
In.....	49	0.511	412.5	409	0.448	441	427
Sn.....	50	0.485	424	417	0.427	452	436
Sb.....	51	0.449	441	426	0.396	468	445
Ba.....	56	0.383	476	468	0.334	511	490
La.....	57	0.373	483	477	0.326	517	498

The measurements of Moseley and of Siegbahn and Stenström agree with each other much more closely than either of them agrees with Malmer's measurements. Where more than one observer has measured the wave-length of the same line the means of the different measurements are taken in the table. All the wave-lengths for elements whose atomic numbers are above 47 are exclusively from Malmer's data.

In Table II., the same data are given for the principal line of the L-radiation as are given in Table I. for the two lines of the K-radiation. The wave-lengths used are taken from Moseley's tables and from a paper by Siegbahn and Friman in Phil. Mag., 31, 403 (1916). The values of Q in column five are calculated from the equation $Q = 3.65(N - 7.4) 10^{-10}$.

TABLE II.

L—Radiation.

Atom.	No.	λ .	Q .	
			$\frac{2.95 \cdot 10^{-12}}{\sqrt{\lambda}}$.	$3.65 (N - 7.4) 10^{-10}$.
Zr.....	40	6.091	119.43	118
Nb.....	41	5.749	122.92	122.7
Mo.....	42	5.432	126.6	126.3
Ru.....	44	4.861	133.8	133.6
Rh.....	45	4.662	136.6	137
Pd.....	46	4.385	141.2	141
Ag.....	47	4.170	144.7	144.6
Sn.....	50	3.619	155	155.2
Sb.....	51	3.458	158.6	159
La.....	57	2.676	180	181
Ce.....	58	2.567	184.5	184.5
Pr.....	59	2.471	188	188
Nd.....	60	2.382	191.4	192
Sa.....	62	2.208	199	199
Eu.....	63	2.130	202	203
Gd.....	64	2.057	203.5	206
Ho.....	66	1.914	214	214
Er.....	68	1.790	220	221
Ta.....	73	1.525	238.7	239
W.....	74	1.486	242	242.5
Os.....	76	1.397	250	250
Ir.....	77	1.354	254	254
Pt.....	78	1.316	256	257
Au.....	79	1.287	258.5	261
From the data of Siegbahn and Friman.				
Au.....	79	1.287	261	261
Hg.....	80	1.243	267	265
Tl.....	81	1.209	268	268.4
Pb.....	82	1.179	272	272
Bi.....	83	1.144	276	276
Th.....	90	0.971	302	301
U.....	92	0.919	308	308.4

The relation of the nuclear charges to the atomic numbers which are shown in Tables I. and II. are shown graphically for the K α line in Fig. 1 and for the principal line in the L-radiation in Fig. 2.

It will be seen from the tables, as well as from the formulæ from which

they are computed, that the positive charges within the orbits of the electrons which give the L-radiation are less than those inside the orbits of the electrons which emit the K-radiation in the same atom.

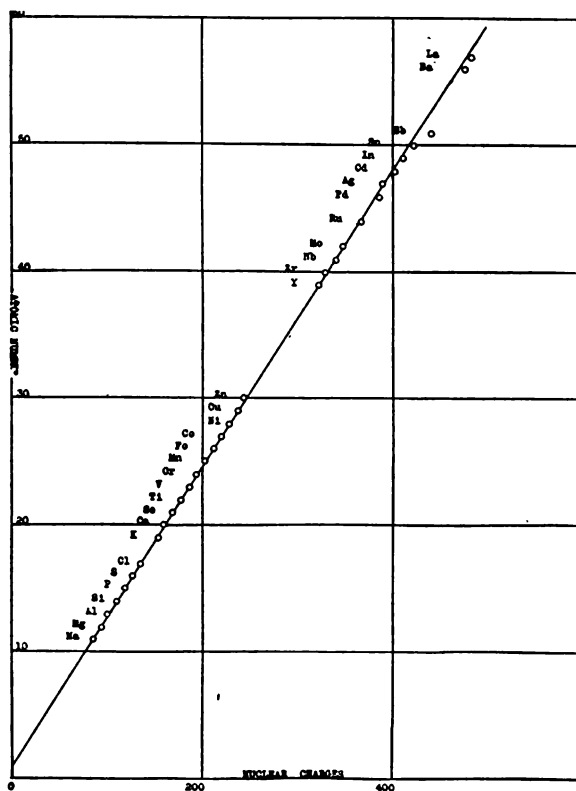


Fig. 1.

This can be explained on the assumption that the electrons which give the K-radiation, and presumably other electrons, lie within the orbits of those which emit the L-radiation. If this be the correct explanation, the difference in the nuclear charges computed from the two kinds of radiation for the same element should be some integral multiple of the electronic charge.

There are nine elements for which both the K and L wave-lengths are given in the table. When the differences of the computed charges for these nine elements are taken, they are found to be in every case very nearly integral multiples of the electronic charge, so that $Q_K - Q_L = e(N + 5)$, where e has a mean value for the nine elements of $4.75 \cdot 10^{-10}$ E.S.U. This relation is shown in Table III.

The orbital radii of the revolving electrons whose wave-lengths are

TABLE III.

Atom.	No.	Q_K .	Q_L .	$Q_K - Q_L$.	$\frac{Q_K - Q_L}{(N + 5)}$.
Zr.....	40	332	118	214	4.76
Nb.....	41	340.5	122.7	217.8	4.74
Mo.....	42	349	126.3	222.7	4.74
Ru.....	44	366	133.6	232.4	4.74
Pd.....	46	383	141	242	4.75
Ag.....	47	392	144.6	247.4	4.76
Sn.....	50	417	155.2	261.8	4.76
Sb.....	51	426	159	267	4.77
La.....	57	477	181	296	4.77

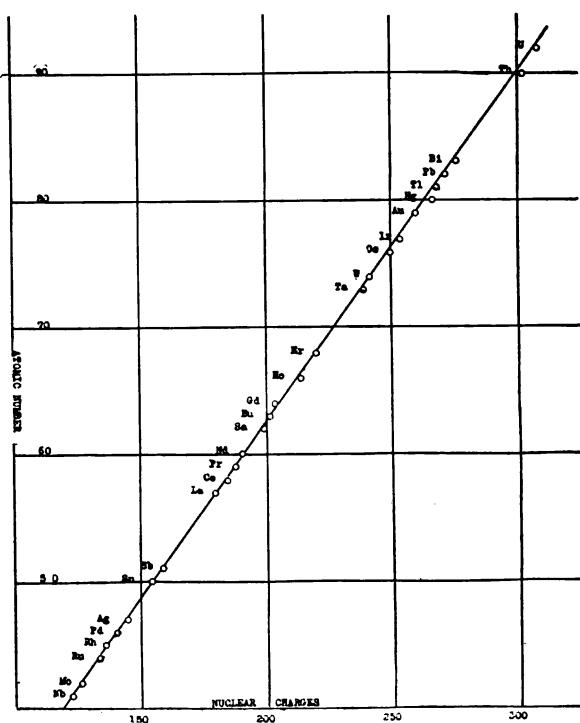


Fig. 2.

known may be calculated from the equation $R = 3.542 \lambda 10^{-6}$. They vary in the case of the K-radiation from $12.25 \cdot 10^{-10}$ for sodium to $2.17 \cdot 10^{-10}$ for lanthanum. It has been shown in a previous paper by the present writer¹ that the atomic radii calculated in this way from the convergence wave-lengths in spectral series give values of the same order of magnitude and of the same relative size as when calculated by other methods.

¹ Astrophysical Journal (Nov., 1916).

It may also be seen that the orbital radii vary inversely as the nuclear charges. If this decrease of the effective positive charge within the electronic orbits with the increase of the orbital radii is due to the number of electrons surrounding the positive nucleus, it would seem to follow that these electrons must be spaced at equal intervals outwards from the center.

In a paper on "The Specific Inductive Capacity of Certain Metals,"¹ the present writer has called attention to the fact that in order that the Einstein equation may apply to a system of electrons revolving in circular orbits at different distances from the center they must be held by centripetal forces which vary as the inverse cubes of their respective distances from this center. It was suggested that this result might be accomplished if the specific inductive capacity of the system increased proportionally as the distance from the center of the system. It may also be accomplished if the negative charges within the electronic orbit increase directly as the radius of the orbit.

Since the β lines of the K-radiation are of shorter wave-length than the α lines, it would seem that they must be used in computing the true nuclear charges of the atoms. If these electrons are really the closest electrons to the central nucleus, and if the nuclear charge may be calculated from the Moseley equation, using Q instead of $\sqrt{\nu}$, then the nuclear charges of all the atoms of higher atomic weight than helium must be integral multiples of the charge on the helium nucleus, and presumably of the charge of an α -particle. It has generally been assumed that the charge of an α -particle has exactly twice the magnitude of the electronic charge, viz. $9.54 \cdot 10^{-10}$ E.S.U. The mean values from the determinations of Rutherford and Geiger and of Regener give $9.4 \cdot 10^{-10}$ E.S.U. Since the nuclear charge which is the constant factor determined from the β lines of the K-radiation is only $8.9 \cdot 10^{-10}$ E.S.U., it is about 5.3 per cent. less than the values given by the above measurements.

If, as some believe, there is a radiation of still shorter wave-length than the K-radiation, its wave-lengths should be used in computing the true nuclear charge, and these would give a larger value for the nuclear charge of helium than do the wave-lengths of the K-radiation. The J-radiation, if it exists, should be looked for in elements of low atomic weight and with cathode rays of higher speed than those required to excite the K-radiation. If the value of A used in computing the nuclear charge from Moseley's equation for the J-radiation should prove to be the estimated value of the charge on the α -particle, a voltage of 1325 should be required to excite the J-radiation in sodium, and the wave-length of the J-radiation for sodium should be $1.45 \cdot 10^{-8}$.

¹ *Phys. Rev.*, VIII., 89 (July, 1916).

Since the radioactive elements in their transformations give off α -particles and electrons and form new elements, the temptation is great to assume that all elements are built up from some one or two small nuclei by the addition of α -particles and electrons. In that event, leaving the electrons entirely out of consideration, the nuclear charges should increase with the atomic weight only about half as fast as they seem to increase. Thus the atomic weight should increase by four units while the nuclear charge increases by $8.9 \cdot 10^{-10}$ E.S.U.

Notwithstanding this difficulty, and whether a J-radiation having a higher nuclear charge than the K-radiation be found, or not, the important relation which Moseley has shown between the frequency of the x -rays and the atomic numbers of the respective atoms are explained in a very simple manner if the radiating electrons are moving in circular orbits about the positive atomic nucleus. The further fact that in the "corpuscular radiation," induced when x -rays are stopped by metals, electrons are driven off with velocities equal to the orbital velocities of the electrons which induce the exciting x -rays seems to suggest the possibility that x -rays are really produced by electrons which are captured for short periods by the positive nuclei when cathode rays are shot through the atoms.

That charges of similar magnitude to those computed here are actually associated with the positive sub-atoms may be shown from other considerations. Attention has been called by the present writer in a number of papers to what he has called the characteristic ionic charges in electrolysis. These charges are calculated on the assumption that the ions which carry charges in electrolysis are free for very short intervals of time, and that their movement in the solution is accelerated throughout their entire free path by the electromotive force which causes the current. This being true, the acceleration of an ion would be proportional to its charge and inversely proportional to its mass. If the ionic mobilities are proportional to these accelerations, which will be the case if the ions of different elements are free for the same time periods when in solutions of the same molecular concentration, the ionic charges will be proportional to the products of the ionic mobilities into the ionic masses.

Assuming the charge on the hydrogen ion to be $5 \cdot 10^{-10}$ E.S.U., charges have been computed for all the ions whose electrolytic mobilities are known. For the alkali metals and the chlorine group of elements these charges are as follows:

When the ionic charges calculated in this way are compared with the nuclear charges calculated from x -ray wave-lengths, they are found to be closely proportional to them within each group of similar elements.

Ion.	Charge $\times 10^{10}$.	Ion.	Charge $\times 10^{10}$.
Na.....	15.7	F.....	14.6
K.....	39.6	Cl.....	36.5
Rb.....	91.2	Br.....	84.9
Cs.....	142	I.....	132.5

Thus the nuclear charges for the L-radiation as calculated from the formula $Q = 3.65(N - 7.4)$ may also be calculated for Na, K, Rb and Cs from the equation $Q = 1.22(E - 5)$, where Q represents the nuclear charges and E the ionic charges in electrolysis. For the elements F, Cl, Br and I, the formula is $Q = 1.362(E - 10.5)$. These relations are shown graphically in Figs. 3 and 4. A similar relation exists in the case

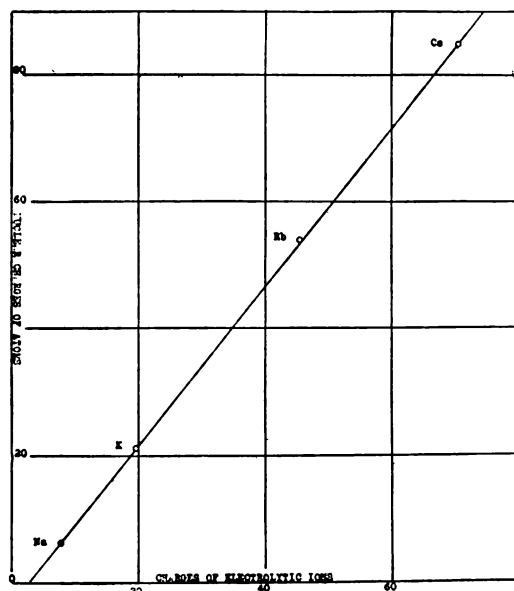


Fig. 3.

of the elements Mg, Ca, Sr and Ba. Thus the ionic mobility of these elements may be calculated from the wave-lengths of their x-ray spectrum as accurately as from electrolysis.

The charges on the positive ions given off by hot metals have been calculated from Richardson and Hulbirt's values of e/m given in Phil. Mag., XX., 545 (1910), and by other methods. These values for the charges of Ni, Pd and Pt have been taken as follows: Ni, 51.4; Pd, 88.9; Pt, 157. They may be used to calculate the nuclear charges in the L-radiation from the formula $Q = 1.7(E - 5)$ to within one per cent. of the above values. This relation is shown graphically in Fig. 5.

Since practically all the properties of the elements which depend upon cohesion have been shown to be closely related to the ionic charges cal-

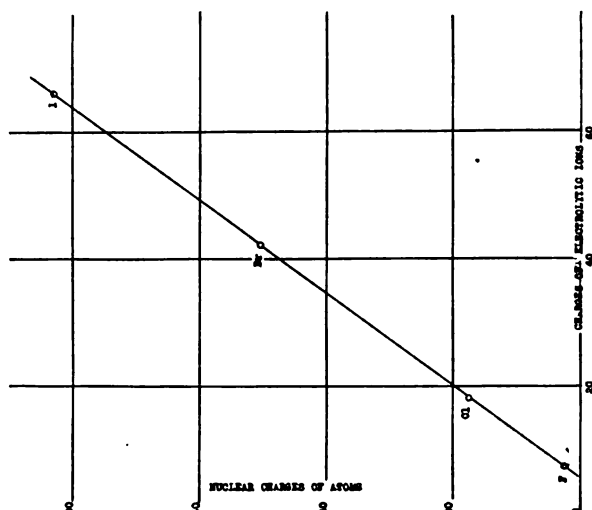


Fig. 4.

culated from electrolysis, and since it is now shown that the characteristic x -rays given off by the atoms may also have their wave-lengths

computed from the mobilities of their ions in electrolysis, it seems to be definitely established that these charges are the most fundamental characteristics of the elements which have yet been observed. Conversely, the evidence of the relation between the ionic charges and the nuclear charges calculated on the assumption that the x -radiation is produced by electrons revolving in orbits about a central positive nucleus seems to make this assumption very probable.

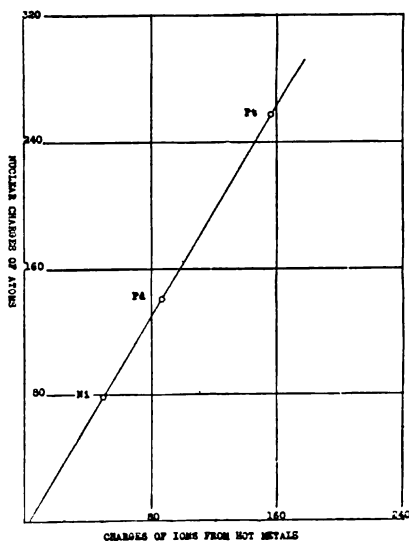


Fig. 5.

SUMMARY.

Assuming, as seems very definitely established, that the Einstein photoelectric equation $Ve = h\nu$, applies to characteristic x -rays and their exciting cathode rays, it is shown that if the electrons which emit the

α -radiation are revolving in orbits about a central positive nucleus, the charge of this nucleus and the orbital radius of the revolving electron may be calculated from equations of the same form as Moseley's equations for the X-ray frequency.

The nuclear charges, when computed, are found to be integral multiples of a primary charge, which in the case of the K-radiation is the nuclear charge of helium, and which for the β lines of the K-radiation has a value of $8.9 \cdot 10^{-10}$ E.S.U.

The van den Broek hypothesis is sustained to the extent that the nuclear charge is found to increase with the atomic number.

It has previously been shown that the orbital radius calculated in this manner from the convergence wave-lengths in known spectral series agrees very closely with the atomic radius calculated from other considerations.

In the case of the nine elements for which both the K and L characteristic radiations are known, it is shown that the nuclear charges calculated from the two classes of radiation differ in every case by an integral number of electronic charges, so that this difference may be calculated from the equations $Q_K - Q_L = e(N + 5)$.

It is shown that the nuclear charges calculated from α -ray wave-lengths may also be calculated, within groups of similar elements, from the ionic charges calculated from the mobility of the ions in electrolysis and from the charges of positive ions given off by hot metals. It is thus possible to calculate the ionic mobilities from the α -ray wave-lengths, and *vice versa*.

STANFORD UNIVERSITY,
November 25, 1916.

THE MAGNETIC PROPERTIES OF IRON, NICKEL AND COBALT ABOVE THE CURIE POINT, AND KEESON'S QUANTUM THEORY OF MAGNETISM.

BY EARLE M. TERRY.

IN 1905, Langevin¹ proposed a kinetic theory of magnetism by means of which he was able to account in a remarkable way for many of the thermo-magnetic phenomena of both para- and dia-magnetic bodies. Assuming that the molecules of a paramagnetic substance are, as in a gas, free to orient themselves in all directions and that, with respect to an external field, the distribution of directions of their magnetic axes under the influence of thermal agitation follows the Maxwell-Boltzmann law, he derived, by purely statistical methods, the law previously obtained experimentally by Curie,² that the susceptibility is inversely proportional to the absolute temperature.

Introducing the idea of an internal magnetic field proportional to the intensity of magnetization, Weiss³ has extended the method of Langevin to ferro-magnetic substances, and obtained a relation between intensity and temperature, which, when put in the form of corresponding states, represents the experimental facts with a fair degree of accuracy. He has further shown that in the region of "induced ferro-magnetism," *i. e.*, in the temperature interval immediately above the magnetic transformation point, called the "Curie Point," the susceptibility should follow a modified form of Curie's Law, namely, inversely proportional to the excess of temperature above the transformation point where the constant of proportionality is the same as would have held for the substance, if, by suppression of the mutual actions between molecules, it had remained paramagnetic.

As a result of their study of magnetic properties at very low temperatures, Weiss and Kammerlingh Onnes⁴ found that the saturation values of the magnetic moments per gram atom for Fe, Ni, and Co are to one another in simple ratios; namely, 11 : 3 : 9. To the aliquot part common to the magnetic moment of the gram atoms, Weiss has given

¹ Langevin, *Ann. de chem. et phys.* (8), 5, p. 70, 1905.

² Curie, *Oeuvres*, p. 255, 1908.

³ Weiss, *Arch. des Sci. phys. et nat.* (4), Vol. 31, p. 5, 1911.

⁴ Weiss and Kammerlingh Onnes, *Comm. phys. Lab. Leyden*, No. 114, p. 3, 1910.

the name of gram-magneton and this quantity, divided by the Avogadro number, gives the moment of the ultimate magnetic corpuscle which he has called the magneton. This he supposes, for magnetism, to play a rôle analogous to the electron for electricity. His theory further shows that the number of magnetons per molecule is related to the Curie constant in a simple manner, so that it is easy to test the existence of the magneton by determining whether or not the number per molecule is an integer. Using the results of Pascal,¹ Weiss and Foëx,² Leibknecht and Wills,³ Miss E. Feytis⁴ and others, he found that the ferromagnetic substances at high temperatures and their salts at ordinary temperatures give, for the number of magnetons per molecule, numbers which rarely depart from integers by more than one or two per cent. Similar results were obtained for a large number of alloys of iron, nickel and cobalt.

On the other hand, Honda⁵ and his co-workers, after examining a large number of substances, both elements and compounds, have amassed a large amount of data showing that Curie's law is not a general law of nature and that the existence of the magneton, at least as based on the simple theory of Weiss, is open to serious question.

In the cryogenic laboratory at Leyden, Kammerlingh Onnes⁶ and his co-workers have extended their thermo-magnetic studies to very low temperatures with a view to testing the validity of Curie's law in this region and have found that, as regards their departures therefrom, substances may be divided into three general groups:

1. The relation, $\chi T = \text{constant}$, holds down to the temperatures of liquid nitrogen, but below this, χT continually decreases.
2. The product χT diminishes at once starting from room temperature. The relation $(T + \Delta) = \text{constant}$ holds with fair accuracy as far as liquid nitrogen temperatures but below this, unsystematic deviations appear.
3. Susceptibility changes very little with temperature and, in the interval between liquid nitrogen, and liquid hydrogen, is almost independent of temperature.

Kunz⁷ has shown that the elementary magnet in the Weiss-Langevin theory is not the magneton mentioned above but is a much larger aggregate consisting of a definite number of atoms: *e. g.*, Fe, 2; and Ni, 6.

¹ Pascal, *Ann. de Chim. et Phys.*, Vol. XVI., p. 531, 1909.

² Weiss and Foëx, *Archiv. des Sci. phys. et nat.*, Vol. XXXI., 1911, p. 4 and 89.

³ Liebknecht and Wills, *Ann. des Phys.*, I., p. 178, 1900.

⁴ Miss E. Feytis, *Comptes rendus*, Vol. CLII., p. 708; 1911.

⁵ Honda, *Science Reports Tohoku Univ.*, Vol. 4, p. 215, 1915.

⁶ K. Onnes, Perrier, *Oosterhuis, Leyden Comm.* No. 5, 116, 122a, 129b, 132e, 134d.

⁷ Kunz, *Phys. Rev.*, Vol. XXX., p. 359; 1910.

Stifler¹ has studied cobalt and found, for the atoms per elementary magnet, four.

The classical work of Curie was carried out at a time when the subject of pyrometry was in its infancy and his temperature scale is in error by at least twenty degrees. Weiss and Foëx could not extend their work to melting point temperatures of the ferro-magnetic substances, because oxidation and errors from the effects of the hydrogen atmosphere which filled their furnace chamber make their results open to question. The same criticisms may be made of the work of Honda whose experimental equipment was quite similar except that he used an atmosphere of nitrogen. Moreover, in the work of Weiss and Foëx, certain deviations from Curie's law are apparent in the case of iron, nickel, and cobalt. Since it is of extreme importance, from the standpoint of the magneton theory, to have accurate information regarding the thermo-magnetic behavior of these cardinal magnetic substances, the present investigation was undertaken with these objects in view:

1. To extend the work up to the melting point.
2. To eliminate errors due to effects of oxidation by working in a good vacuum.
3. To eliminate the effects of occluded gases by melting the specimens in a vacuum before testing.
4. To improve previous temperature measurements.

Method of Measurement.—The method which seemed best adapted for the purpose is that of Faraday in which the magnetic susceptibility is deduced from the mechanical force exerted upon a specimen in a non-uniform field. For an object of small dimensions placed in the plane of symmetry between the poles of a magnet, the force F in a direction X at right angles to the field H is given by the expression

$$F = m\chi H \frac{dH}{dx}, \quad (1)$$

where m is the mass of the specimen, and χ its susceptibility referred to unit mass. If this is to be an absolute method, it is obviously necessary to determine both the field strength and its gradient over the volume occupied by the specimen. However, if a body of known susceptibility is available, the product $H(dH/dx)$ may be determined for a particular place in the field by substituting it for the test specimen and measuring the force upon it. This method was followed in the present work.

The force F was measured by means of a modified form of the Curie balance as shown in Fig. 1. The specimen m was supported at the desired

¹ Stifler, *PHYS. REV.*, Vol. XXXIII., p. 268; 1911.

point in the field by means of a slender porcelain tube carried by an aluminum arm and counterpoised by an adjustable threaded lead nut *W*. When the magnet *NS* is energized, the specimen is urged toward the stronger part of the field and the couple due to this force is counter-

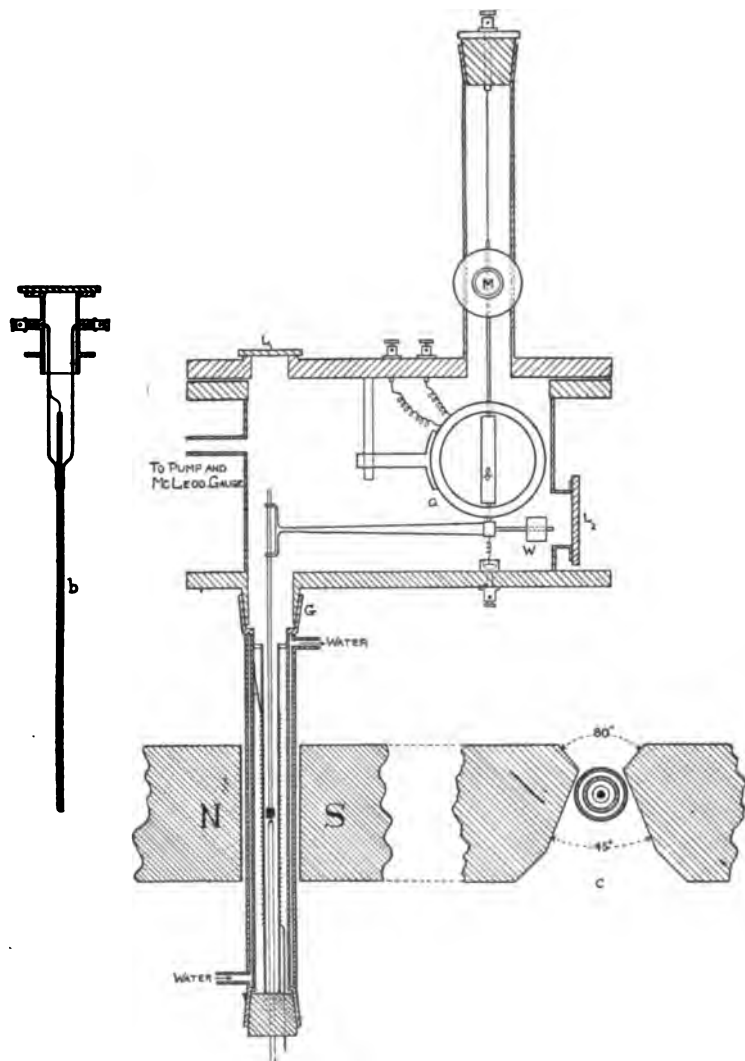


Fig. 1.

balanced by the electrodynamic action between the currents supplied to the coils *a* and *b*, the former of which is fixed while the latter is mounted on the moving system. A spot of light focused by the concave mirror *M* on a distant scale indicates when a balance has been obtained. The

substitution of the coils and optical lever for the torsion fiber and divided head constitutes the principle modification of the original Curie balance. Since the torque is proportional to the current in each coil, this form of instrument may be used for large ranges of forces and has the advantages of high sensitivity and easy control when mounted within a vacuum chamber. Current is led to the moving coil by means of the suspending phosphor bronze ribbon, and away by a small copper wire dipping into a mercury cup below.

Experimental Details.—The furnace consisted of a tube of Berlin porcelain 8 mm. internal diameter and 20 cm. in length closely wound over a distance of 15 cm. with a .5 mm. tungsten wire. Outside of this tube there was placed another porcelain tube, not shown in the figure, and the whole mounted inside a water-cooled jacket. This container was supported from the bottom of the vacuum chamber by means of the ground joint *G*. An input of 750 watts sufficed for a temperature of 1500° C.

Temperature was measured by means of an Heraeus platinum-platinum-rhodium thermo couple supported by a porcelain tube mounted upon the plug which closed the lower end of the furnace chamber. The junction was held immediately below the specimen. In its calibration, the following melting points were used: Cadmium, 320.9° C.; antimony, 630° C.; gold, 1062° C.; and nickel¹ 1452° C. In using the first two of these, the test specimen was swung to one side of the furnace and a tiny crucible of pure magnesia containing a small particle of the melt specimen was lowered through the window *L*₁, till it hung beside the test specimen. A vacuum was then obtained and the temperature gradually raised. The specimen was illuminated by light from a nearby arc reflected into the furnace by means of a prism placed upon *L*₁ and observed by a telemicroscope. The gold and nickel points were obtained by means of the apparatus shown in *b*, Fig. 1. The melt specimen, in the form of a narrow strip of foil, was supported by platinum wires at the end of a slender porcelain tube inserted through *L*₁ and made tight by means of a ground joint. The foil was used as a fuse in an electric circuit containing a relay which actuated a buzzer when the melt occurred. It is to be observed that the thermocouple was thus calibrated in the same furnace and under exactly the same conditions as when used for the measurement of temperature.

In the early part of the work, a tungsten-iron couple was used with good results. By properly adjusting the current and voltage, it is possible by forming an arc between them, to weld tungsten and iron wires in the

¹ Burgess and Waltenberg, Bulletin Bureau Standards, Vol. 10, p. 79, 1914, have shown that nickel possesses a very sharp melting point and, for work in a vacuum, it furnishes a very satisfactory standard temperature.

air. Such a thermo element has an inversion point near 300° C. and gives a thermoelectric power about twice that of platinum platinum-rhodium. It is satisfactory for work in a vacuum or hydrogen.

The magnet employed was a powerful one weighing 800 lbs. and required 4 K. W. for full excitation. It had pole pieces 10 cm. in diameter and was mounted upon a carriage which rested upon two rows of steel balls running in grooves. It could thus be easily moved to and from the test specimen by means of a slow motion screw operated from the observation table two meters distant. Since the pull upon the specimen depends upon its location in the field, it is necessary to work with it always at the same position. The point selected for this purpose was the one for which the pull is greatest, since it is easy to find and gives maximum sensitivity. As the force is proportional to both the magnitude and gradient of the field, it is desirable to have their product constant over as large an interval as possible to minimize the error due to variations in location of the specimen. By giving the pole faces the peculiar shape shown in C, Fig. 1, it is possible to obtain an X - F curve with a very broad peak. A careful study of the particular field used in this work, carried out by means of a small travelling flip coil, showed that the slope of the H^2 - X curve varied by less than one per cent. over a distance of 1 cm. in the neighborhood of the position of maximum pull. Since the specimens used never exceeded 4 mm. in diameter, the error due to faulty location is very small. The method followed in each case was to energize the magnet at the normal working field, usually 9,000 gauss, and measure the force for a series of magnet positions on each side of the position desired and from the curve determine the point of maximum F .

The vacuum was obtained by a molecular pump and read on a McLeod gauge. Since the porcelain furnace tubes occlude large quantities of air and water vapor, it was necessary to bake them out at low temperatures for a long time, *e. g.*, several days. By taking proper precautions in this respect, it was possible to operate at the melting point of iron with a pressure which seldom exceeded .0005 mm. of Hg. The pump was run continuously during all measurements. Two dishes containing P_2O_5 were placed within the vacuum chamber near the opening to the furnace—a precaution absolutely essential to the life of a tungsten furnace and the constancy of a platinum thermo-couple used within it.

Current was supplied to both the magnet and furnace from storage batteries and controlled by water-cooled rheostats, which made possible the maintenance of very constant fields and temperatures.

Standardization of the Curie Balance.—As standard substances, four solutions, nickel nitrate, nickel sulphate, cobalt nitrate, and cobalt

sulphate, were used, whose susceptibilities were determined by the Quincke absolute method. For this purpose, the magnet was placed with its yoke in a vertical position and pole tips of 4 cm. diameter attached with a gap of 6 mm. The field strength was measured by three small independent flip coils with different numbers of turns and different diameters wound on carefully turned vulcanite spools. Their diameters were measured by means of a travelling microscope. The ballistic galvanometer was standardized in terms of a mutual inductance wound on a marble spool and certified by the U. S. Bureau of Standards. The change in level of the solution was read by a microscope travelling vertically with the apparatus so arranged that the capillary tube in which the liquid under test was placed travelled with it, which enabled the readings to be taken always through the same spot in the tube to eliminate errors due to unequal capillarity. A jacket through which water of constant temperature circulated surrounded the capillary tube and maintained the temperature of the solution under test invariable. The outstanding discrepancies in the constant of the Curie balance, as determined by these four different solutions, never exceeded one half of one per cent.

The Test Specimens.—The iron used in this investigation was Burgess electrolytic and was part of the same deposit used in a previous investigation¹ and is known to have a purity, as regards other metals, of 99.98 per cent. The nickel and cobalt were obtained from Kahlbaum in the form of powders guaranteed to be cobalt and nickel free respectively. All specimens were freed from occluded gases by melting in vacuo in crucibles cut from plates of pure magnesia from the Berlin Porcelain Factory. This is believed to be a very important precaution, as a large evolution of gas at the melting point is evidenced by a vigorous boiling which lasts for several seconds and then subsides. In fact, experience has shown that it is quite impossible to obtain consistent results at high temperatures unless gases are thus eliminated. This has also been noted by Burgess² in his work on the temperatures of the Ar_2 and Ar_3 points for iron. Several samples of electrolytic iron melted in vacuo prepared by Yensen³ were also tested.

Results.—The results for beta iron are given in Table I. and shown graphically in Fig. 2.

It is to be noted that iron in this region obeys Curie's law only approximately, for, when the reciprocal of susceptibility is plotted against

¹ Terry, *Phys. Rev.*, Vol. XXX., p. 133.

² Burgess, *Bull. Bur. Stds.*, Vol. 10, p. 315, 1914.

³ Yensen, *Bulletin, Univ. of Ill.*, No. 72.

temperature, instead of a straight line, a curve with a marked upward concavity is obtained.

TABLE I.

Beta iron. Susceptibility as a function of temperature.

Temp.	$\chi \cdot 10^{-6}$	$\frac{1}{\chi} \cdot 10^4$	Temp.	$\chi \cdot 10^{-6}$	$\frac{1}{\chi} \cdot 10^4$
796	1,514	.0661	860	379.0	.2638
806	1,111	.0900	874	327.2	.3056
815	862.5	.1160	878	297.6	.3360
826	701	.1426	889	267.5	.3739
840	532	.1879	896	241.4	.4141
849	456	.2193	907	205.4	.4861
854	427	.2342	916	180.7	.5534

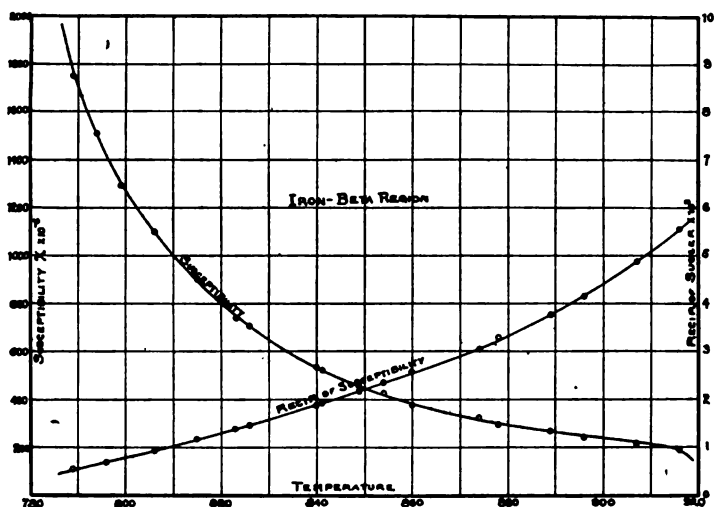


Fig. 2.

Electrolytic Iron Melted in Vacuo, Beta Region.

The results for iron in the gamma and delta regions are given in Table II. and plotted in Fig. 3.

It is to be noted that in the gamma region, Curie's law is not even approximately obeyed. We have here an interval of nearly five hundred degrees throughout which the susceptibility remains almost constant. At 1407° C. there is an abrupt increase when the transformation to the delta form occurs. Throughout the delta region, which extends to the melting point (1530° C.), the susceptibility decreases approximately linearly with the temperature. Electrolytic iron, as deposited, shows slightly greater values of susceptibility in the gamma and smaller in the delta regions than the same iron after being melted in vacuo. The

melting point is less sharply marked also, for fusion apparently begins at 1500° C. and extends over an interval of about twenty-five degrees. These specimens consistently clung to the supporting porcelain rod after melting so that observations on molten iron were obtained over a small interval. The susceptibility shows a marked decrease on melting and

TABLE II.
Iron. Gamma and Delta Regions.

Temp.	$\chi \cdot 10^{-6}$	Temp.	$\chi \cdot 10^{-6}$	Temp.	$\chi \cdot 10^{-6}$
929	26.95	1197	25.54	1409	37.60
961	26.81	1224	25.37	1412	37.96
991	26.71	1245	25.19	1417	37.82
1026	26.40	1285	24.98	1425	37.69
1056	26.38	1343	24.58	1432	37.20
1071	26.32	1359	24.55	1442	36.72
1086	26.20	1370	24.39	1464	35.44
1109	25.96	1374	24.30	1483	34.38
1125	25.93	1400	24.40	1502	33.04
1145	25.87	1400	24.83	1515	31.93
1151	25.85	1400	29.87	1532	30.49
1164	25.76	1403	35.58		

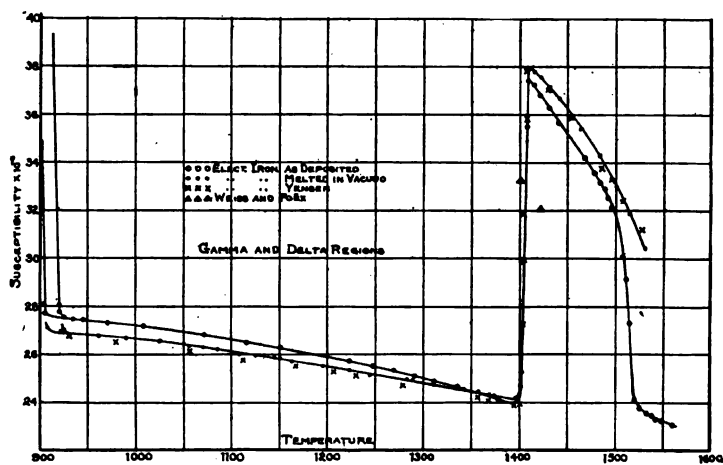


Fig. 3.

Gamma and delta iron. Susceptibility as a function of temperature.

has values approximately corresponding to an extension of the curve for the gamma region. Iron which had previously been melted in vacuum showed a sharp melting point and always dropped from the support.

A special study was made of the transformation from the beta to gamma and gamma to delta regions, the results of which are shown in Fig. 4.

These were carried out by causing the temperature to change very slowly while susceptibility changes were followed closely. As water cooled rheostats were used and the furnace and magnet currents were supplied by storage batteries, it was possible to control conditions very accurately. The rate of change of temperature seldom exceeded 10

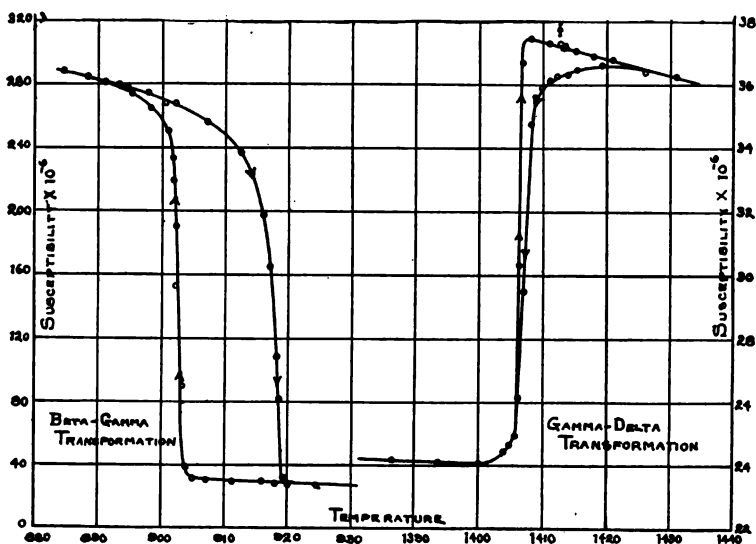


Fig. 4.

Beta-gamma and gamma-delta transformation in iron.

degrees per half hour in the neighborhood of a transformation point. To insure accurate temperature readings, the thermo couple was calibrated during each run. For example, at the beta-gamma transformation, a gold melt specimen was introduced beside the test specimen as explained above. The susceptibility curve for rising temperature was then obtained and extended up to the melting point of gold (1062° C.), after which the curve for decreasing temperature was taken. For the gamma-delta transformation, the thermo couple was checked at the nickel point in the same manner.

It is to be observed that these transformations are of entirely different characters magnetically. For the former, there is a marked hysteresis. The steepest part of the curve occurs at 918° C. on rising and at 903° C. on falling temperatures. For the latter, no real hysteresis can be detected. The steep part of the curve on falling coincides with that on rising within the limits of experimental accuracy. A marked peculiarity, however, is the fact that on cooling the return to the gamma state is not abrupt but begins rather gradually. This peculiarity was always found,

even though the specimen had been heated to within a few degrees of the melting point. The results given here for the beta-gamma transformation correspond qualitatively to those obtained by Burgess¹ for the heat changes. He gives for the temperature at which maximum rates of absorption and evolution occur 898° C. and 909° C. respectively.

The results for nickel are given in Table III. and plotted in Fig. 5. Of the three metals studied, nickel alone shows a tendency to obey Curie's law, but for limited regions only. Starting at the transformation point,

TABLE III.

Temp.	$\chi \cdot 10^{-6}$	$\frac{I}{\chi} \cdot 10^6$	Temp.	$\chi \cdot 10^{-6}$	$\frac{I}{\chi} \cdot 10^6$
381	820.	.1219	782	13.06	7.650
383	723.	.1383	807	12.53	7.980
385	678.	.1478	820	11.72	8.530
387	611.	.1638	849	11.12	9.000
391	498.	.2008	866	10.61	9.425
393	377.	.2653	890	10.13	9.870
403	283.	.3530	924	9.495	10.53
411	219.	.4570	961	8.845	11.31
416	183.7	.5460	1010	8.214	12.18
420	177.2	.5645	1033	7.842	12.75
423	149.2	.6700	1051	7.734	12.93
435	126.7	.7890	1074	7.550	13.24
445	102.3	.9800	1099	7.318	13.66
467	74.90	1.334	1107	7.275	13.74
491.5	56.85	1.760	1147	6.968	14.35
508	47.90	2.046	1172	6.774	14.76
519	44.00	2.274	1187	6.730	14.86
556	32.56	3.068	1217	6.532	15.31
562	31.70	3.155	1242	6.388	15.64
612	23.98	4.170	1261	6.304	15.86
674	18.88	5.390	1295	6.110	16.37
687	17.57	5.690	1329	5.968	16.75
723	15.43	6.480	1357	5.854	17.07
747	14.51	6.890	1378	5.752	17.38
755	13.94	7.175	1407	5.645	17.70
777	13.52	7.390	1431	5.540	18.04

370° C., the curve for the reciprocal of susceptibility shows a marked concavity upward till about 500° C. when there is an interval of 400° for which it is remarkably straight. Between 900° C. and 1000° C. it curves downward after which it continues straight up through the melting point. On this basis, there appear to be three distinct regions of beta nickel. It is interesting to observe that on passing through the melting point (1452° C.) there is no change whatever in the magnetic behavior.

¹ Burgess, Bulletin, Bureau of Standards, Vol. 10, p. 315, 1914.

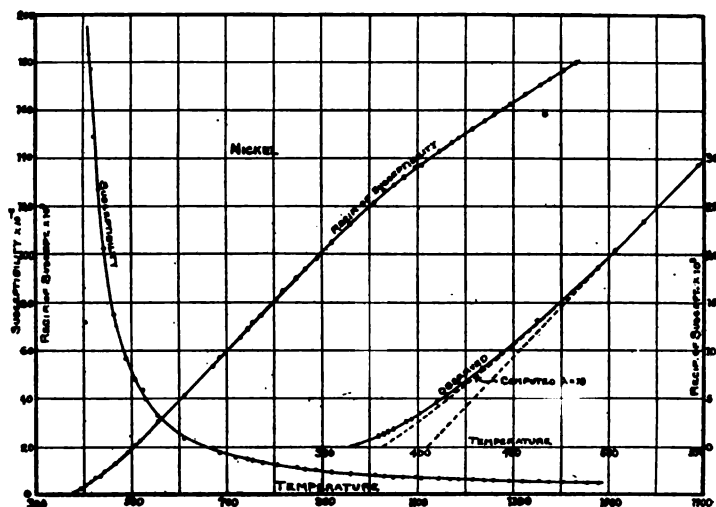


Fig. 5.

Susceptibility as a function of temperature for nickel.

The results for cobalt are given in Table IV. and plotted in Fig. 6. Beta cobalt is similar in its magnetic behavior to beta iron. Starting

TABLE IV.

Temp.	$\chi \cdot 10^{-4}$	$\frac{1}{\chi} \cdot 10^4$	Temp.	$\chi \cdot 10^{-4}$	$\frac{1}{\chi} \cdot 10^4$
1100	6010	.01663	1450	54.48	1.836
1144	777	.1287	1479	48.02	2.167
1178	403	.2482	1490	35.42	2.824
1206	294.5	.3395	1517	32.10	3.130
1237	224.7	.4450	1528	29.10	3.436
1267	161.2	.6205	1533	25.95	3.854
1296	133.2	.7508			
1327	104.7	.9615			
1331	102.0	.9800			
1362	88.4	1.131			
1396	72.08	1.387			
1416	63.92	1.564			

with the relatively high transformation point of 1100° C. the curve for the reciprocal of susceptibility has a gradual upward curvature throughout the entire region up to the melting point (1485° C.). At this point, the susceptibility curve shows a slight knick, continuing regular, however, as far as the observations were carried.

Discussion of Results.—These results are in marked contrast to those

obtained by Weiss and Foëx,¹ who found Curie's law to hold in all cases. For beta iron, cobalt, and nickel at low temperatures they found, however, that the points for the reciprocal of susceptibility against temperature do not lie on a single straight line but on two straight lines with well defined points of intersection; *e. g.*, 828° C. for iron, 412° C. for nickel, and 1241° C. for cobalt, and they accordingly speak of beta₁ and beta₂ regions. My experiments have been repeated a great many times and in no case have I ever found evidence of two straight lines but always a smooth curve. For gamma iron, their departures from Curie's law were

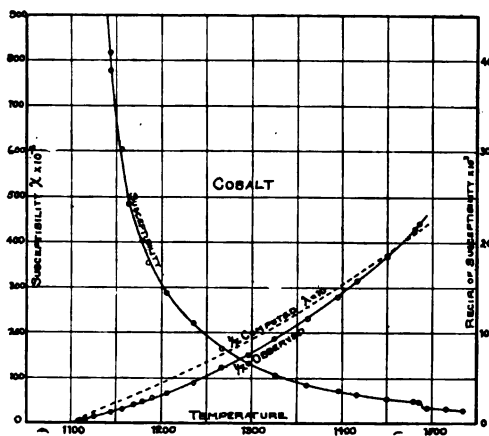


Fig. 6.

Susceptibility of cobalt as a function of temperature.

greatest but there seemed to be a tendency to obey it while my results show that not only is Curie's law not obeyed but the small curvature in the susceptibility plot is in the opposite direction. Delta iron was beyond the temperature range of their apparatus and they were unable to determine the shape of the curve in this region. The transformation temperature which I have obtained, 1407° C., is eight degrees higher than the value given by them. It is obviously then impossible to deduce a Curie constant for these metals or assign a definite number of magnetons to the atom according to the Weiss theory.

Keesom's Quantum Theory of Magnetism.—Keesom,² following the method of Debye³ for specific heats, has introduced the quantum theory into the theory of magnetism and obtained an expression for the variation of susceptibility with temperature which, while it does not represent the facts accurately, gives a much better agreement than any other theory thus far proposed. He supposes that the amount of thermal energy which the molecules possess at a given temperature T is to be obtained from a consideration of the equilibrium of a system of rotating molecules with black radiation at that temperature and assumes that Planck's formula, with the addition of a zero point energy, gives the rotational energy in the same manner, as for a linear oscillator. He supposes

¹ Weiss and Foëx, *Arc. des Sc. Phys. et Nat.*, Vol. XXXI., p. 89, 1911.

² Communications, *Phys. Lab. Leyden*. Sup. No. 32.

³ Debye, *Ann. der Phys.*, Vol. 39 (4), p. 789, 1912.

further that the molecular rotational energy in a substance can be resolved into a system of stationary waves with a finite number of frequencies and that the high frequency limit is obtained by putting the total number of modes of vibration equal to the number of degrees of freedom, and that the velocity of propagation for each mode of vibration is proportional to the square root of its energy. Taking Planck's equation for the energy of rotation of a single molecule of frequency ν , where by frequency is understood the number of rotations per second,

$$U_{rv} = \frac{1}{2} \left\{ \frac{h\nu}{e^{h\nu/KT} - 1} + \frac{1}{2}h\nu \right\} \quad (2)$$

we have for the mean rotational energy for two degrees of freedom about two equivalent axes

$$U_r = \frac{3}{\nu_{\max}^3} \int_0^{\nu_{\max}} \left\{ \frac{h\nu}{e^{h\nu/KT} - 1} + \frac{1}{2}h\nu \right\} \nu^2 d\nu. \quad (3)$$

Introducing the auxiliary variables

$$\xi = \frac{h\nu}{KT}, \quad x = \frac{h\nu_{\max}}{KT}, \quad \theta = \frac{h\nu_{\max}}{K}, \quad \theta_0 = \frac{h\nu_{\max}^0}{K}, \quad (4)$$

the following equations are obtained which determine U_r as a function of T .

$$\left(\frac{U_r}{U_{r_0}} \right)^{\frac{1}{2}} = 1 + \frac{8}{x^4} \int_0^x \frac{\xi d\xi}{e^\xi - 1}, \quad (5)$$

$$\frac{T}{\theta_0} = \frac{1}{x} \left(\frac{U_r}{U_{r_0}} \right)^{\frac{1}{2}}, \quad (6)$$

where

$$U_{r_0} = \frac{3}{8} K \theta_0. \quad (7)$$

The right-hand member of (5) may be developed in the following power series which hold respectively for the larger and smaller values of x :

$$\left(\frac{U_r}{U_{r_0}} \right)^{\frac{1}{2}} = 1 + \frac{8\pi^4}{15} \frac{1}{x^4} - 8 \sum_{n=1}^{\infty} e^{-nx} \left\{ \frac{1}{nx} + \frac{3}{n^2 x^2} + \frac{6}{n^3 x^3} + \frac{6}{n^4 x^4} \right\}, \quad (8)$$

$$\left(\frac{U_r}{U_{r_0}} \right)^{\frac{1}{2}} = \frac{8}{3} \frac{1}{x} \left\{ 1 + \frac{1}{20} x^2 - \frac{1}{1680} x^4 + \frac{1}{90720} x^6 - \dots \right\}. \quad (9)$$

In the introduction of the quantum theory into the Weiss theory, it is supposed that the only change brought about is that in discussing the statistics of the orientations of the elementary magnets, the expression RT for the energy of thermal agitation used in the relation deduced by Langevin is to be replaced by the value U_r for the rotational energy.

Thus the Langevin expression for the intensity of magnetization of a paramagnetic gas

$$\frac{I_v}{I_{v_0}} = \frac{\cosh a}{\sinh a} - \frac{1}{a} \quad \text{where}^1 \quad a = \frac{\mu H}{RT} \quad (10)$$

is modified only by putting a equal to $\mu H/U_r$. In the above expression, I_{v_0} is the intensity of magnetization supposing no thermal agitation present and μ is the moment of a single rotating magnetic molecule.

The particular region in which we are interested in this work, namely, that immediately above the Curie point, has been called by Weiss the region of induced ferromagnetism and is characterized by the fact that the molecular field is brought into existence only by the action of an external field and is not so large that the external field may be neglected in comparison to it as is the case in the ferromagnetic region. Accordingly, we have, for the total field of equation (10)

$$H = H_e + NI_v, \quad (11)$$

where N is the molecular field coefficient which is considered constant for a given substance.

The spontaneous magnetization I_v which corresponds to an external field $H_e = 0$, is obtained by the well-known Weiss construction by obtaining the intersection of the curve of equation (10) and the straight line

$$\frac{I_v}{I_{v_0}} = \frac{U_r}{\mu NI_{v_0}} a, \quad (12)$$

which corresponds to the particular value of T chosen. The magnitude of the rotational energy U_{rc} at the Curie point is obtained by equating the slope of the curve at the origin with that of the straight line. Accordingly,

$$\frac{U_{rc}}{\mu NI_{v_0}} = \frac{1}{3}, \quad (13)$$

whence

$$U_{rc} = \frac{\mu NI_{v_0}}{3} = \frac{n\rho\mu^2 N}{3}, \quad (14)$$

¹ Before proceeding further it is necessary to define clearly certain of the terms entering into the discussion which are frequently confused by writers in this field.

I_v equals vol. intensity of magnetization, = magnetic moment/volume,

I_m equals mass intensity of magnetization, = magnetic moment/mass,

K equals vol. susceptibility, = I_v/H_e .

χ equals mass susceptibility, = I_m/H_e ;

accordingly

$$I_v = \frac{M}{v} = \frac{\rho M}{m} = \rho I_m$$

and $k = \rho\chi$.

Where ρ equals density. In equation (11) it is the volume intensity which is meant.

where n is the number of rotating molecular magnets per gram and ρ the density of the substance.

The relation between the energy of rotation and the susceptibility is obtained as follows:

$$\chi = \frac{I_m}{H_s} = \frac{I_m}{H - H_m} = \frac{I_v}{\rho(H - NI_v)}, \quad (15)$$

$$\frac{1}{\chi} = \rho \left\{ \frac{H}{I_v} - N \right\} \quad (16)$$

but from (10) for this particular region

$$\frac{H}{I_v} = \frac{3U_r}{I_v\mu} = \frac{3U_r}{n\rho\mu^2}. \quad (17)$$

Therefore

$$\frac{1}{\chi} = \frac{3}{n\mu^2} \{U_r - U_{rc}\}. \quad (18)$$

In the first approximation, namely, for equipartition, and also in the second approximation when, in the development according to positive powers of x , of the second member of equation (5) only the first powers are used, this expression goes over into Weiss's law for the susceptibility above the Curie point. When, however, the complete development is used, the relation between χ and T is as shown in Fig. 7.

Application of Theory to Experimental Results.—Before the theory can be put to test it is necessary to determine the equation for the curve of Fig. 7. The problem algebraically consists in obtaining U_r as a function of T from equations (5) and (6) and substituting in equation (18). While

it is impossible to write out this equation in analytical form, one may compute the theoretical curve by means of the power series (8) and (9). For simplicity write equation (18) in the form

$$\chi^{-1} = A U_r - B, \quad (19)$$

where $A = 3/n\mu^2$, and $B = N\rho$.

Put

$$\left(\frac{U_r}{U_{rc}}\right)^{\frac{1}{2}} = 1 + \frac{8}{x^4} \int_0^x \frac{\xi^3 d\xi}{e^{\xi} - 1} = f(x). \quad (20)$$

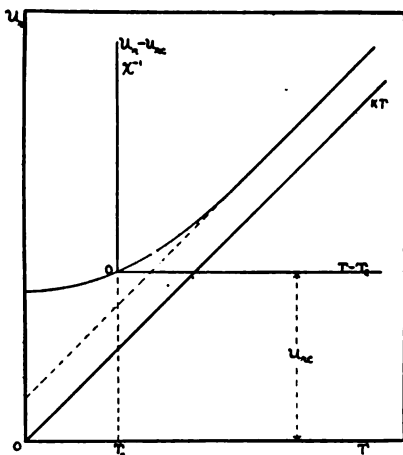


Fig. 7.

Then

$$\chi^{-1} = \alpha f^2(x) + \beta, \quad (21)$$

where $\alpha = A U_{r_0}$, $\beta = -B$.

$$T = \gamma \frac{f(x)}{x}, \quad (22)$$

Whence

$$\frac{\chi^{-1} - \beta}{\alpha} = f^2(x) = n(y), \quad (23)$$

$$\frac{T}{\gamma} = \frac{f(x)}{x} = y. \quad (24)$$

Thus the experimental $\chi^{-1} - T$ curve is directly related to the experimental $\phi(y) - y$ curve by means of the three parameters α , β , and γ . From the developments of equations (8) and (9), the following table was computed.

TABLE V.
Calculation of Theoretical Curve.

x .	$f(x)$.	$f^2(x)$.	$\frac{f(x)}{x}$.
.250	10.702	114.5	42.81
.500	5.486	30.09	10.97
1.000	2.799	7.838	2.799
1.250	2.297	5.275	1.838
1.429	2.052	4.213	1.436
1.667	1.815	3.292	1.088
2.000	1.588	2.523	.794
2.500	1.378	1.897	.5508
3.000	1.252	1.566	.4172
3.200	1.216	1.478	.3800
3.333	1.1956	1.428	.3588
3.400	1.1856	1.406	.3488
3.600	1.1604	1.346	.3225
3.800	1.1388	1.297	.2998
4.000	1.1212	1.257	.2804
4.200	1.1056	1.221	.2632
5.000	1.0628	1.131	.2127
6.667	1.0238	1.049	.1535
10.000	1.0051	1.010	.1005

The y , $\phi(y)$ curve was then plotted, which is, in reality, the theoretical curve of Fig. 7. The direct method of obtaining the constants α , β , and γ , is by means of the slope of the asymptote and its intercept on the $\phi(y)$ axis, and this method was at first employed. A simpler method, sufficiently accurate for the purpose, is to determine γ by means of the point on the curve at which departure from the asymptote begins which then fixes the correspondence of T and y . Then, by getting corresponding

values of χ^{-1} and $\phi(y)$ for two different values of T , two equations in α and β are obtained and their values computed therefrom.

When the calculations were carried out in this manner and applied to the results for nickel, it was found that, although the theoretical curve possesses the same general shape as the experimental one, the departure from the asymptote is not sufficiently rapid. An attempt was then made to see whether by the assumption of a larger zero point energy than is taken in Planck's equation, a better agreement with experimental results could be obtained. This was done by introducing in equation (2) the variable parameter λ as follows:

$$U_{rv} = \frac{1}{2} \left\{ \frac{h\nu}{e^{h\nu/KT} - 1} + \frac{\lambda}{2} h\nu \right\}. \quad (25)$$

When this modified expression is used, and the analytical work carried through in a manner analogous to that employed by Keesom, the following modified developments result:

$$\left(\frac{U_r}{U_{r_0}} \right)^{\frac{1}{2}} = 1 + \frac{8\pi^4}{15\lambda x^4} - \frac{8}{\lambda} \sum_{n=1}^{\infty} e^{-nx} \left\{ \frac{1}{nx} + \frac{3}{n^2 x^2} + \frac{6}{n^3 x^3} + \frac{6}{n^4 x^4} \right\}, \quad (26)$$

$$\left(\frac{U_r}{U_{r_0}} \right)^{\frac{1}{2}} = 1 - \frac{1}{\lambda} + \frac{1}{\lambda} \frac{8}{3x} \left\{ 1 + \frac{1}{20} x^2 - \frac{1}{1680} x^4 + \frac{1}{90720} x^6 \dots \right\}, \quad (27)$$

which were then used in connection with equations (5) and (6).

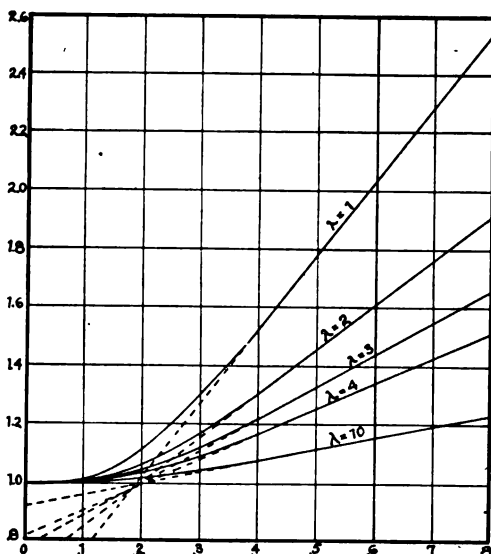


Fig. 8.

By assigning different values to λ the family of $y, \phi(y)$ curves shown in Fig. 8 was computed. These curves were then transformed so as to

have the same asymptote with their points of departure from the asymptote made to coincide, and the family shown in Fig. 9 was computed. Taking λ equal to ten and fitting once more to the experimental curve for nickel, the curve shown dotted in Fig. 5 was obtained. It is to be observed that the agreement is still unsatisfactory. A better agreement

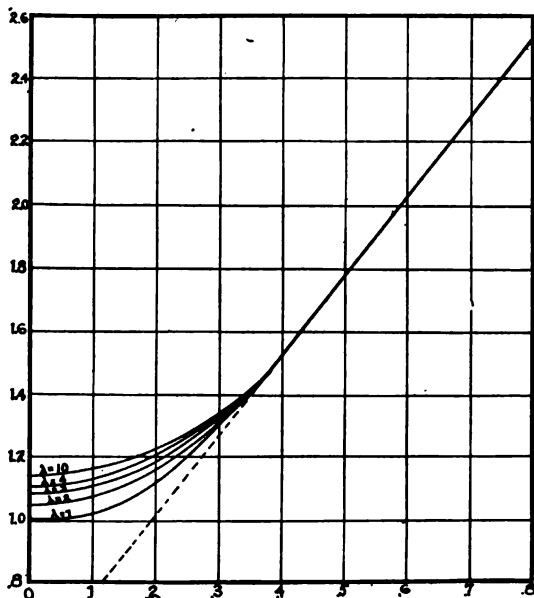


Fig. 9.

would of course result if a still larger value of λ were used; but with the value employed, the theory is so badly distorted that it does not seem worth while. The experimental curve for cobalt shows no tendency to approach an asymptote below the melting point hence it is difficult to attempt to check it. However, selecting the region of the theoretical curve for which the curvature is greatest, to give the theory the best possible advantage, the dotted curve in Fig. 6 was obtained. The departures between the theoretical and experimental curves for beta iron are so great that no calculation was made.

SUMMARY

1. The magnetic behavior of iron, nickel, and cobalt has been studied from the temperatures of their transformation points up through their melting points.
2. With the exception of nickel in limited regions, Curie's law is not obeyed.

3. In the regions immediately above the transformation points, for all three metals, the curve for the reciprocal of susceptibility is a smooth one with a marked upward concavity, instead of two intersecting straight lines as obtained by Weiss and Foëx.

4. Keesom's quantum theory of magnetism fits the experimental results in a qualitative way, but requires further modification.

5. The assumption of a larger zero point energy in Planck's equation is not sufficient to bring the theory into agreement with experimental facts.

PHYSICAL LABORATORY,
UNIVERSITY OF WISCONSIN,
December, 1916.

THE MEASUREMENT OF MAGNETIC PERMEABILITY.

BY ARTHUR WHITMORE SMITH.

IT is well known that when a ring of iron is magnetized by a current in a surrounding solenoid the iron does not instantly reach its full magnetization, but after the first large increase the magnetization continues to increase, in some cases, for several seconds. Ewing¹ observed this "viscous lagging" which "occurs in the softest iron and near the beginning of the steep part of the magnetization curve." Again he states² that this effect is more marked in soft iron than in hard iron, that it depends on the size of the sample tested, and that the cause is obscure. He also gives curves³ showing the effect of increasing the magnetizing force by small steps. At each step there was an immediate increase of magnetization, followed by a slow increase to the final value.

Lord Rayleigh⁴ tried to balance the effect on a magnetometer needle due to the magnetization of a bar of iron against the effect due to the current in a coil. A steady balance was impossible. "Most of the anomalous action was over in 3 or 4 seconds—the final magnetic state was not attained until after 15 or 20 seconds."

Wilson⁵ has noticed a large creeping up in the magnetic flux, even after one second. He states that the "effect is peculiar to the iron itself" and "might be influenced by induced currents." He computes (erroneously, I think) that the eddy current effect had entirely subsided in 0.9 second. More recently Jouaust⁶ has shown that the step-by-step method may give results as much as 12 per cent. in error, and he gives the warning that the sample tested should not be too thick. Rucker⁷ has found considerable difference between measurements by the ring ballistic method and magnetometer measurements. Using electrolytic iron Taylor⁸ has shown that owing to "magnetic viscosity" the step-by-

¹ Phil. Trans., Vol. 176, p. 569, 1885.

² Ewing, *Magnetic Induction in Iron*.

³ Proc. Roy. Soc., Vol. 46, p. 269, 1889.

⁴ Phil. Mag., Vol. 23, p. 230, 1887.

⁵ Proc. Roy. Soc., Vol. 62, p. 375, 1897-8.

⁶ Comp. Rend., Vol. 139, p. 272, 1904.

⁷ Zeit. für Instrumentenkunde, Vol. 25, p. 354, 1905.

⁸ Phys. Rev., Vol. 23, p. 95, 1906.

step method gives values for the magnetic induction considerably lower than those obtained by the method of reversals.

The question naturally arises, how much of the changing flux is measured by the ballistic galvanometer in the usual magnetic measurements? Such a galvanometer, having a period of about 16 seconds, will have completed its first throw in 4 seconds, and certainly cannot measure any change occurring after that time. At the very least, doubt is cast upon measurements made with this instrument under such conditions. The present paper shows that even the method of reversals with an ordinary ballistic galvanometer gives results which are too low, and the amount of this discrepancy for a ring of Swedish iron is shown in Fig. 2.

Some time ago there came into my hands a ballistic galvanometer having one remarkable characteristic. It is of the D'Arsonval type, period 40 seconds, and about as sensitive as other high-grade ballistic galvanometers, the remarkable property being that it is critically damped on a circuit of 70,000 ohms. When used on a circuit of a few ohms it is so greatly overdamped that it can move but slowly, and when once deflected it requires about 13 minutes to drift half way back to its zero position. It will therefore stand practically at rest at any point where it may be left, although it has a perfectly definite zero to which it will return in time. The action of this galvanometer is particularly interesting when it is connected to a few turns of wire around a bar magnet. When the magnet is slowly withdrawn, the spot of light on the galvanometer scale also moves slowly. If the magnet stops, so does the spot of light. When the motion is resumed the spot of light follows, and if the magnet be suddenly returned to its original position the spot of light returns as quickly and stops as suddenly. Every movement of the magnet is reproduced as faithfully as though the mirror were carried on the magnet itself. Of course, if the galvanometer is kept deflected too long there is the slow drifting back towards zero that was mentioned above, but if the movement is stopped with the galvanometer near its natural resting point it will remain stationary indefinitely. Perhaps the action of this galvanometer is best explained by saying that for every maxwell of flux cut by the wire around the magnet the coil of the galvanometer must move enough to cut one maxwell in the opposite sense. The instrument is, then, a fluxmeter rather than a galvanometer.

With the ordinary ballistic galvanometer the sudden reversal of the flux to be measured gives an impulse to the moving coil, and the scale is read at the end of the first throw. In using the new galvanometer the magnetic flux to be measured is reversed as usual, giving a quick deflection, at the end of which is a slow increase combined with the

return drift towards zero. After it has drifted back for half a minute or longer and has reached some definite line on the scale, the magnetic flux is reversed again, deflecting the galvanometer back to zero or a few spaces beyond. The slow increase is noticeable but the drift is negligible because the galvanometer is so near its resting point. After allowing about half a minute for the flux to reach its final value the scale is read, and this return throw is called the "deflection."

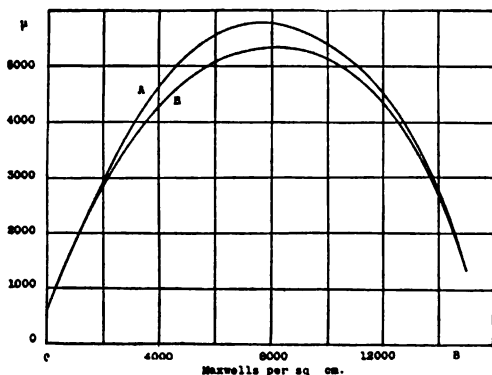


Fig. 1.

Permeability of Swedish iron. A. By new galvanometer. B. By ballistic galvanometer.

ured by each galvanometer, using first one and then the other. Both galvanometer scales were carefully calibrated over the range in which they were used, the readings for the calibration being interspersed with the other readings to avoid any error due to a change in the sensitiveness of the galvanometers.

At the highest magnetization each galvanometer showed the same values for the magnetic flux. At lower magnetizations the values of the flux as measured by the new galvanometer were considerably larger than the values given by the ordinary galvanometer. The values of the

permeability computed from these two sets of measurements are shown by the curves A and B, Fig. 1. The higher curve is doubtless correct, and the lower curve shows that much is missed by the old type of gal-

In order to compare this new galvanometer with a D'Arsonval ballistic galvanometer of the usual type, both instruments were used to measure the same flux. A ring of Swedish iron was magnetized beyond saturation and the magnetizing current then reduced by small steps, it being slowly reversed 10 or 12 times at each step. At each point the flux through the ring was measured

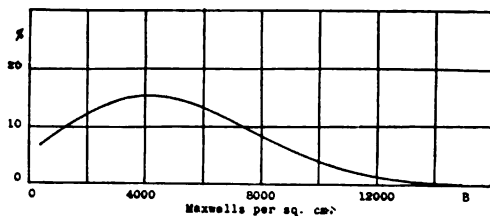


Fig. 2.

Percentage of magnetic flux not measured by the ballistic galvanometer (for a given value of H). For a ring of Swedish iron, 2 sq. cm. in cross section.

vanometer. The same thing is shown in a different way in Fig. 2, where the amount of flux not measured by the ballistic galvanometer is plotted as a percentage of the whole flux, and it is seen to be over 15 per cent. for values of B about 4,000 maxwells per square centimeter.

The measurements were repeated at different times and the same values were obtained at each trial. The amount missed by the ballistic galvanometer is approximately equal to that measured by the galvanometer when its key is closed later than the reversal of the current by a period equal to the time required for the first swing of the galvanometer.

There are two principal reasons why the magnetization lags thus behind the reversal of the magnetizing current. In the first place the primary current does not reach its full value instantly owing to the large self-inductance of the circuit with its many turns around the iron ring. When there is little other resistance in the circuit, and a battery of low E.M.F. is used, this slow rise of the current can be easily observed with an ordinary ammeter. "The time needed to establish a current of given strength in the coil of a large electromagnet with a solid core may be several minutes."¹

But in the results shown by the curves above the current was furnished by a storage battery of 35 volts and rose to its full value in a small fraction of one second. However, it was very evident that a considerable part of the magnetization was late, and occurred after the current had reached its steady value.

In the case of solid pieces of iron it is certain that there will be some current induced in the mass of the iron itself. This current will be greatest when the magnetization is increasing the fastest, and it will circulate in a direction opposite to that of the primary current. The effect of this eddy current (while it lasts) is, then, to reduce the resultant magnetic field which is impressed upon the iron, and of course the magnetic flux cannot increase faster than the impressed field.

To obtain more information regarding the cause of this slow magnetization a thin ring was cut from the side of the ring used above. All of the measurements were repeated with this smaller ring, which had one eighth the cross section of the former ring, but now both galvanometers gave the same results. The permeability curve is shown at A , Fig. 3. Only one curve is shown as the width of the line is sufficient to cover both curves obtained by the two galvanometers. This result was expected, but it was not expected that the permeability of the small ring should drop to half that of the piece from which it was cut. It was therefore unwound and heated to a dull red and allowed to cool slowly.

¹ B. O. Peirce, *Am. Acad. Arts and Sci.*, Vol. 43, 1907, p. 116.

After this annealing it was again tested in the same manner as before and the results are shown by curve *B*. As before the width of the line covers the two curves obtained by the two galvanometers. The permeability has nearly been restored and doubtless would be larger if

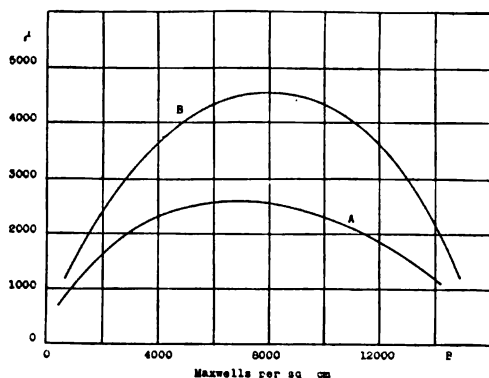


Fig. 3.

Permeability of Swedish iron. Points by both galvanometers are covered by the width of the lines. *A*. Thin ring cut from larger one. *B*. Same ring after annealing.

greater care had been taken in the annealing. There was some creeping of the magnetization, but the galvanometer key had to be closed one fifth of a second after the reversal of the current to detect as much creeping as was found after four seconds with the larger ring. It therefore seems reasonable to suppose that the slow increase of the magnetization is due to eddy currents in the iron of the ring, these currents tending to shield the interior portions of the iron from the changes in the field

impressed by the current in the surrounding solenoid. This agrees with the usual comment that the amount of the creeping depends upon the size of the sample studied and is absent in fine wires.

It is also interesting to see what very marked changes are produced in the permeability by the mere cutting of the iron in the lathe, an effect which must be present more or less in every sample that is thus prepared.

It follows from this comparison of galvanometers that the ordinary ballistic galvanometer does not give the correct measurement of magnetic flux even when the sample tested is only two or three square centimeters in cross section, especially if the permeability of the iron is high. In this case it is necessary to use an instrument which will respond quickly to changes of magnetic flux in order that the observations may not be too tedious, but which at the same time will have a period considerably longer than the time required for the iron to reach complete magnetization.

PHYSICAL LABORATORY,
UNIVERSITY OF MICHIGAN,
January, 1916.

TIME LAG IN MAGNETIZATION.

BY ARTHUR WHITMORE SMITH.

WHEN a steady magnetic field is suddenly impressed on a bar of iron the outside of the iron becomes magnetized first, while the center of the bar follows later in something the same way that the temperature would rise when the bar is plunged into a hot bath. Let us consider a long bar of iron, magnetized by a steady current flowing through a uniform winding around the bar. Let B' , H' , μ' denote the values of B , H , and μ at the center of the bar, where the time lag would certainly be the greatest.

The magnetizing force due to the current is $H_0 = 4\pi I$, where I denotes the C.G.S. current turns per cm. If there is also an eddy current of J C.G.S. units per cm. length of the bar the resultant field at the center is

$$H' = 4\pi(I - J). \quad (1)$$

To find J let us consider a cylindrical portion of the bar, of radius r and of thickness dr . The current circulating in unit length of this tube is

$$dJ = \frac{e dr}{2\pi r \sigma}, \quad (2)$$

where σ denotes the specific resistance and e the E.M.F. induced in this tube. This E.M.F. is due to the changing magnetic flux within the tube.

The simplest case is that in which the value of B is the same at all points within the bar. This would give for the flux within any given tube,

$$\Phi = \pi r^2 B' \quad (3)$$

and the E.M.F. induced in the tube is, then,

$$e = \frac{d\Phi}{dt} = \pi r^2 \frac{dB'}{dt}. \quad (4)$$

This gives

$$J = \frac{1}{2\sigma} \frac{dB'}{dt} \int_0^a r dr = \frac{a^2}{4\sigma} \frac{dB'}{dt}, \quad (5)$$

since dB'/dt is independent of r .

Substituting this value of J in (1)

$$H' = H_0 - \frac{\pi a^2}{\sigma} \frac{dB'}{dt} \quad (6)$$

or, since

$$\frac{dB'}{dt} = \frac{dB'}{dH'} \frac{dH'}{dt} = \mu' \frac{dH'}{dt}, \quad (7)$$

$$H' = H_0 - \frac{\mu' \pi a^2}{\sigma} \frac{dH'}{dt}. \quad (8)$$

An approximate solution of this differential equation can be obtained by taking μ' as constant, in which case

$$H' = H_0(1 - e^{-(\sigma t / \mu' \pi a^2)}). \quad (9)$$

This derivation is of interest as showing why there is an eddy current, and upon what factors it depends. Particularly should it be noticed that μ' is a kind of differential permeability, its value being given by the slope of the curve which shows the relation between B' and H' . The approximations used above result in too small a value for the numerical coefficient of the exponent, and owing to the eddy current thus produced H will not be uniform across the bar.

Using the more general equations of Maxwell's theory, Lord Rayleigh¹ showed that the eddy current in a cylindrical conductor decreases with time as $e^{-(1.44\sigma t / \mu' \pi a^2)}$, which gives 1.44 as the coefficient of the exponent. This result is often quoted to show that the effect of eddy currents dies out too quickly to account for much of the observed "magnetic viscosity," although it is generally remarked that this effect is greater in iron of large cross section and is most noticeable in the region of maximum permeability.

J. J. Thomson² derived a similar expression and computed that for an iron rod of 1 cm. radius and $\mu = 1,000$ the induced current will fall to $1/e$ of its maximum value in $2/9$ of a second. For soft iron μ' may be as large as 12,000, in which case 1 per cent. of the eddy current would still remain after 10 seconds, and of course there will be some eddy current as long as B is changing.

Ewing explains³ this time lag of magnetization as largely due to the smaller relative amount of surface on the larger bars and the longer chains of molecular magnets. An outlying molecule is first upset; then its neighbors, weakened by the loss of its support, follow suit and the action spreads from molecule to molecule. The surface molecules are least securely held. In fine wire there is much surface in proportion

¹ B. A. Report, 1882, p. 446.

² Recent Researches, pp. 352-6.

³ Magnetic Induction, p. 335.

to the amount of iron, and the breaking up of molecular communities starts at so many points that it is completed very quickly.

In order to examine further the effect of eddy currents, four rings of Swedish iron were made. The cross sectional area of each was about 1 cm. by 1.2 cm. Two of the rings had an external diameter of 15 cm., while the internal diameter of the other two was a little larger. When placed together the four rings formed a ring of larger cross section (2.1 cm. by 2.5 cm.), but with the four parts separated by air gaps of about 1 mm. in width. These air gaps could have no effect upon the magnetization of the ring, but they do prevent any eddy current circulating through the ring as a whole; there can only be smaller eddies in the four parts. The magnetic characteristics of this composite ring are shown by the hysteresis curve *A*, Fig. 1. The hysteresis curves for each of the four separate rings are likewise represented, by this same curve, which was obtained by the method of direct deflection¹ but using the fluxmeter-galvanometer described in the previous paper.

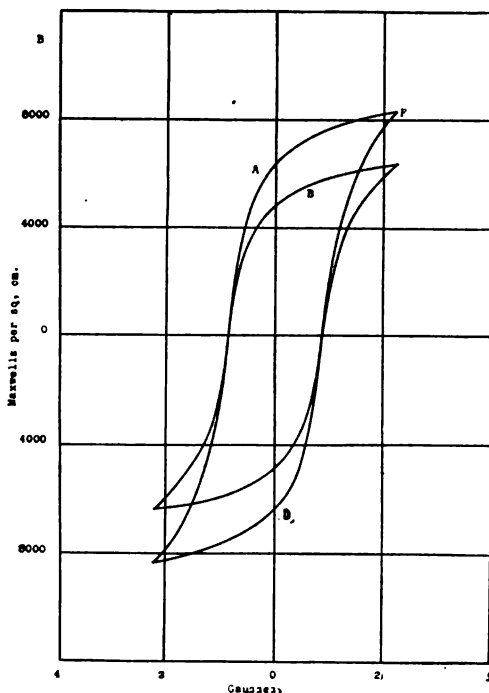


Fig. 1.

Hysteresis of Swedish iron. *A*. Four separate rings.
B. After soldering together.

The time lag was studied by means of a pendulum apparatus which closed the galvanometer key at any desired interval after closing the battery circuit. A storage battery of 125 volts was used to reduce the time lag of primary current to a negligible amount. The magnetizing current of 0.25 ampere ($H = 2.2$ gauss) was reversed slowly (once every ten seconds) many times, and the circuit left open. The pendulum then swung, closing the battery circuit so as to continue the magnetic cycle from *D* to *F*, Fig. 1, and then, after a predetermined interval, closing the galvanometer key. The galvanometer deflection measures the amount of change in the magnetic flux after the key is closed, that

¹ A. W. Smith, *Electrical Measurements*, p. 182.

is, the amount by which the flux lacked of reaching its final value at the moment the key was closed. As this galvanometer responded promptly to every change of flux, but moved almost not at all under its own restoring forces, the total change of flux was accurately measured even when spread over considerable time. The growth of the flux is shown by the curve *P*, Fig. 2, the width of the line covering all of the

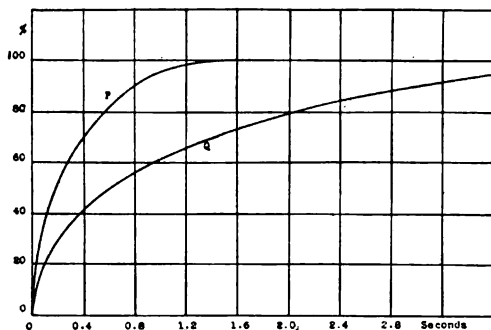


Fig. 2.

Time lag of magnetization. ($H = 2.2$ gauss.) *P*. Four separate rings. *Q*. Four rings soldered together.

fourteen observed points. The ordinates are expressed in per cent. of the final value, and the abscissæ are the times in seconds after closing the primary circuit. The curve shows that the magnetic flux has reached its final value after about 1.4 seconds.

Now if this lag in magnetization is due to the magnetic viscosity of the iron it should make no difference if the air gaps separating the four rings should become conducting electrically. Neither would this change the amount of iron surface bounding the four rings if the conducting gaps are non-magnetic.

In carrying out this idea copper rings were fitted tightly between the iron rings and the whole soldered together into one solid ring. This ring was rewound with the same number of turns as before, and all of the measurements were repeated. Evidently the ring had received some treatment which lowered its permeability and this must be taken into account if the measurements made on it are to be comparable with those shown by curve *P*. The hysteresis curve is shown at *B*, Fig. 1, and shows the decreased permeability. In spite of this, the time lag of magnetization was very much more marked than before. The results are shown by curve *Q*, Fig. 2, in which the ordinates are per cent. of the final value as before, and the abscissæ are the observed times corrected to give the same values of t/μ' (see (9)) as for curve *P*. It is assumed that the specific resistance of the iron is the same as before. For a straight bar of the same kind of iron this was found to be 11,000 C.G.S. units.

Comparing these two curves it is seen that in *P* the magnetization rises to 0.8 of its final value in 0.58 second, while in *Q* it rises much slower, taking 2.05 seconds to reach 0.8 of the final value. This is nearly a fourfold increase, which was expected since the cross section is increased four times. It is evident that these two curves are not strictly of the form ϵ^{-kt} , and this is not to be expected since μ has a considerable range while the iron is going over the quarter cycle from *D* to *F*. The maximum value of *B* for this cycle is 8,300, corresponding to $H = 2.2$; this would give 3,760 for the ordinary value of the permeability. But the total change in *B* corresponding to the change in *H* from 0 to 2.2 is 14,700, giving $\mu' = 6,700$. As any given portion of iron is carried along the cycle from *D* to *F*, the value of μ' at *D* is about 2,000, increasing at the steepest part of the curve to 14,000 and then decreasing to about 2,200 at *F*. This is also shown in the curves as they rise at first more rapidly than exponential curves, then more slowly, and at the end they quickly reach the final value. Moreover the iron at different points in the ring is at different stages of magnetization and at any given instant no value of μ could be appropriate for all portions of the iron. These curves show the increase in the total flux for the entire ring. Equation (9) with the value of $\mu' = 4,000$ agrees with the middle portions of these curves. The magnetic induction at the center of the iron rises much more slowly, probably not more than half as fast, and the average value of μ' could easily be as much as 8,000.

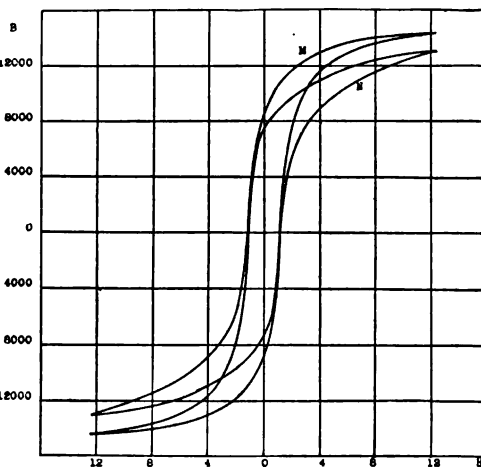


Fig. 3.

Hysteresis of Swedish iron. *M*. Four separate rings.
N. After soldering together.

A ring of cast iron was examined in the same way and almost no time lag was found. The formula would also lead one to expect such a result, since μ is small for cast iron and whatever time effect there might be would be over in a very short time.

But it is not necessary to use a different kind of iron to obtain a small value of μ . The same rings of Swedish iron were subjected to a magnetizing force of 12.4 gauss, and the hysteresis curves are shown in Fig. 3. While the steepest part of the curve is much like that in Fig. 1, a large

part of it is flatter and at the end $\mu' = 300$. Thus on the whole μ' is less and the time lag of H' would be less than in the former case. Moreover, the last 10 per cent. change in H' would not have a corresponding effect in changing either B' or the total flux measured by the galvanometer. The observed time lag for the four separate rings is shown by curve R , Fig. 4, in which the abscissæ are the observed times in seconds,

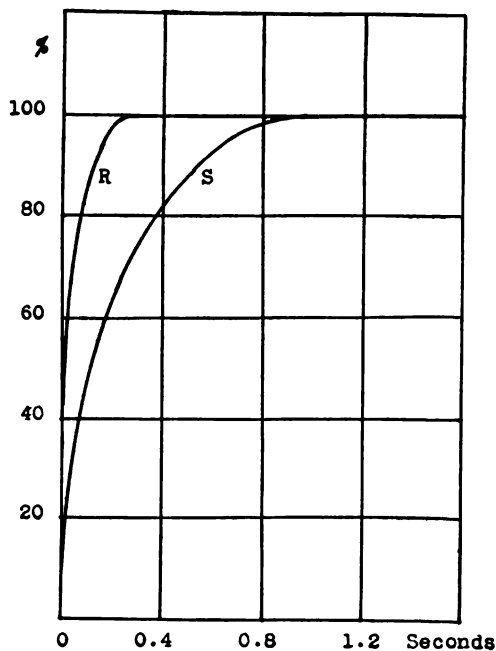


Fig. 4.

Time lag of magnetization. ($H = 12.4$ gauss.) R . Four separate rings. S . Four rings soldered together.

the same as in curve P for the same rings at lower magnetization. This curve corresponds to an average value of $\mu = 700$, and if H' rose half as fast as this an average value of $\mu' = 1,400$ is not difficult to explain on the basis of the actual hysteresis curve M .

The hysteresis curve for these rings after they were soldered together, with copper filling the intervening spaces, is shown by N , Fig. 3. The lowered permeability is again in evidence but not as much as before. The time lag for this ring when $H = 12.4$ is shown by S , Fig. 4, where the curve has been made comparable with R by increasing the observed abscissæ inversely as the permeabilities. The

relation between R and S is about the same as between P and Q , and the relation between R and P , for the same rings, is what would be expected in view of the change in the average value of the permeability.

With more mathematical analysis it would be possible to show more precisely the relation between the flux through a bar of iron at any instant and the time which has elapsed since the steady field was applied. But the purpose of the present paper has been fulfilled by showing that the observed time lag in these rings can be accounted for by the eddy currents which the formula indicates should circulate in the iron. Further effects of these eddy currents will be given in another paper.

A STUDY OF THE ACCELERATED MOTION OF SMALL DROPS
THROUGH A VISCOUS MEDIUM.¹

BY W. A. SHEWHART.

NOT long ago Professor E. P. Lewis² developed a new method of determining the amplitude of a sound wave in air. It was necessary to assume that if we have a small particle falling freely through a viscous medium under the action of weight, and at the same time acted upon by a simple harmonic force at right angles to the vertical fall, the equations of the vertical and horizontal components of the motion may be written respectively,

$$(1) \quad mg = k \frac{dy}{dt},$$

$$(2) \quad m \frac{d^2x}{dt^2} - k \frac{dx}{dt} = A \sin \omega t,$$

where m represents the mass of the particle, x and y the displacement at any instant, A the amplitude of the simple harmonic force, g the acceleration of gravity and k a constant based upon the assumption that the viscous resisting force, in both cases, bears the same proportion to the first power of the velocity. Three general questions are involved. In the first place what are the limitations under which equation (1) holds if the particle moves under the influence of a constant force in one dimension only? Stokes³ has stated this problem as that of the motion of an incompressible fluid, infinite in extent and of uniform density, moving with a small velocity and without slipping, past the surface of a small sphere, and he arrives at the well-known formula,

$$(3) \quad F = 6\pi\mu av,$$

where F is the resultant viscous force, μ the coefficient of viscosity of the medium, a the radius of the sphere and v the terminal velocity. The fall of solid and fluid spheres in air and in liquids has been very carefully studied experimentally by Millikan,⁴ Nordlund,⁵ Jones,⁶ Allen,⁷

¹ Accepted in partial satisfaction of the requirements for the degree of doctor of philosophy in the University of California.

² Lewis and Farris, *PHYS. REV.*, N.S., Vol. VI., p. 492.

³ Stokes, *Math. and Phys. Papers*, Vol. III., p. 57.

⁴ Millikan, *PHYS. REV.*, Vol. 4, Apr., 1911, p. 349; Vol. 2, Aug., 1913, p. 109.

⁵ Nordlund, *Zeitschrift für Physikalische Chemie*, 87, 1914, p. 40.

⁶ Jones, *Phil. Mag.*, 37, p. 45, 1914.

⁷ Allen, *Phil. Mag.*, 1900, p. 323.

Zeleny,¹ Arhold,² Ladenburg,³ Silvey,⁴ Ellis,⁵ and others, and the theoretical development given by Stokes has been extended and modified to fit certain conditions not contained in the earlier treatment. The work of these men shows that equation (1) represents the motion of a particle with a uniform linear velocity V , so long as the quantity $\frac{Va}{\nu}$ is small, where a is the radius of the particle and ν is the kinematic coefficient of viscosity.⁶

In the second place, no record of experimental evidence has been found by the writer in justification of using equation (2) as an expression of the horizontal vibratory motion, even though the particle be confined to move in one dimension. Although in the early theoretical work of Stokes, he arrives at an equation for the force acting on a pendulum bob, assuming that it oscillates in a straight line through a viscous medium, and later, after investigating the problem of the small sphere moving in a similar medium, he points out that the expression for the resisting force on the spherical bob reduces to that for the force acting on a small sphere, which moves with uniform velocity, when the period of the pendulum approaches infinity. Thus far the two equations (1) and (2) have been discussed independently one of the other, as expressions representing only a one-dimensional motion, and it has been pointed out that we cannot justly assume equation (2) without further experimental evidence. There still remains the third question, *i. e.*, granted that equations (1) and (2) hold separately, when the motion is linear, will they still serve to express the component relations when the motion is two-dimensional? The experiment described below was designed to answer this third question, and to determine to what extent, if any, the terminal vertical velocity component of a small sphere will be changed if a simple harmonic force is applied at right angles to the original motion.

Charged particles are made to fall between two condenser plates connected to the terminals of a Thoradson transformer, and illuminated in such a way that a photographic trace of the path of each small sphere may be obtained. In this case equation (2) becomes

$$(4) \quad m \frac{d^2x}{dt^2} + k \frac{dx}{dt} = neX \sin \omega t,$$

where m and k have the same meaning as previously and X is the potential gradient of the field, e the elementary charge and n the number of

¹ Zeleny, *PHYS. REV.*, Vol. 30, May, 1910, p. 535.

² Arnold, *Phil. Mag.*, 22, 6, p. 755.

³ Ladenburg, *Ann. der Physik*, 22, p. 287 (1907), and 23, p. 447, 1907.

⁴ Silvey, *PHYS. REV.*, N. S., Jan. 1916, p. 106.

⁵ Ellis, *Phil. Mag.*, 6th Series, Vol. 29, p. 526.

⁶ Rayleigh, *Phil. Mag.*, Vol. 36, 1893, p. 365.

elementary charges on a given particle. The complete solution of this equation is

$$(5) \quad x = \frac{neX}{k\omega} \sin \delta \sin (\omega t - \delta) + Be^{-(kt/m)} + B_1,$$

where δ is the phase difference between the motion of the particle and the varying simple harmonic disturbance and B and B_1 the integration constants.

The second term soon becomes negligible in the actual case, whereas the third term, which is a constant, merely determines the point about which oscillations take place, depending upon the phase of the driving force at the instant the field is turned on. After a short time, under these conditions, the particles would vibrate back and forth in a horizontal line, but for the fact that they are falling at the same time so that the trace becomes an approximate sine curve.

From equation (5) the maximum displacement from the mean position $x = B$ is seen to vary directly as the number of elementary charges on the falling drop and directly as the magnitude of the field gradient X , providing of course, that k remains a constant for such motion. Thus it is possible to test in a very simple way the involved assumption in regard to k , since the displacement, x and y , may be read from the plate and ne and X may be determined experimentally. From equation (5), neglecting the last two terms, the horizontal motion, after a very short interval of time, is practically expressible as

$$(6) \quad V = A \sin \omega t, \quad \text{where} \quad A = \frac{ne X \sin \omega t}{k\omega}.$$

The experimental arrangement was as follows: Two parallel brass plates, each 17.78 cm. $\times$ 3.81 cm. $\times$.5 cm., were supported on an ebonite base. The distance between them was accurately fixed at 3.8 cm. by means of ebonite bars, and grooves in the base. The plates were then connected to the terminals of a transformer and the air condenser thus formed was set parallel to, and about 7 cm. from the lens of the camera. The particles were illuminated by a carbon arc enclosed in a sheet iron cylinder, and the angle between the direct rays of the arc and the perpendicular to the face of the camera was adjusted to approximately 28° . A large sheet of paper in which a narrow slit 2 mm. $\times$ 10 mm. had been cut, was arranged about an inch above the condenser so that when a water or oil spray was made with an atomizer above the paper, the small drops fell into the illuminated field near the center of the condenser.

Needle points were placed at the top of the condenser so that as the drops fell past them they would pick up one or more elementary charges

and then when the drops came between the plates they would oscillate back and forth as they fell, thus tracing a sine-like path. After careful focusing on a glass fiber, held for the instant at the center of the condenser but withdrawn after final adjustment, a spray was made above the paper and so soon as the first droplets entered the field the shutter was opened from $\frac{1}{2}$ to 5 seconds. Since the image on the plate was very narrow, each plate could be withdrawn a small distance at a time and thus made to serve for several exposures.

A lens of 1.5 cm. aperture and .9 cm. focal length was used. It was found necessary to employ the very sensitive Seed Graflex plate and the largest stop was used for most of the work, but a very faint narrow trace could be obtained with a stop .5 mm. in diameter.

To exclude all light coming from other sources, it was found helpful to place a screen immediately in front of the condenser. In every case, the best photographic results were obtained when the particles fell so as to be viewed against a background in which there was no reflecting material of any kind. For instance, if the screen was placed too near the condenser, some light would be reflected from its surfaces into the camera and fog the plates, even though all screens and the condenser itself was painted black.

Owing to convection currents, very small particles soon drifted out of focus, so that in the present work only comparatively large drops were used. At first an X-ray tube was employed to furnish a constant supply of ions between the plates in the hope that the particle would pick up charges as they fell, which would, as we have seen from equation (6), result in a change ΔA of the amplitude A , which would bear an integral relation to the previous amplitude. The average potential maintained between the condenser plates, as determined by the sparking distance between two needle points was about 28,000 volts, and calculation shows that for an increase of only one elementary charge the increase in A would be small, so that it would be very difficult to measure the amplitudes with a sufficient degree of accuracy to test the ratio $\Delta A/A$.

In view of this fact it was decided to increase the field gradient in a known manner, keeping the charge on the particle constant. This was done by making the distance between the plates at the top different from that at the bottom. A heavy brass strip 3.81 cm. $\times$ 1.27 cm. $\times$ 8.9 cm. was screwed to the lower edge of the condenser, so that with this in place the two field gradients were in the ratio 2 to 3. The same ratio should, therefore, exist between the amplitudes of the particle's motion at the top and the bottom of the condenser.

Four representative photographs are reproduced. In each of these,

traces of a large number of particles are shown. Many of the drops drift out of focus and in the measurements which follow these were discarded. The data for thirty-six particles is recorded in

TABLE I.

Drop No.	No. of Waves at Top of Plate.	Dist. in Mm. on Plate.	No. of Waves at Bottom of Plate.	Dist. in Mm. on Plate.	Ave. λ_t .	Ave. λ_b .	Amp. at Top in Mm.	Amp. at Bottom.	Ratio of Amplitudes.
1	4	32.5	5	45.3	8.12	9.06	2.4	3.6	.666
2	4	37.5	4	39.5	9.37	9.87	2.6	3.5	.666
3	3	34.2	4	47.2	11.40	11.80	1.6	2.6	.616
4	3	33.5	4	47.7	11.16	11.92	1.6	2.0	.800
5	2	39.0	2	40.0	19.50	20.00	.6	1.0	.600
6	3	33.0	3	33.5	11.00	11.16	1.5	2.3	.652
7	3	46.0	3	48.7	15.33	16.23	1.3	1.7	.764
8	4	26.7	6	43.2	6.67	7.20	.8	1.0	.800
9	4	32.6	5	41.6	8.15	8.32	.5	.8	.633
10	6	41.6	6	43.5	6.93	7.25	1.0	1.5	.666
11	3	45.5	2	30.9	15.16	15.45	2.0	2.7	.740
12	5	47.2	4	38.7	9.44	9.67	1.3	2.0	.650
13	3	42.7	2	28.5	14.23	14.25	1.0	1.5	.666
14	11	50.0	9	38.7	4.54	4.30	1.9	2.7	.705
15	4	47.2	3	37.2	11.80	12.40	1.3	2.0	.650
16	3	38.2	3	41.5	12.73	13.83	2.1	3.2	.656
17	5	31.2	6	41.0	6.24	6.83	1.0	1.5	.666
18	6	50.5	7	59.3	8.42	8.47	1.8	2.4	.750
19	5	39.5	6	45.5	7.90	7.60	1.8	2.5	.720
20	6	48.5	6	52.0	8.08	8.66	2.0	3.0	.666
21	4	40.0	6	61.5	10.00	10.25	.8	1.5	.537
22	5	54.4	5	55.0	10.88	11.00	1.4	2.0	.700
23	5	30.0	4	24.0	6.00	6.00	2.3	3.5	.656
24	4	32.3	5	40.5	8.07	8.10	1.6	2.0	.800
25	6	45.5	2	13.2	7.58	7.60	1.9	2.5	.760
26	3	34.0	4	45.0	11.30	11.25	1.0	1.8	.555
27	4	34.0	6	52.5	8.50	8.75	1.0	1.8	.555
28	4	47.0	4	47.0	11.75	11.75	2.0	2.8	.714
29	5	49.0	6	59.7	9.80	9.95	.6	1.0	.600
30	5	56.5	5	58.3	11.30	11.66	1.3	2.0	.650
31	7	49.5	6	46.5	7.07	7.75	1.0	1.5	.666
32	8	61.5	8	65.5	7.71	8.19	.9	1.5	.600
33	8	60.0	8	62.5	7.50	7.81	1.5	2.1	.715
34	5	54.5	5	58.5	10.90	11.70	1.0	1.5	.666
35	8	54.0	9	65.0	6.77	7.22	.6	1.0	.600
36	9	45.0	12	60.0	5.00	5.00	1.0	1.6	.625

Columns 5 and 6 show the mean wave-lengths for the top and the bottom of the condenser, a wave-length being taken as the distance between successive maximum and minimum in the approximate sine curve reproduced on the plate. The frequency of the 60-cycle alter-

nating current feeding the primary of the transformer was found to be approximately constant at night when most of the work was done, therefore we may compare the wave-lengths of the top and bottom, which should be equal, if the terminal vertical component of velocity is not changed at the lower part of the condenser where the gradient becomes greater.

It will be noted that with three exceptions corresponding to drops Nos. 14, 19 and 26 the wave-lengths are longer in the region where the field gradient is a maximum, and careful observation of the plates showed that in each of these three cases the particles had drifted out of focus so that at the lower end of the fall, the distance measured on the plate was only the projection of the actual motion, which accounts for the apparent discrepancy in the results. We should therefore conclude that a particle falls faster when it is simultaneously vibrating at right angles to the line of fall.

The objection might be raised that the particles under consideration had not reached a constant vertical velocity when they first entered the region of the condenser and so to make sure that this error did not occur photographs were taken of particles falling between parallel condenser plates. The results for six representative particles are shown in the following table and indicate that the terminal vertical velocities had been reached before the particles came under observation.

TABLE II.

Drop No.	No. of Waves at Top of Plate.	Wave-length in Mm.	No. of Waves at Bottom of Plate.	Wave-length in Mm.
1	3	11.29	3	11.29
2	3	12.06	3	12.03
3	4	9.20	4	9.20
4	5	8.02	5	8.01
5	6	8.00	4	8.01
6	3	11.86	3	11.88

Furthermore, the field was somewhat distorted near the middle of the condenser where the half plate terminates. The force lines are curved upward at this point and thus the amplitude of oscillation begins to increase slightly above this region, but this distortion should not give rise to the qualitative increase in fall in the part of the condenser where the field intensity is strongest.

In reference to the amplitudes of the particles, it is interesting to note that a large number of the ratios given in Table I. are approximately two thirds, which would indicate that k is a constant. It is

impossible, however, to draw any definite conclusions from these ratios, for it was observed that the particles usually picked up additional charges near the middle of the condenser, owing to the constant discharge taking place from the sharp edge of the half plate.

If the motions in the two perpendicular directions are independent of each other, we may write the component displacements as

$$x = a \sin \omega t \quad \text{and} \quad y = ct,$$

or the resultant path is given by the equation

$$x = a \sin \frac{\omega y}{c},$$

where c is a constant. The resultant velocity at any instant is therefore

$$v = \sqrt{\left(\frac{dy}{dt}\right)^2 + \left(\frac{dx}{dt}\right)^2} = \sqrt{c^2 + a^2 \omega^2 \cos^2 \omega t},$$

and makes an angle with the vertical equal to

$$\frac{dx}{dy} = \frac{a\omega}{c} \cos \frac{\omega y}{c}.$$

If we assume a frictional viscous force $F = kv^n$ tangent to the path and resisting the motion of the particle it will have a normal component F_y equal to

$$F \cos \theta = kc(c^2 + a^2 \omega^2 \cos^2 \omega t)^{n-1}.$$

If $n = 1$ or if the resistance is proportional to the first power of the velocity, then $F \cos \theta = kc = \text{const.}$ If $n = 1$ then the mean value of $F \cos \theta$ for a half period interval is constant, so that the mean vertical component velocity should be constant within the same interval, and this mean value should diminish as the amplitude increases for the vertical component of the force becomes larger. This would produce a result opposite to the one actually observed.

The maximum horizontal component velocity of the particles was from 10 cm. to 75 cm. per sec., values far in excess of the limit within which Stokes's law holds, so that the resistance in this direction varied as some higher power of the velocity.

If F_x and F_y represent the viscous forces, where each component motion is taken separately, it is evident that they do not represent the component forces when the displacement is a resultant of the separate displacements.

In conclusion it is pointed out that if we assume the resultant viscous force to be exerted tangential to the path at any point, then the terminal vertical component will be decreased rather than increased, so that it is

necessary to assume a turbulent motion of the medium in order to explain this discrepancy from observed results.

If F_y is the resistance offered to a small particle moving with a constant velocity c in the y direction and F_x is the viscous resistance offered to a particle oscillating in the x direction, then it does not follow that F_x and F_y are the components of the viscous tangential resistance when both motions are compounded, unless this resistance depends upon the first power of the velocity.

The experimental results of this paper indicate that if a small sphere, moving through a viscous medium with a terminal velocity v under a constant force is suddenly caused to vibrate at right angles to the original line of motion the vertical velocity component v is increased.

Certain problems¹ relating to the two-dimensional flow of viscous liquids have been solved, but the solution under the boundary conditions set in the present problem involves certain mathematical difficulties. The general equations of motion of an incompressible fluid is of the type

$$\frac{\partial u}{\partial t} = X - \frac{1}{\rho} \frac{\partial p}{\partial x} - v \frac{\partial u}{\partial y} - \omega \frac{\partial u}{\partial z} + \sqrt{\Delta^2} u$$

where u, v, ω, x, y, z , are the components of velocity and of body force respectively, p the mean pressure, ρ the density, and $\sqrt{\Delta^2}$ the kinematic coefficient of viscosity. Two general types of solution have been obtained, one in which terms involving the second power of velocity, such as $u(\partial u/\partial x)$ are omitted, and the other in which the terms $\sqrt{\Delta^2} u$, etc., are neglected. Thus, for steady one dimensional motion, there is a large range of velocities for which no solution has been found to apply. It should be expected, therefore, that the broader problem of motion of a small sphere in two dimensions should present greater difficulties.

A study of the experimental and theoretical problem suggests three possible explanations of the results observed in this paper.

1. "Cavitation" at the edge of the sphere would give rise to the phenomenon observed, since even a small decrease in pressure on the lower side of the drop would give rise to a greater vertical component of velocity.

2. From the experiments of Mr. Ellis Williams² the stream lines for large values of Va/v indicate that for a comparatively long distance in the wake of the particle the liquid has the same velocity as the small sphere. If the particle suddenly stops as it does at the end of the oscillation and starts to return the pressure will evidently be greater on the upper side and will increase the vertical fall.

¹ Jeffery, *Phil. Mag.*, 29, 455, 1915.

² *Phil. Mag.*, 29, 526, 1915.

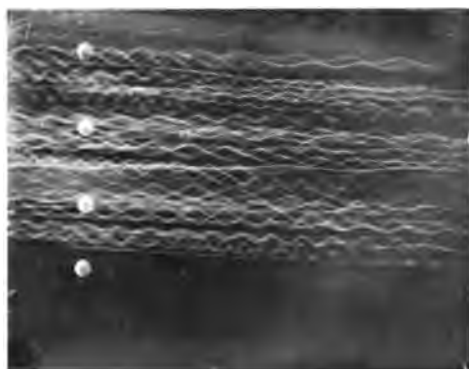


Fig. 1.

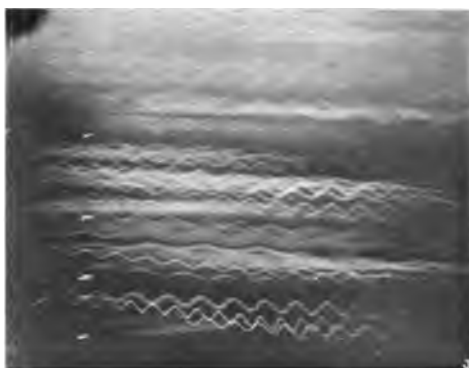


Fig. 2.

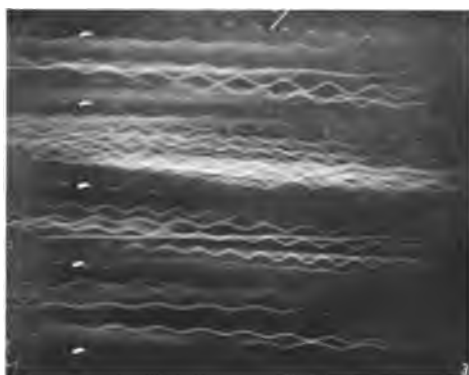


Fig. 3.

W. A. SHEWHART.

3. If the spherical drop of liquid suffers a distortion when it enters the electric field and becomes flattened, it would tend to set itself with the longer axis at right angles to the line of motion. The effect of the field would be to keep the flat side perpendicular to the sides of the condenser. The resultant effect upon the shape of the drop would be complicated, but it would undoubtedly affect the velocity.

In conclusion, I wish to thank Professor E. P. Lewis for his many helpful suggestions and counsel throughout these experiments and for the facilities placed at my disposal.

LABORATORY OF PHYSICS,
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June, 1916.

THE USE OF THE EINTHOVEN GALVANOMETER AND A DETECTOR OF THE AUDION TYPE FOR MEASURING THE STRENGTH OF RADIOTELEGRAPHIC SIGNALS.

BY LAURENS E. WHITEMORE.

ONE of the important determinations to be made in radiotelegraphy is the measurement of signal strength at a receiving station. Measurements of this kind are valuable in determining (1) the effectiveness of various types of antennæ and transmitters, and (2) the effect of weather conditions and of the nature of the region over which propagation takes place.

Among the experiments which have been made in the past to measure quantitatively the phenomena of radio transmission may be mentioned those of Austin,¹ Taylor and Blatterman,² and Marchant.³

In general, two methods of measurement are in use. (1) Noting the deflection of a galvanometer placed in series with a crystal detector or in shunt around a thermo-junction. (2) Shunting a telephone receiver with a variable resistance which can be adjusted until the signals are just barely audible, that is, until dashes can just be distinguished from dots. The second method has frequently been shown to be unsatisfactory.

For making quantitative measurements it is very convenient to record the signals by means of an Einthoven galvanometer and a suitable photographic device. A permanent record is thereby obtained which can be measured accurately at the leisure of the observer. A very convenient control board for use with this galvanometer is now being manufactured by Leeds and Northrup.⁴

The crystal detectors proving unreliable and the thermo-junctions too insensitive, we may turn to the more recently developed vacuum bulb detector which is sensitive and not easily thrown out of adjustment. A difficulty arises in its use from the fact that a battery of large electromotive force is used in the plate circuit. A current of about a milliampere

¹ Bul. Bur. of Stand., Vol. 7, 1911, Reprint No. 159; Vol. 11, 1914, Reprint No. 226.

² Proc. Inst. Radio Eng., 4, 131, 1916. (Good bibliography.)

³ El. (London), 75, 267, 309, 1915.

⁴ H. B. Williams, Am. Journ. Physiol., April 1, 1916.

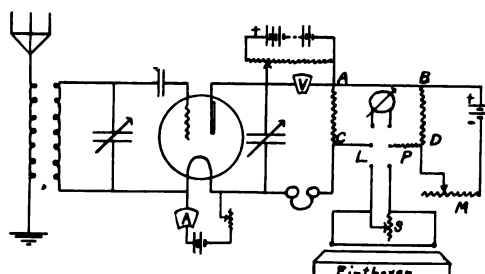


Fig. 1.

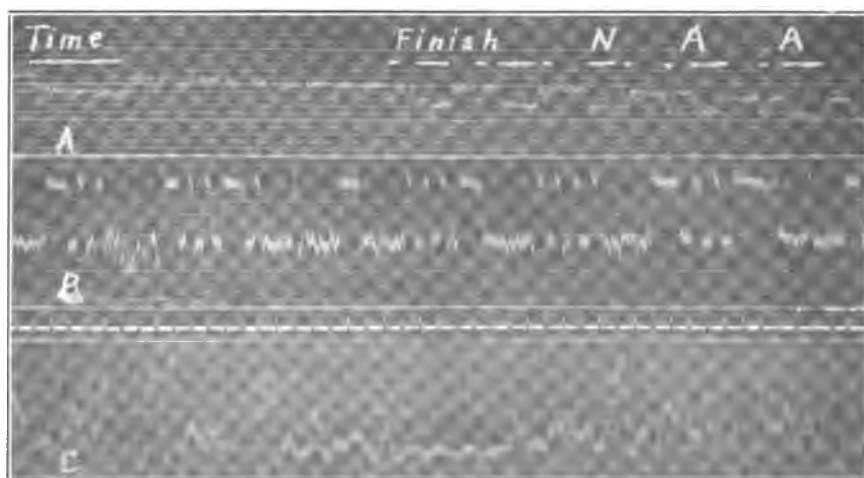


Fig. 2.

flows in this circuit whenever the detector is in use, thus making it impossible to use a sufficiently sensitive galvanometer directly in the receiver circuit. To avoid this difficulty the arrangement shown in the accompanying diagram, Fig. 1, was used.

The current which is continually flowing through the plate circuit causes a potential drop to exist across the resistance $A-C$ in the circuit. This is balanced by an equal potential drop across a resistance $B-D$ in an auxiliary circuit. The two circuits are connected together at the point $A-B$ and the galvanometer is placed between the two circuits at $C-D$. When no signals are being received the galvanometer should show no deflection. When signals are received the potential drop $A-C$ changes, thereby creating a difference of potential between C and D and causing a deflection of the galvanometer. If this galvanometer is of the Einthoven type it will follow the variations of the current with the reception of dots and dashes. It was found convenient to keep both of the resistances $A-C$ and $B-D$ constant at about 2,000 ohms and to secure a balance by varying an additional resistance, M , in series with the auxiliary battery.

When the filament of the detector is first lighted it is necessary to wait a few minutes until the current in the plate circuit becomes steady. For the preliminary adjustment of the potentiometer circuit it is very convenient to use a D'Arsonval galvanometer of the box type. It can then be replaced by the Einthoven by means of the switch, L , Fig. 1.

For quantitative work it is essential that the detector be put in the same condition of sensitiveness for the successive tests. The detectors of the vacuum bulb type are well arranged in this respect, for it is possible by means of an ammeter in series with the filament and a voltmeter across the battery of the plate circuit to reproduce the conditions in these circuits whenever desired. It is also important to keep the tuning and coupling conditions constant.

A check can be had on the detector by setting up an auxiliary or substitute antenna circuit and exciting this with a constant frequency buzzer in order to give in the detector circuit a current of about the same magnitude as that due to the received signals.¹

Even after a comparatively steady current has been set up in the receiver circuit (observable with the D'Arsonval galvanometer) a very slow increase or decrease may take place due, perhaps, to fluctuations in the temperature of the bulb, resulting in a drift in the line of photographic signals as in Fig. 2, A . In order to bring the galvanometer back to zero

¹ A most excellent arrangement of circuits for this purpose is given by Marchant in the *Journal of the (British) Institution of Electrical Engineers*, 53, 329, 1915. Abstract, *El.*, 74, 621, 1915.

it is necessary to adjust the series resistance, M , which consists, conveniently, of a dial type resistance box in connection with a fairly large sliding contact, or otherwise continuously variable, resistance. The deflection caused by the signals is not affected by this adjustment.

The sensitiveness of the apparatus may be decreased for recording strong signals or atmospherics by shunting the galvanometer as at S or including a series resistance, P .

Attention may be called to the fact that detector bulbs with lime cathodes have been found rather irregular in their behavior.¹ The newer bulbs of the tubular type seem to be very stable if the filament is not maintained at too high a temperature.

The photographs reproduced in Fig. 2 were obtained by the author by the use of the arrangement described in this paper. In this figure time increases toward the right and the deflection of the galvanometer is down, the zero position being above. A is a photograph of the time dash and signature signals sent out at 10 P.M., eastern time, May 24, 1916, by the U. S. government station (NAA) at Arlington, Virginia, whose distance from this station (9XP) is about 1,000 miles. B shows the beginning of the weather report sent out from Arlington on the evening of April 4, 1916. It reads, USWB — S m(issing). Atmospherics, at times making the signals somewhat difficult to translate, may be seen in A and B , while in C (11:15 P.M., March 21, 1916) the atmospherics are so strong as to completely overshadow any signals. The shadow of the time indicator marking fifths of seconds may be seen between B and C . The parallel lines, one millimeter apart, are for convenience in measurement and are caused by rulings on the cylindrical lens in front of the shutter of the camera.

This method of measurement seems especially applicable for taking data such as is being gathered by the British Association for the Advancement of Science, through its committee for radiotelegraphic investigation, for obtaining quantitative information regarding the influence of the weather on atmospherics and signal strength.

I am indebted to Dr. T. Townsend Smith, following whose suggestion the arrangement above described was developed, and to Dr. Ida Hyde, of the department of physiology of the University of Kansas, who was so kind as to put the Einthoven galvanometer at my disposal.

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¹ Willows, El., 74, 742, 1915.

PROCEEDINGS
OF THE
AMERICAN PHYSICAL SOCIETY.

A SIMPLE METHOD FOR DETERMINING THE AUDIBILITY CURRENT OF A
TELEPHONE RECEIVER.¹

BY EDWARD W. WASHBURN.

THE use of the telephone receiver as a detector for small alternating currents in wireless telegraphy, in many electrical measurements with the alternating current bridge, and for other purposes makes it desirable to have a simple and reliable method for determining the audibility current of a telephone. By audibility current is meant the current which must pass through the telephone in order to produce a sound which can just be recognized. The curve connecting the audibility current of a telephone with the frequency of the impressed E.M.F. is a valuable indication of the field of usefulness of the instrument and its availability for the above purposes.

The usual method of determining such a curve is first to measure the volt sensitivity of the telephone at various frequencies. This is usually done with the aid of a suitable slide wire connected in shunt across a non-inductive branch of an alternating current circuit provided with a hot wire ammeter.² Having measured the volt sensitivity in this way, the audibility current is then obtained by dividing the audibility voltage at each frequency by the impedance of the telephone for that frequency. Since, however, both the effective resistance and the inductive reactance of the telephone depend upon the frequency, it is necessary in this method to measure each of these quantities for each frequency. Moreover, as pointed out by Austin,³ it is doubtful whether the values thus obtained are strictly applicable for the purpose in hand, since they are measured with the aid of currents very much larger than the audibility current of the telephone. This uncertainty can be avoided and the whole determination much simplified by means of the following method based upon the equation of the Wheatstone bridge.

The set-up is shown in the accompanying figure. The resistances R and R' should be substantially free from both inductance and capacity. For low

¹ Abstract of a paper presented at the Chicago meeting of the American Physical Society, Dec. 2, 1916.

² Cf. *Wien. Ann. Phys.*, 4, 456 (1901).

³ *N. B. S. Bull.*, 5, 155 (1908).

resistance telephones Curtis coils would fulfill this condition sufficiently well, but for high resistance telephones especially at the higher frequencies, film resistances¹ would be preferable. The two resistances marked R' are connected by a stretched wire of suitable resistance (R_w), provided with a scale and a sliding contact. The telephone is connected to this sliding contact and to a point between the two resistances R as indicated. A small variable air condenser connected where it is required is employed to insure the exact balancing of reactances. The resistance of that portion of the slide-wire, lying between its central point and the position occupied by the sliding contact

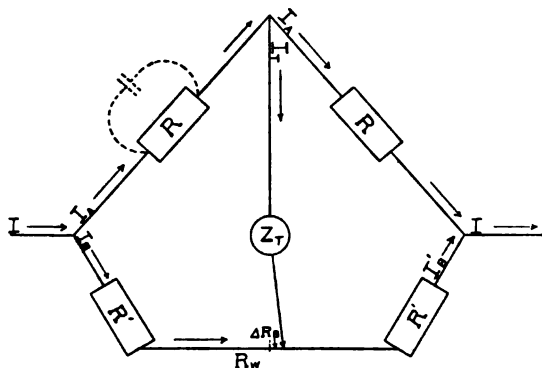


Fig. 1.

when the audibility current I_T is passing through the telephone, will be designated by ΔR_B .

The current I comes from a high frequency generator whose frequency can be controlled and kept constant at any desired value. It enters the bridge network at the left and divides as indicated. In series with the bridge is placed a vacuum thermocouple (not shown in the figure) connected to a sensitive millivoltmeter. This instrument indicates the magnitude of the current I . For high resistance telephones a hot wire ammeter could be employed.

The procedure consists in adjusting the current I until the range of silence on the slide-wire is found to have a convenient measurable value. This range of silence is obviously equal to $2\Delta R_B$ ohms. The value of I_T is then obtained from the equation of the Wheatstone bridge which is as follows:²

$$I_T = \frac{2\Delta R_B R I}{A(A + 2Z_T)},$$

where A is written in place of $R' + \frac{1}{2}R_w + R$.

This method would seem to involve a knowledge of Z_T , ($= R_T - jx_T$) the impedance of the telephone, but this can obviously be avoided by making A so large that $2Z_T$ is negligible in comparison with it, or, so large that the D.-C. resistance of the telephone may, with sufficient accuracy, be employed in place of its impedance. For low resistance telephones there will be no

¹ Jour. Amer. Chem. Soc., 35, 179 (1913).

² Maxwell, Electricity and Magnetism, I., 477.

difficulty in doing this because the impedance of a low resistance telephone is ordinarily not more than two or three times its resistance even at fairly high frequencies. In the case of some of the very high resistance wireless telephones, where the impedance at 1,000 cycles may run up as high as 16,000 ohms it will, of course, be necessary to determine and employ an approximate value of its impedance. For an electrically tuned telephone Z_T in the above equation reduces to R_T , the effective resistance.

As an example of the determination of the audibility current of a telephone receiver by this method, the following values may be cited:

The high frequency generator was operated at a frequency of 990 cycles per second. Resistance boxes of Curtis coils were employed for obtaining the two resistances indicated by R , each box being set at 5,000 ohms. The value of $R' + \frac{1}{2}R_w$ was 125 ohms. The range of silence on the slide-wire was found to be $2\Delta R_B = 0.025$ ohm. The vacuum thermocouple indicated a current $I = 0.4$ milliampere. The D.-C. resistance of the telephone was 170 ohms and its effective resistance at 1,000 cycles was 220 ohms, its impedance at this frequency being 290 ohms. For A we have, therefore, 5,125 ohms. The value of $2Z_T$ could evidently be neglected in comparison with 5,125 ohms; or, the value of $2R_T$, that is, 340 ohms could be used in place of it, since in either case the error involved would be less than ten per cent. Substituting these values in the above equation, we find that the audibility current of this telephone is 2.10^{-6} milliampere at 1,000 cycles.

DEPARTMENT OF CERAMIC ENGINEERING,
UNIVERSITY OF ILLINOIS,
November 15, 1916.

NEW BOOKS.

First Course in General Science. By F. D. BARBER, Professor of Physics, Illinois State Normal, M. L. FULLER, Forecaster, U. S. Weather Bureau, J. L. PRICER, Professor of Biology, Illinois State Normal, and H. W. ADAMS, Professor of Chemistry, Illinois State Normal. New York: Henry Holt and Co., 1916. Pp. vii + 607.

This book is another addition to the books which have been planned for use as texts in a first course in science, to be given in the first year of a high-school course. Wisely there is no attempt in this book to cover the entire field of elementary science; instead those things with which the pupil is already more or less familiar have been selected. The authors believe that a first course of this kind should deal almost wholly with physical sciences. The only biological part is a chapter on microorganisms. On the whole the subject matter seems to be well selected.

O. M. S.

General Physics. By W. S. FRANKLIN AND B. MACNUTT. New York: McGraw-Hill Publishing Co., 1916. Pp. viii + 604. Price \$2.75.

The authors state in the preface that this book has been prepared to meet the needs for a more rigorous treatment of the subject, such as used in the teaching of mathematics. Further, the notation of the differential calculus is used to some extent. This may be somewhat misleading, for the text is really intended as a first course for students that have not had calculus. Mechanics and some parts of heat may be more difficult than in some of the other recent texts. In part this is due to the fact that some of the simpler things, familiar topics already understood by the student, are omitted. Simple machines, the pulley, inclined plane, etc., are not mentioned. Some of the more difficult topics in electricity have been left out. No discussion of the magnetization of iron, with a distinction between the intensity of a magnetic field, H , and magnetic induction, B , is given. Permeability is not mentioned. The term potential does not appear, instead of difference of potential, as for example, between the plates of a condenser, electromotive-force is used.

The treatment of heat is very interesting. It is approached from the standpoint of thermodynamics with considerable emphasis on the law of entropy.

There is among engineers considerable confusion in the use of the word "pound." Is it a unit of force, or a unit of mass, or is it to be used for both mass and force? Textbook writers on physics, probably wish that engineers would answer this question, and possibly engineers wish that the physics

textbook writers would settle it. The authors of this book use *pound* as a unit of mass, and as unit of force, the *poundal*, and for work, the *foot-poundal*. The result, of course, is that an engineer using this book for reference and wanting an answer in foot-pounds may have trouble, for example, in computing the kinetic energy of either translation or rotation. But why *poundal*, and *foot-poundal*? Are they used any place outside of textbooks? Certainly engineers do use forces a great deal, but on the other hand they seldom use the concept of mass. Hence the common practice among them is to use the pound as the unit of force.

However no brief review of this book can do justice to it. There are here many ingenious methods of handling difficult topics, and it abounds with helpful analogies.

O. M. S.

Engineering Applications of Higher Mathematics. By V. KARAPETOFF. New York: Wiley and Sons, 1916. Part II., Problems on Hydraulics. Pp. v + 101. Part III., Problems on Thermodynamics. Pp. v + 113. Part IV., Problems on Mechanics of Materials. Pp. v + 81. Part V., Problems on Electrical Engineering. Pp. vii + 65. Price, \$.75 each.

The problems contained in these small volumes appear to be very well chosen to fulfill the purpose of the author as stated in the prefaces: to assist "the student or the engineer who wishes to review calculus or analytics or to acquire facility in applications of higher mathematics to engineering problems." They are "practical" in a legitimate sense of that much misused word. The engineering student who goes through one or more of these books, even though he be of very ordinary mathematical ability, can hardly escape the conviction that the calculus is a real and powerful tool of his trade and not an educational frill devised mainly for his discomfort.

The problems of different types are introduced by brief and clear statements of the physical and mathematical principles involved and this part of the work is, on the whole, admirably done. Sometimes, especially in the volume on thermodynamics, the brevity of the explanation leads to inadequate or even erroneous definitions and statements of the facts of nature which underly the formulæ and methods used. A student would gain a very imperfect comprehension of absolute temperature or entropy from this volume, but it is to be remembered that it is not designed to be a textbook but a sort of laboratory manual in applied mathematics and that it should be accompanied by a book which is much fuller on the theoretical side.

The problems are of about the same grade of mathematical difficulty as those ordinarily found in textbooks of mechanics which assume a knowledge of the elementary calculus; but the author very sensibly introduces certain valuable methods not usually contained in such books. Thus in Chapter III. of Part III. and in Chapter VI. of Part IV. much practice is provided in dealing with empirical equations, determining their constants and improving their

form; the method of least squares is utilized without much theoretical introduction but with insistence upon what is usually the only practicable way of using the method, viz., by applying it to find small corrections to approximate values of the constants.

Apart from their regular use with a class, these little books are a valuable storehouse of good problems which ought to be of service not only to teachers of engineering and of mathematical physics but perhaps even more to teachers of analytical geometry and the calculus. There can be little doubt that the average college student misses the real point in a good deal of his mathematical study; a great many men get their first adequate and rational idea of the calculus when they come to apply it to mechanics or physics, as every teacher of these subjects knows. There is much to be said in favor of a greater admixture of "practical" problems in the mathematical courses themselves and these books will be helpful to teachers who desire to do this.

H. A. B.

The Emission of Electricity from Hot Bodies. By O. W. RICHARDSON. New York: Longmans, Green and Co., 1916. Pp. vi + 304. Price, \$2.75.

After a brief history of the subject, general statements concerning the theory of ions and ionization, the conductivity of gases drawn from the neighborhood of hot bodies, the ratio of the charge to the mass of an ion and the electron theory of metallic conduction, and brief descriptions of experimental methods of obtaining gas-free bulbs and of measuring temperatures, the author discusses the theory of the emission of electrons from hot bodies. This he does from the point of view of thermodynamics, the kinetic theory of gases and the quantum hypothesis, developing two principal formulas, which later he puts to experimental tests. This part of the book also contains sections on contact differences of potential, the liberation of electrons by chemical action, photoelectric emission by the complete spectrum (black body radiation), etc.

Later chapters are devoted to the effect of gases on the emission of electrons from platinum, tungsten and other substances, the energy transformations connected with the emission of electrons, the emission of positive ions from hot metals and the influence of gases on it, the emission of ions by heated salts and ionization due to chemical reactions, bubbling, splashing, etc.

The book does not contain descriptions of the important practical applications of the subject in wireless telegraphy and telephony, X-radiation, long-distance telephony, etc.

To a large extent (although not entirely) the book represents researches carried on by the author and his co-workers. A scientific investigator has taken the time to write out a connected account of his own researches and related matter, and for this reason, as well as on account of the importance and interest attached to this new subdivision of science, the book should find a place in every library that claims to have an approximately complete set of works on physical science.

Problems in Physics. By WILLIAM D. HENDERSON. New York: McGraw-Hill Book Company, Inc., 1916. Pp. viii + 205. Price, \$1.50.

The book contains 1,025 problems in mechanics, sound, heat, electricity and light, designed for engineering students who are taking a first course in university physics, but who have not yet reached the calculus stage.

The solutions of most of the problems involve the simple substitution of numerical values for the algebraic symbols in the equations that represent some of the fundamental and at the same time elementary laws of physics. These equations are to be found in brief statements of fundamental scientific principles, that precede the various subdivisions into which the book is divided.

A few examples are solved in the text. Answers are not, in general, given.

The book will doubtless be of considerable value to those (both teachers and students) who are engaged in first-year engineering physics.

W. D.

The Physical Properties of Collodial Solutions. By E. F. BURTON. New York: Longmans, Green and Co., 1916. Pp. vii + 200. Price \$1.80.

The subject of collodial solutions is one which has become most attractive by virtue of its extensive applications. In this connection it is necessary to mention only such fields of investigation as dyeing, the properties of clay, the manufacturing of soaps, etc. The subject, however, has been almost repellent owing to its complication and more specially to the desire of different investigators to introduce so many different words. The first investigations were of course those of Graham, about 1861. For a long time there were comparatively few additions to the knowledge obtained from his experiments. Then came an enormous revival of interest and at the present time the literature is practically overwhelming. As a matter of fact a distinctly new science has arisen, with its textbooks, its special journals and special societies.

This book by Dr. Burton, of the University of Toronto, is one of the Monographs on Physics, edited by Sir J. J. Thomson and Dr. Horton. In every respect it is admirable. It states clearly the various terms used by different writers and with great success brings order out of chaos in the analysis of different types of solutions. The references to the literature are wonderfully ample. The statement of theories is in every case clear and the conclusions drawn are as definite as one could expect under the circumstances. The two indexes are most helpful. In short, no book has yet appeared in English or any other language which brings so many facts together, so well coördinated and so well described. It is indispensable to any one interested in the modern developments of this branch of physical chemistry.

The scope of the book and the width of its point of view are best shown in a few words by giving the list of the titles for the different chapters. They are as follows: Preparation and Classification of Collodial Solutions; The Ultra-Microscope; The Brownian Movement; The Optical Properties of Collodial Solutions; Measurements of the Sizes of Ultra-Microscopic Particles; Motion

of Collodial Particles in an Electric Field; The Coagulation of Colloids; Theory of the Stability of Colloids; Practical Applications of the Study of Colloidal Solutions. It will be apparent from this that the author has brought to bear on the subject his extensive knowledge of physics, mathematics and chemistry; and the combination has resulted in a most satisfactory treatise.

J. S. A.

THE
PHYSICAL REVIEW.

SOME APPLICATIONS OF THE ELECTROMAGNETIC EQUATIONS TO MATTER, SUPPOSED TO BE CONSTITUTED OF POSITIVE AND NEGATIVE ELECTRONS.

BY ALBERT C. CREHORE.

THERE is good reason to believe that all so-called gross matter in its neutral state and in the passive condition, which constitutes the very large majority of the matter making up the mass of the earth, is composed of atoms in a practically stable condition, which endure for long periods of time, and that each of these atoms is made of positive and negative electrons, the negative revolving in approximately circular orbits about a center determined by the positive electrons.

The initiative in the following investigations is the belief that all the phenomena of nature will ultimately find their complete and adequate explanation in terms of simpler or more fundamental conceptions by means of an application of the proper theory of the electromagnetic field to the individual electrons making up the atoms. The theorist demands first of all the simplest possible conditions subject to the smallest number of variations, and hence the steady states of these atoms and the matter composed of them are obviously the first to be treated.

There is a tendency among scientists at the present time to take the view that electromagnetic theory is insufficient to account for the atomic phenomena which have been observed, particularly those connected with radiation. This temporary laying aside of the so-called classical dynamics has certainly resulted in much that is good. It has, as one instance, pointed out the existence of a new constant, Planck's " h ," which seems to be universally connected with atomic phenomena. But, let us hope that the laying aside of the electrodynamics is only temporary. The failures of the theory really form the pivotal point for future progress, for it is at such points only that we may discover at what point the theory departed from the facts. There are known to be other possible

forms of the potential function, for example, which would be quite as much in harmony with electromagnetic theory as the one on which the most work has been done up to the present time. It is by a closer examination of those cases which fail that we may look for an inspiration to point out the way to more correct ideas. If one believes in the reality of an aether and regards the true electromagnetic equations merely as the expression of its properties in formal terms, it becomes very difficult to conceive that the individual electrons can escape from its domain, and increases the belief that they must be the very first to come under its sway.

Although the derivation of the general equation for the instantaneous mechanical force with which one moving charge affects another is given in the modern textbooks that deal with this subject, yet, since it is our purpose to trace back to their source the origin of any particular term in the final expression for the mechanical force, we will give the derivation in more detail than would otherwise be desirable.

If we desire first of all to know something of properties of matter as a whole, or in the gross, its rigidity, and tendency to maintain a fixed volume under great changes of pressure, and the possibility of forming crystalline structures, the problem resolves itself in its ultimate analysis into that of finding the average mechanical force which one single negative electron exerts upon a second electron, each revolving at uniform angular velocities in circular orbits. The actual forces between the atoms are merely the sums of the various component parts making up the atom.

The fundamental conceptions of an electromagnetic theory are expressed in terms of the electric and magnetic forces by the following equations. $\mathbf{E}$ and $\mathbf{H}$ are the electric and magnetic forces respectively.

$$c \operatorname{curl} \mathbf{E} = - \dot{\mathbf{H}} \quad (1)$$

and

$$\operatorname{div} \mathbf{H} = 0. \quad (2)$$

In view of (2) it is possible to write

$$\mathbf{H} = \operatorname{curl} \alpha, \quad (3)$$

and α may be defined as the vector potential. Substituting this in (1) we get

$$\operatorname{curl} \left(\mathbf{E} - \frac{1}{c} \dot{\alpha} \right) = 0, \quad (4)$$

whence

$$\mathbf{E} = - \nabla \phi - \frac{1}{c} \dot{\alpha}. \quad (5)$$

This latter equation may be regarded as defining the scalar potential ϕ .

It is recognized that α is not completely determined by (3), and that a large number of different forms for it are possible. The most of the work in the modern theory has been based upon the following condition imposed upon the potentials ϕ and α ,

$$\text{div. } \alpha + \frac{1}{c} \dot{\phi} = 0. \quad (6)$$

To obtain the values of the potentials from this use is made of the fundamental equations of definition

$$\text{div. } \mathbf{E} = 4\pi\rho \quad (7)$$

and

$$c \text{ curl } \mathbf{H} = \dot{\mathbf{E}} + 4\pi\rho\mathbf{q}, \quad (8)$$

whence the assumption concerning the potentials (6) leads to the two equations

$$\nabla^2\phi - \frac{1}{c^2}\ddot{\phi} = -4\pi\rho \quad (9)$$

and

$$\nabla^2\alpha - \frac{1}{c^2}\ddot{\alpha} = -4\pi\rho\frac{\mathbf{q}}{c}. \quad (10)$$

Integrals of these two equations have been obtained by Lorentz, namely

$$\phi = \int [\rho] d\Omega/R \quad (11)$$

$$\alpha = \int \left[\rho \frac{\mathbf{q}}{c} \right] d\Omega/R, \quad (12)$$

in which the square brackets indicate that the quantities within them are to be taken at a time $t - (R/c)$ and not at the actual time t when we want the potentials. When the charges are to be considered as point charges it has been shown that the integration of these latter equations leads to the simple expressions for the potentials

$$\phi = \frac{e_2}{\left[R \left(1 - \frac{\mathbf{q}_2 \cdot \mathbf{R}}{cR} \right) \right]}, \quad (13)$$

$$\alpha = \frac{e_2 \mathbf{q}_2}{c \left[R \left(1 - \frac{\mathbf{q}_2 \cdot \mathbf{R}}{cR} \right) \right]}. \quad (14)$$

An additional fundamental equation due to Larmor and Lorentz is required, to derive the mechanical force exerted by one charge upon the other*

* The vector notation of Gibbs is employed throughout.

$$\mathbf{F} = e_1 \left(\mathbf{E} + \frac{1}{c} \mathbf{q}_1 \times \mathbf{H} \right), \quad (15)$$

which gives from (3) and (5)

$$\mathbf{F} = e_1 \left(-\nabla\phi - \frac{1}{c} \frac{\partial\alpha}{\partial t} + \frac{1}{c} \mathbf{q}_1 \times \mathbf{H} \right), \quad \text{where } \mathbf{H} = \nabla \times \alpha. \quad (16)$$

Performing the operations upon the potentials for point charges (13) and (14) indicated, we obtain the final values of the mechanical force in terms of the velocities of the charges, their accelerations and the distance between them. But it should be remembered that in differentiating the retarded potentials use must be made of the forms

$$\frac{\partial t_1}{\partial t_2} = \left[1 - \frac{\mathbf{q}_2 \cdot \mathbf{R}}{cR} \right], \quad (17)$$

$$\frac{\partial[\psi]}{\partial t} = \frac{1}{\left[1 - \frac{\mathbf{q}_2 \cdot \mathbf{R}}{cR} \right]} \left[\frac{\partial\psi}{\partial t} \right], \quad (18)$$

$$\frac{\partial[\psi]}{\partial x} = \left[\frac{\partial\psi}{\partial x} \right] - \left[\frac{\partial\psi}{\partial t} \right] \left[\frac{\frac{x}{cR}}{1 - \frac{\mathbf{q}_2 \cdot \mathbf{R}}{cR}} \right]. \quad (19)$$

The final result may be most compactly expressed as in (15), in which

$$\mathbf{E} = -\frac{e_2(1 - \beta_2^2)}{R^3 \left(1 - \frac{\mathbf{q}_2 \cdot \mathbf{R}}{cR} \right)^3} \left\{ \mathbf{R} - \frac{R}{c} \mathbf{q}_2 + \frac{\left\{ \mathbf{f}_2 \times \left(\mathbf{R} - \frac{R}{c} \mathbf{q}_2 \right) \right\} \times \mathbf{R}}{c^2(1 - \beta_2^2)} \right\}, \quad (20)$$

and

$$\mathbf{H} = \frac{e_2(1 - \beta_2^2)}{R^3 \left(1 - \frac{\mathbf{q}_2 \cdot \mathbf{R}}{cR} \right)^3} \left\{ -\frac{\mathbf{R} \times \mathbf{q}_2}{c} + \frac{\mathbf{R} \times \left\{ \left[\mathbf{f}_2 \times \left(\mathbf{R} - \frac{R}{c} \mathbf{q}_2 \right) \right] \times \mathbf{R} \right\}}{Rc^2(1 - \beta_2^2)} \right\}. \quad (21)$$

But, since we desire to trace back to their origin the different terms, we will write in detail the different parts. The vector in the last term of (20) may be written

$$\left\{ \mathbf{f}_2 \times \left(\mathbf{R} - \frac{R}{c} \mathbf{q}_2 \right) \right\} \times \mathbf{R} = (\mathbf{f}_2 \cdot \mathbf{R}) \mathbf{R} - R^2 \mathbf{f}_2 + \frac{R}{c} (\mathbf{q}_2 \cdot \mathbf{R}) \mathbf{f}_2 - \frac{R}{c} (\mathbf{q}_2 \cdot \mathbf{R}) \mathbf{q}_2. \quad (22)$$

Hence, putting

$$A = \left(1 - \frac{\mathbf{q}_2 \cdot \mathbf{R}}{cR} \right),$$

the value of $\mathbf{E}$ becomes

$$\mathbf{E} = \frac{e_2}{R^3 A^3} \left\{ \mathbf{R} - \frac{R}{c} \mathbf{q}_2 - \frac{q_2^2}{c^2} \mathbf{R} + \frac{R}{c} \frac{q_2^2}{c^2} \mathbf{q}_2 + \frac{1}{c^2} (\mathbf{f}_2 \cdot \mathbf{R}) \mathbf{R} - \frac{R^2}{c^2} \mathbf{f}_2 \right. \\ \left. + \frac{R}{c^2} (\mathbf{q}_2 \cdot \mathbf{R}) \mathbf{f}_2 - \frac{R}{c^2} (\mathbf{f}_2 \cdot \mathbf{R}) \mathbf{q}_2 \right\}. \quad (23)$$

Returning to the potentials (13) and (14), to find $-\nabla\phi$ and $-\frac{1}{c} \frac{\partial \alpha}{\partial t}$ for substitution in (16), we have

$$-\nabla[\phi] = -[\nabla\phi] + \frac{1}{cRA} \left[\frac{\partial \phi}{\partial t} \right] \mathbf{R} \quad (23a)$$

and

$$-[\nabla\phi] = \frac{e_2}{R^3 A^3} \left\{ A \left(\mathbf{R} - \frac{R}{c} \mathbf{q}_2 \right) \right\}, \quad (24)$$

$$+ \frac{1}{cRA} \left[\frac{\partial \phi}{\partial t} \right] \mathbf{R} = \frac{e_2}{R^3 A^3} \left\{ + \frac{\mathbf{q}_2 \cdot \mathbf{R}}{cR} \mathbf{R} + \frac{1}{c^2} (\mathbf{f}_2 \cdot \mathbf{R}) \mathbf{R} - \frac{q_2^2}{c^2} \mathbf{R} \right\}, \quad (25)$$

$$-\frac{1}{c} \frac{\partial [\alpha]}{\partial t} = \frac{e_2}{R^3 A^3} \left\{ -\frac{R^2}{c^2} A \mathbf{f}_2 - \frac{1}{c^2} (\mathbf{q}_2 \cdot \mathbf{R}) \mathbf{q}_2 - \frac{R}{c^2} (\mathbf{f}_2 \cdot \mathbf{R}) \mathbf{q}_2 + \frac{R q_2^2}{c^2} \mathbf{q}_2 \right\}. \quad (26)$$

But, comparing (15) and (16),

$$\mathbf{E} = -\nabla[\phi] - \frac{1}{c} \frac{\partial [\alpha]}{\partial t}. \quad (27)$$

Hence, by adding (23) and (26) or (24), (25) and (26)

$$\mathbf{E} = \frac{e_2}{R^3 A^3} \left\{ \mathbf{R} - \frac{R}{c} \mathbf{q}_2 - \frac{1}{cR} (\mathbf{q}_2 \cdot \mathbf{R}) \mathbf{R} + \frac{1}{c^2} (\mathbf{q}_2 \cdot \mathbf{R}) \mathbf{q}_2, \text{ or } -[\nabla\phi] \right. \\ \left. + \frac{1}{cR} (\mathbf{q}_2 \cdot \mathbf{R}) \mathbf{R} + \frac{1}{c^2} (\mathbf{f}_2 \cdot \mathbf{R}) \mathbf{R} - \frac{q_2^2}{c^2} \mathbf{R}, \text{ or } + \frac{1}{cRA} \left[\frac{\partial \phi}{\partial t} \right] \right. \\ \left. - \frac{R^2}{c^2} \mathbf{f}_2 + \frac{R}{c^2} (\mathbf{q}_2 \cdot \mathbf{R}) \mathbf{f}_2 - \frac{1}{c^2} (\mathbf{q}_2 \cdot \mathbf{R}) \mathbf{q}_2 \right. \\ \left. - \frac{R}{c^2} (\mathbf{f}_2 \cdot \mathbf{R}) \mathbf{q}_2 + \frac{R q_2^2}{c^2} \mathbf{q}_2, \text{ or } -\frac{1}{c} \frac{\partial [\alpha]}{\partial t} \right\}. \quad (28)$$

It is to be observed that two pairs of terms in this cancel out, leaving the result identical with (23) above, which, therefore, completes the derivation of the electric force $\mathbf{E}$ from the potentials. As an alternative way of writing the fundamental equation for the mechanical force (15) or (16), we may put

$$\mathbf{F} = e_1 \left\{ -\nabla[\phi] - \frac{1}{c} \frac{\partial [\alpha]}{\partial t} + \frac{1}{c} \mathbf{q}_1 \times (\nabla \times [\alpha]) \right\}, \quad (16a)$$

where the operations to be performed are according to (17), (18) and (19). Hence we find

$$\mathbf{F} = e_1 \left\{ \begin{array}{l} -\nabla[\phi] \text{ or } -[\nabla\phi] + \frac{1}{cR} \left(\frac{\partial t_2}{\partial t_1} \right) \left[\frac{\partial \phi}{\partial t} \right] \\ -\frac{1}{c} \frac{\partial[\alpha]}{\partial t} \text{ or } -\frac{1}{c} \left(\frac{\partial t_2}{\partial t_1} \right) \left[\frac{\partial \alpha}{\partial t} \right] \\ +\frac{1}{c} \mathbf{q}_1 \times (\nabla \times [\alpha]) \text{ or} \\ \frac{1}{c} \mathbf{q}_1 \times [\nabla \times \alpha] - \frac{1}{c^2 R} \left(\frac{\partial t_2}{\partial t_1} \right) \left(\mathbf{q}_1 \times \left[\mathbf{R} \times \frac{\partial \alpha}{\partial t} \right] \right) \end{array} \right\}. \quad (16b)$$

Here $[\nabla\phi]$ and $[\nabla \times \alpha]$ involve the differentiation of ϕ and α with respect to the space coördinates but not with respect to the time, and all the other terms involve their differentiation with respect to the time. The force equation is here put into this particular form because it is desirable to separate the differentiations with respect to space and time for reasons that appear in a later section of this paper.

UNIFORM CIRCULAR MOTION.

The above equations are general for any kind of motion of the two charges. We will now introduce the conditions that must be satisfied when the charges each move at constant angular velocity in circular orbits. In a previous communication¹ I have given the solution for the special case of two coaxial orbits only. In dealing with matter in its general form the solution of this problem for the most general possible positions of two circles must be obtained.

To define the circular motion select two sets of rectangular axes i, j , and k for the charge e_1 , and i', j' and k' for e_2 , taking the origin of the former at the center of the orbit of e_1 , and the latter at the center of the orbit of e_2 . The k and k' axes are respectively perpendicular to the planes of the orbits of e_1 and e_2 , the positive directions being such that the rotation of the electrons as viewed from the positive end of the axis are each in the clockwise sense. In general the planes of the two orbits intersect in some line. This common intersection is selected for the direction of the j - and j' -axes, so that for our purposes the prime may be suppressed, the directions of the two axes being common. The positive direction along the j - and j' -axes is defined by the vector $k \times k'$, which always takes the direction of the j -axis.

Let O_1 and O_2 represent the fixed centers of the orbits of e_1 and e_2 , and let $\mathbf{r}_1$ and $\mathbf{r}_2$ be the rotating vectors O_1e_1 and O_2e_2 respectively. These may be defined by the equations

$$\mathbf{r}_1 = a_1(S_1i + C_1j), \quad (29)$$

$$\mathbf{r}_2 = a_2(S_2i' + C_2j'), \quad (30)$$

¹Not yet published.

where a_1 and a_2 are the radii, and S and C represent the expressions

$$S_1 = \sin(\omega_1 t + \theta_1), \quad C_1 = \cos(\omega_1 t + \theta_1), \quad (31)$$

$$S_2 = \sin\left[\omega_2\left(t - \frac{R}{c}\right) + \theta_2\right], \quad C_2 = \cos\left[\omega_2\left(t - \frac{R}{c}\right) + \theta_2\right], \quad (32)$$

in which ω_1 and ω_2 are the angular velocities, and θ_1 and θ_2 the respective phase angles at the time t .

Let $\mathbf{r}$ represent the constant vector from the center O_1 to O_2 , and $\mathbf{R}$ the variable vector from e_2 at time $t - R/c$ to e_1 at time t . Then

$$\mathbf{R} = \mathbf{r}_1 - \mathbf{r} - \mathbf{r}_2. \quad (33)$$

The i' and k' axes may be transformed into the i and k axes by the relations

$$i' = (\cos \alpha)i + (\sin \alpha)k, \quad (34)$$

$$k' = -(\sin \alpha)i + (\cos \alpha)k. \quad (35)$$

Denoting by x , y and z the coördinates of the center O_2 referred to O_1 as origin, and writing all quantities in terms of the rectangular unit axes i , j and k , we have

$$\mathbf{r} = xi + yj + zk, \text{ a constant.} \quad (36)$$

$$\mathbf{r}_1 = a_1(S_1 i + C_1 j), \quad (29)$$

$$\mathbf{r}_2 = a_2(S_2 \cos \alpha i + C_2 j + S_2 \sin \alpha k), \quad (37)$$

$$\begin{aligned} \mathbf{R} = & (-x + a_1 S_1 - a_2 S_2 \cos \alpha)i + (-y + a_1 C_1 - a_2 C_2)j \\ & + (-z - a_2 S_2 \sin \alpha)k. \end{aligned} \quad (38)$$

From this we get the value of R^2

$$\begin{aligned} R^2 = & (x^2 + a_1^2 + a_2^2) + 2(-a_1 x S_1 - a_1 y C_1 + a_2 z \sin \alpha S_2 + a_2 y C_2 \\ & + a_2 x \cos \alpha S_2 - a_1 a_2 \cos \alpha S_1 S_2 - a_1 a_2 C_1 C_2). \end{aligned} \quad (39)$$

Denoting by s^2 the first parenthesis in (39), as being quantities independent of the time, and the rest of (39) by us^2 , u being a function of the time, we have

$$R^2 = s^2(1 + u), \quad (40)$$

where

$$u = 2/s^2 (\text{quantities in second parenthesis of (39)}). \quad (41)$$

The velocities and accelerations of the charges may be obtained by differentiation of (29) and (37), giving

$$\mathbf{q}_1 = a_1 \omega_1 (C_1 i - S_1 j), \quad (42)$$

$$\mathbf{q}_2 = a_2 \omega_2 (C_2 \cos \alpha i - S_2 j + C_2 \sin \alpha k), \quad (43)$$

$$\mathbf{f}_2 = -a_2 \omega_2^2 (S_2 \cos \alpha i + C_2 j + S_2 \sin \alpha k). \quad (44)$$

The above are all the expressions required on the hypothesis of uniform circular motion to find the complete values of $\mathbf{E}$ and $\mathbf{H}$ and the mechanical force $\mathbf{F}$. The process of finding them is long on account of the large number of terms involved but it seems worth giving in full. We have to find several vectors required by the complex expressions in $\mathbf{E}$ and $\mathbf{H}$, (20) and (21), first

$$\begin{aligned} \mathbf{R} - \frac{R}{c} \mathbf{q}_2 = & (-x + a_1 S_1 - a_2 S_2 \cos \alpha - R\beta_2 \cos \alpha C_2) i \\ & + (-y + a_1 C_1 - a_2 C_2 + R\beta_2 S_2) j \\ & + (-z - a_2 S_2 \sin \alpha - R\beta_2 C_2 \sin \alpha) k, \end{aligned} \quad (45)$$

$$\begin{aligned} \mathbf{f}_2 \times \left(\mathbf{R} - \frac{R}{c} \mathbf{q}_2 \right) = & -a_2 \omega_2^2 \{ [-z C_2 - R\beta_2 \sin \alpha \\ & + y S_2 \sin \alpha - a_1 S_2 C_1 \sin \alpha] i + [(z \cos \alpha \\ & - x \sin \alpha) S_2 + a_1 S_1 S_2 \sin \alpha] j + [-y S_2 \cos \alpha \\ & + a_1 C_1 S_2 \cos \alpha + R\beta_2 \cos \alpha + x C_2 - a_1 C_2 S_1] k \}, \end{aligned} \quad (46)$$

$$\begin{aligned} \frac{\left[\mathbf{f}_2 \times \left(\mathbf{R} - \frac{R}{c} \mathbf{q}_2 \right) \right] \times \mathbf{R}}{C^2(1 - \beta_2^2)} = & -\frac{a_2 \omega_2^2}{c^2(1 - \beta_2^2)} \{ (-z - a_2 S_2 \sin \alpha) [(z \cos \alpha \\ & - x \sin \alpha) S_2 + a_1 S_1 S_2 \sin \alpha] i - (-y + a_1 C_1 - a_2 C_2) [-y S_2 \cos \alpha \\ & + a_1 C_1 S_2 \cos \alpha + R\beta_2 \cos \alpha + x C_2 - a_1 C_2 S_1] j + (-x + a_1 S_1 \\ & - a_2 S_2 \cos \alpha) [-y S_2 \cos \alpha + a_1 C_1 S_2 \cos \alpha + R\beta_2 \cos \alpha + x C_2 \\ & - a_1 C_2 S_1] j - (-z - a_2 S_2 \sin \alpha) [-z C_2 - R\beta_2 \sin \alpha + y S_2 \sin \alpha \\ & - a_1 S_2 C_1 \sin \alpha] j + (-y + a_1 C_1 - a_2 C_2) [-z C_2 - R\beta_2 \sin \alpha \\ & + y S_2 \sin \alpha - a_1 S_2 C_1 \sin \alpha] k - (-x + a_1 S_1 - a_2 S_2 \cos \alpha) \\ & - [(z \cos \alpha - x \sin \alpha) S_2 + a_1 S_1 S_2 \sin \alpha] k \}. \end{aligned} \quad (47)$$

The complete part of the force arising from the first two terms $-\nabla\phi - [(1/c)(\partial\alpha/\partial t)]$, or from the first term of (15) may now be found by adding (45) and (47) and multiplying by e_1 , whence after multiplying out the quantities in (47), and putting

$$A = 1 - \frac{\mathbf{q}_2 \cdot \mathbf{R}}{CR},$$

and separating the terms in various powers of R , we obtain

$$\begin{aligned}
F_{E_i} = e_1 E = \frac{e_1 e_2}{R^2 A^3} & [-\beta_2 C_2 \cos \alpha] i \text{ from (45)} \\
& + \frac{e_1 e_2}{R^3 A^3} [-x + a_1 S_1 - a_2 S_2 \cos \alpha] i \text{ from (45), the electrostatic} \\
& \text{part not involving } \beta, \\
& + \frac{e_1 e_2}{R^2 A^3} [+ \beta_2^3 C_2 \cos \alpha] i \text{ from (45)} \\
& + \frac{e_1 e_2}{R^3 A^3} [\beta_2^2 (\underline{x} - a_1 S_1 + a_2 S_2 \cos \alpha)] i \text{ from (45)} \\
& + \frac{e_1 e_2}{R^2 A^3} \left[\beta_2^3 \cos \alpha \left(-\frac{y}{a_2} + \frac{a_1}{a_2} C_1 - C_2 \right) \right] i \text{ from (47)} \quad (48) \\
& + \frac{e_1 e_2}{R^3 A^3} \left\{ \frac{\beta_2^2}{a_2} [(z \cos \alpha - x \sin \alpha) z S_2 + a_1 z S_1 S_2 \sin \alpha \right. \\
& \left. + \underline{a_2 x S_2^2 \sin \alpha \cos \alpha} - \underline{a_2 x S_2^2 \sin^2 \alpha} + a_1 a_2 S_1 S_2^2 \sin^2 \alpha \right. \\
& \left. + y^2 S_2 \cos \alpha - 2 a_1 y C_1 S_2 \cos \alpha - x y C_2 + a_1 y C_2 S_1 \right. \\
& \left. + a_1^2 C_1^2 S_2 \cos \alpha + a_1 x C_1 C_2 - a_1^2 C_1 C_2 S_1 + a_2 y C_2 S_2 \cos \alpha \right. \\
& \left. - a_1 a_2 C_1 C_2 S_2 \cos \alpha - \underline{a_2 x C_2^2} + a_1 a_2 C_2^2 S_1] \right\} i \text{ from (47),}
\end{aligned}$$

$$\begin{aligned}
F_{E_j} = e_1 E = \frac{e_1 e_2}{R^2 A^3} & [+ \beta_2 S_2] j \text{ from (45)} \\
& + \frac{e_1 e_2}{R^3 A^3} [-y + a_1 C_1 - a_2 C_2] j \text{ from (45) (electrostatic part)} \\
& + \frac{e_1 e_2}{R^2 A^3} [-\beta_2^3 S_2] j \text{ from (45)} \\
& + \frac{e_1 e_2}{R^3 A^3} [\beta_2^2 (\underline{y} - a_1 C_1 + a_2 C_2)] j \text{ from (45)} \\
& + \frac{e_1 e_2}{R^2 A^3} \left[\beta_2^3 \left(\frac{x}{a_2} \cos \alpha - \frac{a_1}{a_2} S_1 \cos \alpha + S_2 + \frac{z}{a^2} \sin \alpha \right) \right] j \quad (49) \\
& \text{from (47)} \\
& + \frac{e_1 e_2}{R^3 A^3} \left\{ \frac{\beta_2^2}{a_2} [-x y S_2 \cos \alpha + a_1 x C_1 S_2 \cos \alpha + x^2 C_2 \right. \\
& \left. - a_1 x C_2 S_1 + a_1 y S_1 S_2 \cos \alpha - a_1^2 S_1 C_1 S_2 \cos \alpha \right. \\
& \left. - a_1 x S_1 C_2 + a_2^2 S_1^2 C_2 - \underline{a_2 y S_2^2 \cos^2 \alpha} \right. \\
& \left. + a_1 a_2 C_1 S_2^2 \cos^2 \alpha + a_2 x S_2 C_2 \cos \alpha - a_1 a_2 S_2 C_2 S_1 \cos \alpha \right. \\
& \left. + x^2 C_2 - y z S_2 \sin \alpha + a_2 z S_2 C_2 \sin \alpha - \underline{a_2 y S_2^2 \sin^2 \alpha} \right. \\
& \left. + a_1 a_2 S_2^2 C_1 \sin^2 \alpha + a_2 z C_1 S_2 \sin \alpha] \right\} j \text{ from (47),}
\end{aligned}$$

$$\begin{aligned}
\mathbf{F}_{E_1} = e_1 \mathbf{E}_1 &= \frac{e_1 e_2}{R^2 A^3} [-\beta_2 C_2 \sin \alpha] k \text{ from (45)} \\
&+ \frac{e_1 e_2}{R^2 A^3} [-z - a_2 S_2 \sin \alpha] k \text{ from (45) (the electrostatic part)} \\
&+ \frac{e_1 e_2}{R^2 A^3} [+ \beta_2^2 C_2 \sin \alpha] k \text{ from (45)} \\
&+ \frac{e_1 e_2}{R^2 A^3} [\beta_2^2 (\underline{z} + a_2 S_2 \sin \alpha)] k \text{ from (45)} \\
&+ \frac{e_1 e_2}{R^2 A^3} \left[\beta_2^2 \sin \alpha \left(-\frac{y}{a_2} + \frac{a_1}{a_2} C_1 - C_2 \right) \right] k \text{ from (47)} \quad (50) \\
&+ \frac{e_1 e_2}{R^2 A^3} \left\{ \frac{\beta_2^2}{a_2} [-yzC_2 + y^2 S_2 \sin \alpha + a_1 z C_1 C_2 \right. \\
&\quad - 2a_1 y C_1 S_2 \sin \alpha + a_1^2 C_1^2 S_2 \sin \alpha - \underline{a_2 z C_2^2} \\
&\quad + a_2 y C_2 S_2 \sin \alpha - a_1 a_2 C_2 S_2 C_1 \sin \alpha \\
&\quad - (z \cos \alpha - x \sin \alpha) x S_2 - a_1 x S_1 S_2 \sin \alpha \\
&\quad + a_1 (z \cos \alpha - x \sin \alpha) S_1 S_2 + a_1^2 S_1^2 S_2 \sin \alpha \\
&\quad \left. - \underline{a_2 S_2^2 (z \cos \alpha - x \sin \alpha) \cos \alpha} - a_1 a_2 S_1 S_2^2 \sin \alpha \cos \alpha] \right\} k \\
&\quad \text{from (47)}.
\end{aligned}$$

Before proceeding to find the second part of the force $(e_1/c) \mathbf{q}_1 \times \mathbf{H}$ in (15) arising from the magnetic component, let us examine the above result arising from the electric force. I had been given to understand on the part of several that these modern, sometimes called exact, equations based upon retarded potentials give rise to no terms varying as the inverse square of the distance, and that the inverse cube¹ of the distance would be the lowest order term to appear. It was with considerable surprise, therefore, that terms of all orders of the inverse powers of the distance appeared in the equations just obtained, because I had formerly made use of approximate forms not involving retarded potentials, and there was no reason to question the correctness of the findings of others.

AVERAGE VALUES.

The above expressions are for the instantaneous force upon electron e_2 due to e_1 . To obtain the average over a long time we must average (48), (49) and (50) over a long time. Taking the quantity

$$A = 1 - \frac{\mathbf{q}_2 \cdot \mathbf{R}}{cR},$$

¹ F. R. Sharpe, *PHYS. REV.*, Sept., 1912, p. 231.

which appears in the coefficients of the equations, as sensibly equal to unity when q_2 is a small quantity compared with c , we obtain different results in averaging dependent upon the relative values of the angular velocities of the two electrons. If the angular velocities are incommensurable, which would be the case generally, we obtain the following average force expressed in a series¹ of inverse powers of r , and including the r^{-1} and r^{-2} terms only.

$$\begin{aligned} F_i = e^2 \beta_2^2 \left\{ \frac{\beta_2}{a_2} (-Y \cos \alpha) r^{-1} + \left[\frac{1}{2} X - \frac{1}{2} X \sin^2 \alpha + \frac{1}{2} Z \sin \alpha \cos \alpha \right. \right. \\ \left. \left. - \frac{3}{2} Z^2 \sin \alpha \cos \alpha - \frac{3}{2} X Z^2 \cos^2 \alpha + \frac{3}{2} X Z^2 \sin^2 \alpha + \frac{3}{2} X^2 Z \sin \alpha \cos \alpha \right. \right. \\ \left. \left. - \frac{3}{2} Y^2 Z \sin \alpha \cos \alpha - \frac{3}{2} X Y^2 \cos^2 \alpha + \frac{3}{2} X Y^2 \right] r^{-2} \dots \right\} i, \end{aligned} \quad (51)$$

$$\begin{aligned} F_j = e^2 \beta_2^2 \left\{ \frac{\beta_2}{a_2} (X \cos \alpha + Z \sin \alpha) r^{-1} + \left[\frac{1}{2} Y + 3 X Y Z \sin \alpha \cos \alpha \right. \right. \\ \left. \left. + \frac{3}{2} X^2 Y \cos^2 \alpha + \frac{3}{2} Y Z^2 \sin^2 \alpha - \frac{3}{2} Y Z^2 - \frac{3}{2} X^2 Y \right] r^{-2} \dots \right\} j, \end{aligned} \quad (52)$$

$$\begin{aligned} F_k = e^2 \beta_2^2 \left\{ \frac{\beta_2}{a_2} (-Y \sin \alpha) r^{-1} + \left[\frac{1}{2} Z - \frac{1}{2} Z \cos^2 \alpha + \frac{1}{2} X \sin \alpha \cos \alpha \right. \right. \\ \left. \left. - \frac{3}{2} Y^2 Z \sin^2 \alpha - \frac{3}{2} X Y^2 \sin \alpha \cos \alpha + \frac{3}{2} X Z^2 \sin \alpha \cos \alpha \right. \right. \\ \left. \left. - \frac{3}{2} X^2 Z \sin^2 \alpha + \frac{3}{2} X^2 Z \cos^2 \alpha - \frac{3}{2} X^3 \sin \alpha \cos \alpha \right. \right. \\ \left. \left. + \frac{3}{2} Y^2 Z \right] r^{-2} \dots \right\} k. \end{aligned} \quad (53)$$

The letters X , Y and Z have been introduced as equivalent to x/r , y/r and z/r , the direction cosines of the center of the orbit of e_2 referred to the axes i , j and k with origin at O_1 . α is the fixed angle between the directions of the axes of rotation of the two electrons, and r the fixed distance between the centers.

Had the two electrons been synchronous with each other so that $\omega_1 = \omega_2$, additional terms involving the constant angle of phase difference, γ , would have appeared, but only in the first power of $\sin \gamma$ and $\cos \gamma$, and, since all such terms cancel out in summing up for a ring of two or more electrons, such as we suppose exist in an atom, we do not obtain

¹ In former papers the letters m , n and v were used instead of a_1 , a_2 and r respectively, as denoting the same quantities, the radii of the two orbits and the distances between their centers, the former, however being measured in a small unit a_* instead of in centimeters, and being therefore ratios to this unit. The value of v seldom is less than 100 and the series is evidently rapidly convergent when expressed in inverse powers of v . The convergence is just the same when the equation is written in terms of a and r , although at first sight the convergence is not so evident. The use of the former notation is therefore discarded here as being unnecessary.

any additional terms. It should be remarked that the electrostatic terms in (48), (49) and (50), those which remain when $\beta = 0$, namely

$$\mathbf{F} = \frac{e_1 e_2}{R^3} \{ [-x + a_1 S_1 - a_2 S_2 \cos \alpha] i + [-y + a_1 C_1 - a_2 C_2] j \\ + [-z - a_2 S_2 \sin \alpha] k \},$$

have been omitted in obtaining the average force for the reason that their value has been obtained elsewhere.¹ When the effect of the positive charge of the atom is also included so as to get the force between atoms instead of electrons it has been shown that the lowest order terms which make their appearance due to these electrostatic forces is in r^6 .

The above result shows that in general the force between two electrons in circular orbits is not in the direction of the line joining the centers of the orbits, but if we resolve the force along the direction or $\mathbf{r}$ the inverse first power terms exactly cancel each other, leaving the force-series to begin with r^{-2} as the lowest order term. To show this multiply (51) by X , (52) by Y and (53) by Z and add to resolve the whole force along $\mathbf{r}$, giving

$$\mathbf{F}_r = e^2 \beta^2 \left\{ \frac{1}{2} - \frac{1}{2} (-X \sin \alpha + Z \cos \alpha)^2 \right\} r^{-2}. \quad (54)$$

INTERPRETATION OF THIS RESULT.

This equation expresses that portion of the total force between two electrons revolving in circular orbits resolved along the line joining centers arising from the electric force $\mathbf{E}$, but not including the electrostatic terms as above explained. If we obtain in a similar manner the portion arising from the magnetic force $\mathbf{H}$, another set of inverse square terms is obtained, but the coefficient of all such terms is in the higher orders of the small quantity β , and they are, therefore, negligible in comparison with the terms in (54) arising from the electric force. This equation, therefore, expresses the whole force to a very close degree of approximation between two electrons in circular orbits having incommensurable velocities resolved along the line of centers, when the distance between the centers is very large compared to the radius of either orbit, because the terms in higher powers, r^{-3} , r^{-4} , etc., become inappreciable if the distance r is large enough.

In case the angular velocities are equal, as above explained, this equation applies to rings of electrons when multiplied by the numbers in the rings, and it does not matter in the case of rings whether the velocities are synchronous or non-synchronous. There is no limit to the distances at which these forces are felt, and the equation thus sug-

¹ The average of the electrostatic forces for atom on atom is given by the r^{-6} terms in equations (23), (24) and (25), p. 755, Phil. Mag., June, 1915.

gests that this may be the gravitational force because it varies inversely as the square of the distance. On this account it is worth making a critical examination of this equation. The general appearance is right for the gravitational force for several reasons. First, the force is always an attraction independent of the directions of the axes. Heretofore, the forces between atoms as yet obtained in former papers have depended almost entirely upon the directions of the axes of rotation, being either an attraction or a repulsion according to their directions. We may show that this is not the case with equation (54). Let us designate the ring of electrons, of which e_2 is one, as atom A_2 , and the ring, of which e_1 is one, as atom A_1 . Imagine that the centers of the two atoms are fixed in space, and suppose that the axis of rotation of A_2 is prolonged, and that a perpendicular line from the center of A_1 is drawn to it. The quantities within the brace in (54) then represent $\frac{1}{2}$ the square of the ratio of this perpendicular line to the fixed distance r . The part $-x \sin \alpha + z \cos \alpha$ is the distance from A_2 along its axis to the foot of this perpendicular line, hence $r^2 - (-x \sin \alpha + z \cos \alpha)^2$ is the square of the perpendicular line, and the square of the ratio of this perpendicular to the distance r is $1 - (-X \sin \alpha + Z \cos \alpha)^2$, which quantity appears in (54).

When the distance from A_2 to the foot of this perpendicular vanishes, the whole force reduces to

$$F = \frac{1}{2} e^2 \beta_2^2 r^{-2}. \quad (55)$$

It will thus vanish when the axis of A_2 is perpendicular to the line r , the atom A_1 then lying in the equatorial plane of A_2 . If we apply the theory of probabilities to the case, and suppose that the body, of which A_2 is but a single atom, is either a gas, liquid, or homogeneous solid without crystalline structure, it is then just as likely that the positive pole of any axis will point toward A_1 as that a negative pole will do so, and the position of the average one of a large number of atoms must be such that A_1 lies in the equatorial plane of the average atom in the second mass, and for such an atom the force is that given by (55).

Another way of looking at the matter is to observe that the sum of the squares of the perpendicular lines drawn from the mass in which A_1 is one atom, to the various axes of rotation produced from the atoms in the second mass in no way depends upon the orientation of the axes in the first mass, so that we are at liberty to orient the first mass as we please without affecting the attraction due to the second mass. If this is true of mass No. 1 it must also be true of mass No. 2, for we may make the calculation the other way about interchanging the masses.

A third consideration on the same subject is to fix the attention upon a single atom A_1 , upon which the force is to be obtained. The quantities

z or Z are then approximately fixed and invariant for all atoms in mass No. 2. The independent variables are x or X and α . If we seek for the average values of X and α on the theory of probabilities for a large number of atoms in mass No. 2, the average value of α is 60° , and the average value of X is zero. Hence the total force of all atoms in mass No. 2 on the single atom A_1 , when divided by the number of atoms in mass No. 2, is

$$F = \frac{1}{2}e^2\beta_1^2(1 - \frac{1}{2}Z^2)r^{-2}. \quad (56)$$

If we next consider other atoms in mass No. 1, the average value of Z is zero, and the total force of mass No. 2 upon mass No. 1 divided by the product of the numbers of atoms in the two masses, provided the atoms are of the same kind, is the same as equation (55) above.

ATTRACTION BETWEEN CRYSTALS.

The question now naturally arises as to the gravitational attraction between crystals. Mackenzie¹ has shown experimentally in a rather convincing manner that the variation in the attraction between two crystals of calcspar when oriented in all possible directions is not greater than one part in about 500, or about .2 per cent. This seems to indicate that there is no variation whatever to a first order approximation, and the proper theory of gravitation should give a reason for this. Before we can proceed to examine this question it is necessary to know just how the axes of rotation are directed in any given crystal. I am not acquainted with any investigations of others upon this subject, and the only crystals as yet investigated belong to the cubic or isometric system, but the determination of the directions of the axes of rotation rests upon a very simple conception, namely, that there is a tendency on the part of any atom to turn the axis of any other atom in the crystal so as to become parallel with itself, and, when a second atom has its axis parallel to the first, the turning moment vanishes. This alone is sufficient to determine the directions of the axes of rotation in a crystal, because the sum of all the turning moments acting upon any atom due to all the other surrounding atoms must vanish. This has been shown to be true for the arrangement of the axes in about twenty forms of crystals each belonging to the cubic system. In each case there are but four different directions to the axes of the various atoms making up the whole crystal, and these are parallel to the four medial lines of a regular tetrahedron, or which is the same, to the four diagonals of a cube. These lines make equal angles with each other, about 109° , or exactly so that $\cos \alpha = -\frac{1}{3}$ and $\sin \alpha = \frac{2}{3}\sqrt{2}$.

¹ A. S. Mackenzie, "On the Attraction of Crystalline and Isotropic Masses at Small Distances," *PHYS. REV.*, Vol. II., Mar.-Apr., 1895, p. 321.

Let us calculate the attractive force between two small diamonds, say, at a great distance apart compared with their size by means of (54). It is necessary to show that the attraction is independent of the orientation of the diamonds if it is the gravitational force. We will calculate the force exerted by the second mass upon the first. Draw four lines parallel to the four directions of the axes of all the atoms in diamond No. 2 through the center of its mass, and then draw a perpendicular line from the center of diamond No. 1 upon each of the four lines thus drawn. Since the whole diamond is made up of equal numbers of atoms with axes parallel to these four directions, it is sufficient to find the attraction of a single group of four atoms with axes, one in each of these four directions, and then to multiply the result by the number of such groups. If we can show that one such group exerts an attraction which is independent of the orientation of the group with respect to the position of the first diamond we shall have proved the case. This is equivalent to saying that the sum of the squares of the four perpendicular lines from diamond No. 1 upon the four different axes of the group in diamond No. 2 shall be constant for a fixed distance, r , between the diamonds. This leads to the statement of a geometrical proposition which, if true, proves the case, as follows.

If through any point four lines be drawn making equal angles each with any other, and, if from a second point at a distance r from the first point four perpendiculars be drawn one to each of the said four lines, then the sum of the squares of these perpendiculars is constant for all points at the same distance from the first point. The locus of the second point is the surface of a sphere with the first point as center.

The truth or falsity of this proposition was unknown at the time of arriving at the conclusion that it must be true if this is the proper form for the gravitational force. It has since been proved to be true.¹

Another way of looking at the matter is to show that the force in (54) in no way depends upon the orientation of diamond No. 1, for the perpendiculars from No. 1 upon the axes of No. 2 would not be changed by turning the diamond No. 1. If we calculate the force the other way about it is equally evident that we should be able to turn No. 2 in any direction without changing the force.

THE MAGNITUDE OF THE ATTRACTION.

To calculate the magnitude of the attraction between two diamonds let us place the second diamond so that one of the four axes of the atoms

¹ The truth of this proposition has been established by Prof. F. W. Owens independently, though it is possible that it may have been published somewhere.

in it passes through diamond No. 1, since it makes no difference in the force how they are oriented. The perpendicular upon this axis is then zero, and those upon the other three axes are equal to each other. This perpendicular is $\frac{2}{3}\sqrt{2}r$, and the distance from A_2 to the foot of it is $\frac{2}{3}r$. The sum of the squares of the four perpendiculars is then $\frac{8}{3}r^2$. Hence the total force exerted by a group of four electrons, one in each of four atoms composing a group in diamond No. 2, upon a single electron in diamond No. 1 is

$$F = \frac{4}{3}e^2\beta_2^2r^{-2}. \quad (57)$$

Before we can calculate numerically the total force with which one diamond attracts the other in order to compare the magnitude with that derived from the gravitational constant, it is necessary to know more about the structure of the carbon atom. The diamond has been selected for this purpose because there is but a single kind of atom in it, and the atomic weight of carbon is as small as that of any atom we know which forms crystals without other elements. From previous investigations it is thought that the carbon atom has twelve electrons, consisting of a ring of 8 within which is a ring of 4, and we will proceed with this form of atom. Recently it has been shown that a ring of eight electrons, based upon the fundamental equations (1) to (21) above, which forms the basis of the present discussion, must have a linear velocity so as to make $\beta = .012$ approximately, irrespective of the radius of the orbit. It has also been shown that the ring of four electrons within the eight will not affect the speed of the eight in an appreciable degree. If the ring of four were by itself, it would have a speed such that $\beta = .00846$, but, when within the ring of eight, the mutual forces may bring the speed of the inside ring to the same angular velocity as the outside ring, which really controls the atom. However this may be, it will appear that the inside ring of four has but little effect upon the whole attraction. It has been calculated that the radius of the inside ring is .402 part of that of the outside ring, and, assuming the angular velocity the same as the outside ring, $\beta = .402 \times .012 = .004824$ for the inside ring. For the outside ring $\beta^2 = 1.44 \times 10^{-4}$, and for the inside ring $\beta^2 = .2328 \times 10^{-4}$. To get the sum of β^2 for the carbon atom, multiply the former by 8 and the latter by 4 and add, giving

$$\sum \beta^2 = (11.52 + .93) \times 10^{-4} = 12.45 \times 10^{-4},$$

a value not much greater than for the outside ring alone.

To obtain the average pull between one atom in diamond No. 1 and one in No. 2, multiply (57) by 12 for the number of electrons in atom A_1 ,

and put for β_z^2 the $\sum_F \beta^2$ for the atom A_1 , and divide by 4 because (57) is for a group of four electrons. We thus find the average attraction between two carbon atoms at a distance of 1 cm. apart in the diamond to be

$$F = 4 \times 12.45 \times 10^{-4} e^2 = 49.8 \times 10^{-4} e^2. \quad (58)$$

Taking the value of e in electrostatic units as 4.77×10^{-10} , we find

$$F = 1134 \times 10^{-24} \text{ dynes.} \quad (59)$$

Let us next assume that the two diamond crystals are cubes 10^{-8} cm. on an edge, and that the distance between their centers of mass is 1 cm. The number of atoms that such a cube with edge a contains is $2\sqrt{2}(a^3/l^3)$ where $l = 2.528 \times 10^{-8}$ cm., this being the edge of the elementary tetrahedron. If N is the number of atoms in this cube, when $a = 10^{-8}$ cm.,

$$N = \frac{2\sqrt{2} \times 10^{-18}}{16.16 \times 10^{-24}} = 175,000. \quad (60)$$

Hence the total attraction between the two cubes is N^2 times (59), and

$$F = .03062 \times 10^{12} \times 1134 \times 10^{-24} = 34.72 \times 10^{-12} \text{ dynes.} \quad (61)$$

THE GRAVITATIONAL ATTRACTION.

This value may now be compared with the gravitational pull between these diamonds as calculated from the well-known gravitational constant. The force at 1 cm. distance is

$$F = 666 \times 10^{-10} m^2,$$

where m is the mass of one cube. Since $m = \text{density} \times \text{volume} = 3.51 \times 10^{-18}$, we have

$$F = 666 \times 10^{-10} \times 3.51^2 \times 10^{-36} = 82.1 \times 10^{-44} \text{ dynes.} \quad (62)$$

A comparison between (61) and (62) shows that the theoretical force is 4.23×10^{31} times the actual gravitational force between the same pieces of matter. In other words the calculated pull between the two diamonds is immensely greater than any existing force acting between them. The order of magnitude is so greatly different that there is no question that the magnitude of the calculated force is in error somewhere. Let us examine the case more closely. The only two hypotheses which have been used in obtaining an absurdly large value for the force between two pieces of matter are, first, that the atoms are composed of electrons revolving at uniform velocities in circles: and, second, that the accepted electromagnetic equations are applicable to these electrons. Unless we reject both hypotheses we are forced to decide between the two horns

of this dilemma. We either have to abandon the idea that the electrons are revolving in circles in the atoms, or that the present form of the electromagnetic equations apply to the electrons in atoms. There is little doubt as to which of the alternatives to choose. We will not yet abandon the idea that the electrons revolve in circles, and are driven to the conclusion that the present form of the equations does not apply to the electrons in the atoms. This is the position that the world of physicists has already taken, but not on these grounds. Some have, perhaps, gone a step too far and settled down to the conviction that we must get on somehow without any electromagnetic equations as far as the atoms are concerned. This does not seem to be a necessary alternative when we realize the very great difficulties which the derivation of these equations presents.

The value of the force derived above has the right look in so many respects that there seems to be some hope that this will open the way to a revision of the present form of the equations so as not only to meet all demands already made upon them but so as to include the gravitational force as well. On this account the derivation of these equations has been given here in some detail. Things look as if some important factor had been omitted somewhere which changes the magnitude of the force but not its general characteristics. A search has been made for such a factor and the place to put it without altering other results that are already satisfactorily interpreted by the equations in their present form. In other words it is to be a factor which affects the gravitational or inverse square terms to a large extent, and some of the other terms to a small extent.

If equation (54), the calculated expression for the gravitational pull, were multiplied by a factor $\frac{1}{4} \times .898 \times 10^{-27} \beta_1^2$, where β_1 is the velocity of the first electron in terms of that of light, we obtain the correct value of the gravitational force within about 1 per cent. It seems very natural that the velocity of the first electron as well as that of the second should come in to the final result, but in the present form of the electric force, from which (54) is derived, β_1 fails to appear.

Making the calculation for the two diamonds after introducing this factor we would have obtained instead of (57)

$$F = \frac{1}{4} \times .898 \times 10^{-27} \beta_1^2 \beta_2^2 e^2 r^{-2} \quad (67)$$

and instead of (58), writing $\sum_P \beta_1^2$ for atom A_1 instead of 12, the number of electrons in A_1 ,

$$F = \frac{1}{12} \times .898 \times 10^{-27} \times 12.45^2 \times 10^{-8} e^2 = 11.6 \times 10^{-46} e^2, \quad (68)$$

and instead of (61)

$$F = 80.9 \times 10^{-44} \text{ dynes.} \quad (69)$$

This is the revised value of the attraction between the two diamonds to be compared with the gravitational pull (62), namely 82.1×10^{-44} . This value is in very close agreement with the calculated value.

It is to be remarked that the number $.898 \times 10^{-27}$, which is part of the factor that has been introduced, is the numerical value of the mass of the electron at slow velocities, and, hence, the total factor is not merely a numeric but has a real physical significance, being a property of the electron itself. If the factor had no such value it would not mean so much because some factor could of course be found to correct the final result, but that it has a physical meaning seems very significant, and especially so because it is a property of the electron itself. The revised form of the gravitational terms in (54) may then be put into the following symmetrical form,

$$F = \frac{m_0}{k} e_1 e_2 \frac{\beta_1^2}{2} \frac{\beta_2^2}{2} \left\{ \frac{1}{2} - \frac{1}{2} (-X \sin \alpha + Z \cos \alpha)^2 \right\} r^{-2}. \quad (70)$$

If the electron is that of Lorentz the mass at slow velocities may be expressed in terms of the charge as follows

$$m_0 = \frac{4e^2}{5c^2 a}, \quad (71)$$

and the transverse mass at any velocity as

$$m = m_0 (1 - \beta^2)^{-1/2}, \quad (72)$$

and longitudinal mass as

$$m = m_0 (1 - \beta^2)^{-3/2}. \quad (73)$$

In the modern electromagnetic theory the quantities k and μ the specific inductive capacity and the magnetic permeability are usually suppressed. The product only is known $\mu k = 1/c^2$, but not the dimensions of each separately, and until these are known it seems very doubtful that it is correct to write any expression without either of these factors being present even in the common mechanical units such as force, energy or mass. I have, therefore, introduced the k into equation (70) in order to keep the dimensions of the right hand member the same as that of force, letting the unknown dimensions of k compensate for the new mass factor. Using the Lorentz mass (71) for slow velocities, we may write (70), where e is expressed in electrostatic units,

$$F = \frac{e^4 \beta_1^2 \beta_2^2}{5ac^2 k} \left[\frac{1}{2} - \frac{1}{2} (-X \sin \alpha + Z \cos \alpha)^2 \right] r^{-2}, \quad (74)$$

or in electromagnetic units,

$$F = \frac{\mu e^4 \beta_1^2 \beta_2^2}{5a} \left[\frac{1}{2} - \frac{1}{2}(-X \sin \alpha + Z \cos \alpha)^2 \right] r^{-2}. \quad (75)$$

Comparing the dimensions of the quantities on the right with that of force, we find from (75), in electromagnetic units in the dimensional form,

$$MLT^{-2} = \mu(M^{1/2}L^{1/2}\mu^{-1/2})^4L^{-2} = \mu^{-1}M^2L^{-1}. \quad (75)$$

Hence

$$M = \mu L^2 T^{-2} \text{ in the electromagnetic system,} \quad (76)$$

and similarly from (74) with electrostatic units

$$M = k^{-1} \text{ in the electrostatic system.} \quad (77)$$

In these dimensional forms β is a numeric, being a ratio of two velocities, and does not appear.

Taking these as the dimensions of mass on the two systems, a new system of dimensions becomes possible in which mass does not appear, there being but three dimensions L , T and k or μ instead of four L , M , T and k or μ in the present systems. A reduction of a few of the more common units according to this new system are given in the following table, those units not involving mass on the old systems remaining unchanged.

Name of Unit.	Electrostatic Old.	Electromagnetic Old.	Electrostatic New.	Electromagnetic New.
Quantity of electricity....	$M^{1/2}L^{3/2}T^{-1}k^{1/2}$	$M^{1/2}L^{1/2}\mu^{-1/2}$	$L^{3/2}T^{-1}$	$L^{3/2}T^{-1}$
Quantity of magnetism....	$M^{1/2}L^{3/2}k^{-1/2}$	$M^{1/2}L^{1/2}T^{-1}\mu^{1/2}$	$L^{3/2}k^{-1}$	$L^{3/2}T^{-2}\mu$
Electric potential and E.M.F.....	$M^{1/2}L^{3/2}T^{-1}k^{-1/2}$	$M^{1/2}L^{1/2}T^{-2}\mu^{1/2}$	$L^{3/2}T^{-1}k^{-1}$	$L^{3/2}T^{-2}\mu$
Electric current.....	$M^{1/2}L^{3/2}T^{-2}k^{1/2}$	$M^{1/2}L^{1/2}T^{-1}\mu^{-1/2}$	$L^{3/2}T^{-2}$	$L^{3/2}T^{-2}$
Resistance.....	$L^{-1}T k^{-1}$	$L T^{-1}\mu$	$L^{-1}T k^{-1}$	$L T^{-1}\mu$
Coef. of self-induction....	$L^{-1}T^2 k^{-1}$	$L \mu$	$L^{-1}T^2 k^{-1}$	$L \mu$
Capacity of a condenser ..	$L k$	$L^{-1}T^2 \mu^{-1}$	$L k$	$L^{-1}T^2 \mu^{-1}$
Energy.....	$M L^2 T^{-2}$	$M L^2 T^{-2}$	$L^3 T^{-2}k^{-1}$	$L^3 T^{-2}\mu$
Mass.....	M	M	k^{-1}	$T^{-2}\mu$
Force.....	$M L T^{-2}$	$M L T^{-2}$	$L T^{-2}k^{-1}$	$L^2 T^{-4}\mu$

THE ORIGIN OF THE GRAVITATIONAL TERMS.

Let us endeavor to trace to their origin the terms which contributed to the gravitational expression (54). These came immediately from (51), (52) and (53), and these again from the process of averaging (48), (49) and (50). In these latter equations the only terms which have contributed anything to the r^{-2} terms, that is, to (54), are doubly under-scored. In the i -component there are five such terms only, one of

which, $(e_1 e_2 / R^3 A^3) \beta_2^2 x$, comes from (45), and the rest from (47). This one term is traceable back to the term

$$-\frac{q_1^2}{c^2} \mathbf{R} = -\beta_2^2 \mathbf{R},$$

the last term in second line of (28), arising from the differentiation of the scalar potential with respect to the time $[\partial\phi/\partial t]$. All the rest of the terms are traceable to (47) and thence to (22), which terms appear in (28) either as the differentiation of ϕ or α with respect to the time, but not with respect to the space coördinates.

The search thus seems to lead back to two possible sources, either of which, if slightly modified, might have given results more in conformity with several physical phenomena. First, the imposed condition

$$\text{div. } \alpha + \frac{1}{c} \dot{\phi} = 0, \quad (78)$$

which partly defines the potentials ϕ and α . As Schott remarks on page 3 of his "Electromagnetic Radiation," "Thus α is not completely determined, and a third condition may be imposed upon it. Thus a singly infinite series of different functions α are possible, but *only two have been used*." The first condition to have been used was $\text{div. } \alpha = 0$, and the second one equation (6) above, which assumption is the basis of the modern theory.

Second, the fundamental equation attributed to Larmor and Lorentz,

$$\mathbf{F} = e_1 \left(\mathbf{E} + \frac{1}{c} \mathbf{q}_1 \times \mathbf{H} \right), \quad (79)$$

by which the mechanical force is derived from the above potentials. This equation expresses an assumed connection between charge and aether. A modification of the forms of either of these assumptions is possible, and seems justifiable, without departing from the more fundamental conceptions of an electromagnetic theory. It must have been necessary originally to make these assumed forms conform to experience so far as then known, and as revealed by physical phenomena. Is it not possible that some new phenomena have now been found which are in slight disagreement with the original forms assumed? It seems natural to expect to have to modify these forms when some new phenomena not originally included in the scheme is found. And, similarly, it is difficult to imagine any way to arrive at correct forms except as these are indicated by additional phenomena. This seems to be a logical method of arriving finally at an all-embracing electromagnetic theory.

If we hazard the guess that the form (6) defining the potentials is

correct, and attempt to alter (15) so as to make these potentials give the proper mechanical force, this process seems to require that a factor be introduced into those parts which involve differentiation with respect to the time. The form (16*b*) is the equivalent of (15), and in this the factor is required in three of the terms but not in two, namely neither in $-\nabla\phi$ nor in $\frac{1}{c}q_1 \times [\nabla \times \alpha]$.

A difficulty apparently arises from the fact that, in the current method of attacking the problem by resorting to the potentials, the potential is found at a stationary point in the aether due to a second distant charge in motion, and this potential is then applied to a first charge moving *through the point* at which the potential is found. The only way by which the motion of the first charge contributes anything to the mechanical force is then through the second term of (15), and this does not seem to be sufficient. In some manner the mechanical force seems to demand an integration extended over the volumes of both charges, and one of them at least, being in motion through the region where the force is wanted, should be a function of the time, the same charge not occupying the same element of volume constantly. This integration does not come into the derivation of the potential at a point, and so escapes entirely unless it is included separately in such a relation as (15) connecting the potential with the mechanical force.

It is with regret that no fundamental solution is as yet found that will introduce the required changes in the mechanical force, and the subject has been presented at some length in the hope that some one will have some solution to suggest.

OTHER ATOMIC PHENOMENA.

To show that these changes required for the gravitational force are in complete harmony with other phenomena which also depend upon the same modified electromagnetic equations, we consider below three distinct properties of the diamond which depend upon the higher order terms of the force-series, which become effective at molecular distances in distinction to the great distances heretofore employed. The same treatment applies, of course, to other substances, but it is not included here. They are: (1) The speed of electrons in rings, such as those composing the atoms; (2) the structure of the diamond, and determination of the radius of the orbit of the electrons in the carbon atom; (3) the determination of the bulk-modulus and the force required to produce a given contraction in the volume per unit volume.

Let us first write the complete forms of $\mathbf{E}$ and $\mathbf{H}$ as modified by the

requirements for the gravitational terms above discussed. These may be compared with (20) and (21) above.

$$\mathbf{E} = \frac{e_2(1 - \frac{1}{2}m\beta_1^2\beta_2^2)}{R^3A^3} \left\{ \mathbf{R} - \frac{R}{c}\mathbf{q}_2 + \frac{\frac{1}{2}m\beta_1^2 \left\{ \left[\mathbf{f}_2 \times \left(\mathbf{R} - \frac{R}{c}\mathbf{q}_2 \right) \right] \times \mathbf{R} \right\}}{c^2(1 - \frac{1}{2}m\beta_1^2\beta_2^2)} \right\}, \quad (80)$$

$$\mathbf{H} = \frac{e_2(1 - \frac{1}{2}m\beta_1^2\beta_2^2)}{R^3A^3} \left\{ -\frac{\mathbf{R} \times \mathbf{q}_2}{c} + \frac{\frac{1}{2}m\beta_1^2 \left\{ \mathbf{R} \times \left[\mathbf{f}_2 \times \left(\mathbf{R} - \frac{R}{c}\mathbf{q}_2 \right) \right] \times \mathbf{R} \right\}}{c^2R(1 - \frac{1}{2}m\beta_1^2\beta_2^2)} \right\}. \quad (81)$$

If we follow the same plan with these equations as was done with (20) and (21) above and find the i , j and k components of the mechanical force as in (48), (49) and (50), we obtain a similar expression for $\mathbf{F} = e_1\mathbf{E}$, term for term as in (48), (49) and (50), except that most of the terms have an additional factor $\frac{1}{2}m_0$. The only terms which do not have this factor are those in the first two lines of these equations, as follows for the $\mathbf{E}$ part

$$\begin{aligned} \mathbf{F}_E = e_1\mathbf{E}_i &= \frac{e_1e_2}{R^3A^3} [-x + a_1S_1 - a_2S_2 \cos \alpha]i \text{ (electrostatic part)} \\ &+ \frac{e_1e_2}{R^2A^3} [-\beta_2C_2 \cos \alpha]i, \end{aligned} \quad (82)$$

$$\begin{aligned} \mathbf{F}_E = e_1\mathbf{E}_j &= \frac{e_1e_2}{R^3A^3} [-y + a_1C_1 - a_2C_2]j \text{ (electrostatic part)} \\ &+ \frac{e_1e_2}{R^2A^3} [+ \beta_2S_2]j, \end{aligned} \quad (83)$$

$$\begin{aligned} \mathbf{F}_E = e_1\mathbf{E}_k &= \frac{e_1e_2}{R^3A^3} [-z - a_2S_2 \sin \alpha]k \text{ (electrostatic part)} \\ &+ \frac{e_1e_2}{R^2A^3} [-\beta_2C_2 \sin \alpha]k. \end{aligned} \quad (84)$$

These three equations are equivalent to

$$\mathbf{F} = e_1\mathbf{E} = \frac{e_1e_2}{R^3A^3} \left\{ \dot{\mathbf{R}} - \frac{R}{c}\mathbf{q}_2 \right\}. \quad (85)$$

To get the complete force we have to add to this the terms having the very small factor less than 10^{-30} , the addition of which makes no appreciable difference in (85). If, therefore, we contemplate using these equations at small distances where any one of the terms without the small factor is effective, it becomes the whole force, and we might have put $m = 0$ just as well in the forms (80) and (81) so far as these terms

are concerned. Doing this would have given (85) directly, and instead of (81) we would have

$$\mathbf{H} = \frac{e_1}{R^3 A^3} \left[-\frac{\mathbf{R} \times \mathbf{q}_2}{c} \right]. \quad (86)$$

Had we continued the process of averaging the instantaneous force as was done with (48), (49) and (50) to obtain (51), (52) and (53) expressed as an infinite series in terms of $1/r$, we should have obtained just the expressions given in (51), (52) and (53), which only include the r^{-2} terms plus other terms in higher powers which are there omitted. Now, omitting the electrostatic part as before, the terms in (85) and (86) do not appear at all in the r^{-2} terms, and make their first appearance in the series in the higher powers only. At molecular distances these higher power terms arising from (85) and (86), being the β -terms which do not have the small factor, represent the whole force, the terms in (51), (52) and (53) being entirely ineffective, as well as those in higher powers which contain the small factor.

These equations (85) and (86) will now be applied to the case of a single ring of electrons, which has been calculated elsewhere¹ by means of the original unmodified equations (20) and (21), in order to make a comparison.

It is required to find the force exerted by a second electron in the same orbit as the first, resolved along the tangent line to the orbit of the first. It may be shown that $\mathbf{H}$ has no component along the tangent line, the vector $\mathbf{R} \times \mathbf{q}_2$ in (86) being perpendicular to the plane of the orbit, so that the electric force controls the ring. When the centers of the orbits coincide, the values of $\mathbf{r}$ and its components x , y and z in (36) are zero. And, since the axes of rotation are common, $\alpha = 0$. The radii $a_1 = a_2 = a$, and $\beta_1 = \beta_2 = \beta$. Hence the value of $\mathbf{R}$ in (38) becomes

$$\mathbf{R} = a\{(S_1 - S_2)\mathbf{i} + (C_1 - C_2)\mathbf{j}\} \quad (87)$$

and

$$\mathbf{q}_2 = a\omega(C_2\mathbf{i} - S_2\mathbf{j}). \quad (88)$$

Hence

$$\mathbf{F} = e_1 \mathbf{E} = \frac{e_1 e_2 a}{R^3 A^3} \left\{ \left(S_1 - S_2 - \frac{R\beta}{a} C_2 \right) \mathbf{i} + \left(C_1 - C_2 + \frac{R\beta}{a} S_2 \right) \mathbf{j} \right\}. \quad (89)$$

To resolve this force along the tangent line $\tau = S_1\mathbf{j} - C_1\mathbf{i}$, take the dot product with (89), and find

$$\mathbf{F}_{\tan} = \frac{e_1 e_2 a}{R^3 A^3} \left\{ C_1 S_2 - C_2 S_1 + \frac{R\beta}{a} (C_1 C_2 + S_1 S_2) \right\}.$$

¹ A. C. Crehore, "On the Cause of the Revolution of a Ring of Electrons." As yet unpublished.

Since

$$C_1S_2 - C_2S_1 = \sin \gamma, \quad \text{and} \quad C_1C_2 + S_1S_2 = \cos \gamma, \quad (90)$$

where $\gamma = \theta_2 - \theta_1$, the constant angle of phase difference (see (31) and (32) above), we have

$$\mathbf{F}_{\tan} = \frac{e_1 e_2 a}{R^3 A^3} \left\{ \sin \gamma + \frac{R\beta}{a} \cos \gamma \right\}. \quad (91)$$

This is for clockwise rotation of the electrons. For counterclockwise rotation reverse the sign.

To compare this result from the modified equations with that before obtained from the original forms, we give the former result

$$\mathbf{F}_{\tan} = \frac{e_1 e_2 a}{R^3 A^3} \left\{ (1 - 2\beta^2) \sin \gamma + \frac{R\beta}{a} \cos \gamma - \frac{R\beta}{a} \beta^2 + \beta^2 \sin \gamma \cos \gamma \right\}. \quad (92)$$

In each of these results we have entirely neglected the force which an electron exerts upon itself which has been the subject of much discussion. That is to say, we have neglected the radiation of energy. The justification for this is to be found in the facts. The supply of this energy has been accounted for by some in supposing a slow secular change in the mass of the electron. In the words of Schott¹ "the change of mass need only take place very slowly provided that the velocity of the ring shall be small enough. In order that it should not contradict our experience as to the constancy of the properties of electrons and atoms, it is sufficient that $\dot{m}/m$ be less than, let us say 10^{-18} , or one thirty millionth part per annum, but of course it might be very much smaller." Since we are proceeding to use modified electromagnetic equations, being led to do this for the reasons given above, it seems most probable that the ordinary calculation of the radiation will have to be revised, and may turn out to be of a much smaller order than heretofore supposed. At any rate, experience shows that it is negligible on account of the permanency of atoms, and, if it is clearly stated that the radiation is being neglected here, this should not alter the value of the results obtained if they also agree with experience.

Due to the fact that the orbit is a circle, the equations (91) or (92) require definite numerical values for β for a definite number of electrons in the ring. The additional equation required from the properties of the circle, which makes the solution a definite numerical value, is

$$R^2 = 2a^2(1 - \cos \gamma).$$

Without making the calculation here we will content ourselves with the statement that very approximately the same numerical values of

¹ Electromagnetic Radiation, p. 191.

β are obtained from either the original equation (92), or the modified form (91), with the single exception of a ring of only two electrons. In the case of a ring of four electrons, the value is $\beta = .00846$, and in the case of a ring of eight $\beta = .012$. The equilibrium is very approximately determined by the two electrons adjacent to that on which the force is obtained, one in advance and the other behind the selected electron. The effect of all the other electrons in the ring is comparatively small. To give an idea of the degree of difference due to the use of (92) or (91), when $\beta = .001$ in the case of a ring of four electrons, the sum of the forces due to the two adjacent electrons along the tangent line is $.000,007,247,37(e^2/a^2)$ using (92), and $.000,007,247,06(e^2/a^2)$ using (91), a difference of one part in about 10,000. Adding the force due to the third or opposite electron in the ring does not affect this figure.

The exception noted above in the case of a ring of two electrons is such as to completely change the result. If we use the unmodified equation (92), the force on the one electron due to the other has the opposite sign to that obtained from (91). In the former case there is no equilibrium, or rather it is unstable, but in the latter case the equilibrium is stable and the value of β must be zero. That is to say, this form of the equation permits the two electrons to remain at rest, and, if started in motion due to any cause, the mutual forces bring them to rest again when the disturbance ceases.

CRYSTAL STRUCTURE.

We shall now proceed to derive the average force exerted by one electron e_2 on another e_1 including the higher powers of the force-series, which in (51), (52) and (53) terminated with the r^{-2} term, in order to apply it to the formation of molecules, crystals being regarded as one type of molecule. As pointed out above, the force series, obtained from the complete forms (80) and (81), is the same for our purposes as that obtained from the shorter forms (85) and (86). From these we have for circular motion, omitting the electrostatic part $\mathbf{R}$ in (85) as before,

$$\mathbf{E} = \frac{e_2}{R^3 A^3} \left\{ -\frac{R}{c} \mathbf{q}_2 \right\} = -\frac{e_2 \beta_2}{R^3 A^3} \{ C_2 \cos \alpha \mathbf{i} - S_2 \mathbf{j} + C_2 \sin \alpha \mathbf{k} \}, \quad (93)$$

$$\begin{aligned} \mathbf{H} = \frac{e_2}{R^3 A^3} \left\{ -\frac{\mathbf{R} \times \mathbf{q}_2}{c} \right\} = & -\frac{e_2 \beta_2}{R^3 A^3} \{ [(-x + a_1 S_1 - a_2 S_2 \cos \alpha) \mathbf{i} \\ & + (-y + a_1 C_1 - a_2 C_2) \mathbf{j} + (-z - a_2 S_2 \sin \alpha) \mathbf{k}] \\ & \times [C_2 \cos \alpha \mathbf{i} - S_2 \mathbf{j} + C_2 \sin \alpha \mathbf{k}] \}. \end{aligned} \quad (94)$$

Hence the total force upon e_1 due to e_2 is

$$\begin{aligned}
\mathbf{F} = e_1 \left(\mathbf{E} + \frac{1}{c} \mathbf{q}_1 \times \mathbf{H} \right) = \frac{e_1 e_2 \beta_2}{R^2 A^3} \{ & -C_2 \cos \alpha \, i + S_2 j - C_2 \sin \alpha \, k \} \\
& + \frac{e_1 e_2 \beta_1 \beta_2}{R^3 A^3} \{ [x S_1 S_2 - a_1 S_1^2 S_2 + a_2 S_1 \cos \alpha + y S_1 C_2 \cos \alpha \\
& - a_1 S_1 C_1 C_2 \cos \alpha] i + [-x C_1 S_2 - a_1 C_1 S_1 S_2 + a_2 C_1 \cos \alpha \\
& + y C_1 C_2 \cos \alpha - a_1 C_1^2 C_2 \cos \alpha] j + [z C_1 C_2 \cos \alpha - x C_1 C_2 \sin \alpha \\
& + y S_1 C_2 \sin \alpha + z S_1 S_2 + a_2 S_1 \sin \alpha] k \}.
\end{aligned} \tag{95}$$

THE AVERAGE FORCE.

In a crystal the arrangement of the axes of rotation of the atoms is such that the i and j components of the forces cancel out in summing up to get the force on a single atom due to the surrounding atoms. For this reason we will average only the z - or k -component of the above force. Also, since β is always a small quantity, the quantity A may be taken as unity without sensible error.

We obtain from the electric portion of (95) after averaging

$$\begin{aligned}
\mathbf{F}_k = e^2 \{ & P_1 \sum_{P_2} (a\beta)(+Y \sin \alpha) \} r^{-3} k + e^2 \{ \sum_{P_1} a^2 \sum_{P_2} (a\beta) [-\frac{1}{2} Y \sin \alpha \\
& - \frac{1}{2} Y \sin \alpha \cos \alpha + 3 Y^3 \sin \alpha] + P_1 \sum_{P_2} (a^3 \beta) [-2 Y \sin \alpha \\
& + 3 Y^3 \sin \alpha + 3 Y Z^2 \sin^3 \alpha + 3 X^2 Y \sin \alpha \cos^2 \alpha] \} r^{-5} k;
\end{aligned} \tag{96}$$

and from the magnetic portion

$$\begin{aligned}
\mathbf{F}_k = e^2 \sum_{P_1} (a\beta) \sum_{P_2} (a\beta) \{ & + \frac{3}{4} X \sin \alpha + \frac{3}{2} Z \cos \alpha - \frac{1}{8} [+ X Z^2 \sin \alpha \\
& + X^2 Z \cos \alpha + Y^2 Z \cos \alpha] \} r^{-4} k.
\end{aligned} \tag{97}$$

The series are here complete up to and including the r^{-5} term. It is seen that the electric part is in the odd and the magnetic part in the even powers, there being no terms in (97) in r^{-5} . We do not require any higher terms in this series than r^{-5} because the electrostatic terms, heretofore omitted, make their first appearance in the r^{-6} term. The above equations have the small factor β in the coefficients, and, when the r^{-6} and higher powers are added to the r^{-5} and higher electrostatic terms, the total of such terms does not differ from the electrostatic terms alone in an appreciable degree.

An examination of the result in (96) from the electric force shows that the quantity $Y \sin \alpha$ is a factor of every term. This is significant because the structure of cubic crystals is such that, because of this factor, the total force due to each and every plane of atoms in the crystal

vanishes. This is the case if the axes of the atoms in each plane are as shown in the paper on the diamond, figure 2, page 259, Phil. Mag., Aug., 1915. We conclude from this that the electric force contributes nothing to the total force upon any selected atom.

The magnetic portion (97) is the same as that formerly obtained from different equations with the exception of the numeric 15/8 which was 15/4 in the former result.¹ Following the plan on pages 264-5 of the paper on the diamond referred to, the forces on a selected atom are here recalculated in accordance with (97) for the β or r^{-4} terms, the r^{-6} or electrostatic terms remaining unchanged, being the same as in the papers referred to. That is, the only difference is in the numeric 15/8 instead of 15/4 in part of the r^{-4} terms only.

Plane Atoms

$$\begin{aligned} \text{I } J \quad \mathbf{F}_s = e^2 \{ & -10.66666 \sum_{P_1} (a\beta) \sum_{P_2} (a\beta) l^{-4} \\ & + 142.22222 \sum_{P_1} a^2 \sum_{P_2} a^2 l^{-6} \}, \end{aligned} \quad (98)$$

$$\begin{aligned} \text{I (20-25) } \mathbf{F}_s = e^2 \{ & +0.07530 \sum_{P_1} (a\beta) \sum_{P_2} (a\beta) l^{-4} \\ & + 1.17036 \sum_{P_1} a^2 \sum_{P_2} a^2 l^{-6} \}, \end{aligned} \quad (99)$$

$$\begin{aligned} \text{I (29-31) } \mathbf{F}_s = e^2 \{ & -9.87654 \sum_{P_1} (a\beta) \sum_{P_2} (a\beta) l^{-4} \\ & + 117.6348 \sum_{P_1} a^2 \sum_{P_2} a^2 l^{-6} \}, \end{aligned} \quad (100)$$

$$\begin{aligned} \text{I (32-34) } \mathbf{F}_s = e^2 \{ & -0.08785 \sum_{P_1} (a\beta) \sum_{P_2} (a\beta) l^{-4} \\ & - 2.4363 \sum_{P_1} a^2 \sum_{P_2} a^2 l^{-6} \}, \end{aligned} \quad (101)$$

$$\begin{aligned} \text{3 (35-37) } \mathbf{F}_s = e^2 \{ & -0.9750 \sum_{P_1} (a\beta) \sum_{P_2} (a\beta) l^{-4} \\ & - 5.757 \sum_{P_1} a^2 \sum_{P_2} a^2 l^{-6} \}, \end{aligned} \quad (102)$$

$$\begin{aligned} \text{Total } \mathbf{F}_s = e^2 \{ & -21.5308 \sum_{P_1} (a\beta) \sum_{P_2} (a\beta) l^{-4} \\ & + 252.83 \sum_{P_1} a^2 \sum_{P_2} a^2 l^{-6} \}. \end{aligned} \quad (103)$$

Since there is only one kind of atom in the diamond P_1 and P_2 are the same, and, equating to zero for equilibrium, we find

$$\left[\frac{\sum_P (a\beta)}{\sum_P a^2} \right]^2 = \frac{252.83}{21.53} l^{-2} = 11.73 l^{-2}, \quad (104)$$

¹ Equation (25), p. 755, Phil. Mag., June, 1915.

whence

$$\frac{\sum_P (a\beta)}{\sum_P a^2} = 3.425l^{-1}. \quad (105)$$

Using the same values for the carbon atom as in the gravitational calculation (page 461), and denoting the radius of the outside ring of eight by a_0 , which quantity we did not have to use before, we may obtain from (105) the value of a_0 . We have from the atom

$$\sum_P (a\beta) = (8 + 4 \times .402^2)a_0\beta_0 = 8.6464a_0\beta_0 = .10376a_0, \quad (106)$$

$$\sum_P a^2 = 8.6464a_0^2. \quad (107)$$

Hence

$$\frac{\sum_P (a\beta)}{\sum_P a^2} = \frac{\beta_0}{a_0} = \frac{\omega}{c} = \frac{.012}{a_0} = 3.425l^{-1} \text{ by (105)}. \quad (108)$$

If we use the experimental value of l , the edge of the tetrahedron in diamond, namely 2.528×10^{-8} cm., we find

$$a_0 = \frac{.012}{3.425} l = .885 \times 10^{-10} \text{ cm.}, \quad (109)$$

or

$$l = 285.3a_0, \quad (110)$$

$$\sum_P (a\beta) = 9.18 \times 10^{-12}, \quad (111)$$

$$\sum_P a^2 = 6.77 \times 10^{-20} \quad \text{and} \quad \omega = 5.424 \times 10^{18}. \quad (112)$$

This result is in agreement with the theory of the atom which makes the radius of the orbit of the outside ring of electrons a small fraction, $1/285.3$ of the distance between the centers of the atoms.

It seems incredible that the orbits of the electrons of the atoms in crystals can be any considerable fraction of the distance between the centers of the atoms, because of the interference between adjacent orbits that would result. The dimensions of the orbits demanded by the Rutherford-Bohr theory are incompatible with the above result, and they seem to be inadmissible from the standpoint of crystals alone irrespective of these results.

COMPRESSIBILITY AND BULK MODULUS.

The proper theory of the forces interacting between the atoms in a crystal ought to make it possible to calculate the various mechanical properties of the crystal. This presents more difficulty if we begin

to deform the crystal so as to change the shape of the elementary space lattice formation, but, if we are content to imagine that a pressure is exerted upon all faces of the crystal so as to change its bulk only and leave the angles of the lattice undisturbed, then it is easy to calculate the change of volume produced by any applied pressure. The pressure per unit area divided by the change in volume per unit volume is known as the "bulk modulus," and it has been experimentally determined for a number of substances.

If ΔP denotes the pressure perpendicular to one of the principal planes of the diamond, the (111) plane, and if Δv is the change in volume of a centimeter cube of the substance due to this pressure, the bulk modulus is

$$M = \frac{\Delta P}{\Delta v}. \quad (113)$$

Before any external pressure is applied the crystal is in equilibrium, and the total force upon each atom zero. When the external pressure is applied on all sides so as not to change the formation, a contraction of the whole results until the external pressure is exactly balanced by the elastic forces brought into existence by a change in the size of the space lattice, and until the force upon each atom is again zero. Because of this lattice formation the whole force upon any one atom due to all the others, together with the external pressure, is completely expressed by equation (103), the total force being in the direction of the axis of rotation of each atom, whatever the absolute value of the edge of the elementary tetrahedron. Consider the atoms in a (111) plane. Such planes are each made up of equal numbers of atoms of four different orientations, the first having its axis perpendicular to this plane, and each of the other three groups being with axes of rotation making an angle $\cos^{-1} \frac{1}{3}$ with the normal to this plane. Since the total force on each atom is along its axis, the portion of the pressure due to one atom in each of the latter groups, resolved normal to the (111) plane, is $\frac{1}{3}$ of that due to the atom whose axis is normal to this plane. The resolved portion of the total force of one group of four atoms is, therefore, twice the force due to the perpendicular atom alone. If N is the number of such groups of atoms per square cm., we have

$$N = \frac{\sqrt{3}}{6} \frac{1}{l^2}. \quad (114)$$

The force on one atom per centimeter change in the linear dimensions is dF/dl , and for a small linear displacement Δx , the force per atom with

axis normal to the plane is $(dF/dl)\Delta x$. The force on one group is twice, and the force per sq. cm., ΔP , is $2N$ times this value. Hence

$$\Delta P = 2N \frac{dF}{dl} \Delta x = \frac{\sqrt{3}}{3l^2} \frac{dF}{dl} \Delta x. \quad (115)$$

The volume of the crystal considered is $v = x^3 = 1$ cu. cm. Hence $dv = 3x^2 dx = 3dx$, and $\Delta v = 3\Delta x$. The bulk modulus (113) is, therefore,

$$M = \frac{\Delta P}{\Delta v} = \frac{\sqrt{3}}{9l^2} \frac{dF}{dl}. \quad (116)$$

The value of dF/dl may be obtained from (103) by differentiation, and we find, after substituting in the equilibrium value of l^{-2} from (104)

$$\frac{dF}{dl} = e^2 l^{-3} (\sum_P a\beta)^2 (-43.18). \quad (117)$$

Using the values of these quantities above obtained, we obtain numerically

$$\frac{dF}{dl} = - .0802, \quad (118)$$

whence

$$M = 24.16 \times 10^{12}, \text{ if pressure is dynes per sq. cm.} \quad (119)$$

or

$$M = 24630 \times 10^6, \text{ if pressure is grams weight per sq. cm.} \quad (120)$$

The latter form is in the units usually given in the tables of experimental results. Some experimental determinations are given below for comparison, because the value for the diamond is not given in any of the tables. The figures given here are in the Smithsonian Physical Tables, p. 83.

Substance.	Bulk Modulus, Gms. per Sq. Cm.	Substance.	Bulk Modulus, Gms. per Sq. Cm.
Sylvine.....	138×10^6	Tourmaline.....	$9,140 \times 10^6$
Rock salt.....	246 "	Lead.....	374 "
Quartz.....	387 "	Glass.....	405 "
Barite.....	535 "	Delta metal.....	1,012 "
Fluorspar.....	860 "	Brass.....	1,090 "
Pyrites.....	906 "	Copper.....	1,202 "
Beryl.....	1,384 "	Steel.....	1,518 "
Topaz.....	1,694 "		

The wide range of values for the crystals given in the table makes it impossible to tell the probable value for the diamond. The variation between sylvine and tourmaline is 66.2 times, or 6,620 per cent. The

value obtained theoretically for the diamond in (120) is between two and three times as large as the largest in the table, so that the order of magnitude of the calculated result is right. It may be remarked that Young's modulus for corundum has been measured, and found to be greater than that for any other substance yet tried. Although this is not the diamond, and it is not Young's modulus with which we are dealing, it does serve as a sort of indication that we may expect a large value of the bulk modulus for diamond. Subsequent calculations of this modulus for some of the substances which have been measured are to be desired.

SUMMARY AND REVIEW.

Having a conviction that some form of electromagnetic equations ought to be applicable to the individual electrons of which matter is composed, the first application should obviously be made to the steady states of matter at the absolute zero of temperature. In this condition it is supposed that the orbits of each electron making up the atoms are circles and that the motion in them is perfectly uniform. It is considered that any departure whatever from uniform circular motion will give rise to radiation or absorption of energy, but that this radiation is so small in the steady state of motion, if not exactly zero, that no change in the motion will result due to loss of energy through radiation in a period of time so great that it must entirely escape our observation.

The present form of electromagnetic theory already makes the radiation from a ring of electrons in the steady state very small. It has been supposed by some that the source of this energy is provided by a supposed secular diminution in the mass of the electron. In the words of Schott "the change of mass need only take place very slowly provided the velocity of the ring be small enough. In order that it should not contradict our experience as to the constancy of the properties of electrons and atoms, it is sufficient that $\dot{m}/m$ be less than, let us say 10^{-15} , or one thirty millionth part per annum, but of course it might be very much smaller."

In this paper a change in certain terms in the present form of the electromagnetic equations has been shown to be demanded, if they are to be applied to the atoms. The change suggested does not remove the equations from any connection with electromagnetic theory, and things point to a probable alteration either of the assumed equation of definition of the potentials ϕ and α , or of the assumed connection between charge and aether. If such a change is admitted to be required its effect upon the radiation terms is likely to be such as to introduce an additional factor of something like 10^{-31} into those terms that are already so small

as to cause Schott to make the remark above quoted. This would at once remove a very serious objection to the application of electromagnetic theory to the electrons in the steady states of matter. The source of this energy has been a serious stumbling block to the progress of the theory.

The single assumption is made that the electrons in the atoms of matter are moving at uniform velocities in circles at zero temperature, and the Lorentz form of electromagnetic equations is then applied to them. The fundamental problem to be solved is to find the average mechanical force which one such electron exerts upon another, when their orbits have the most general possible positions with respect to each other. This problem has been solved for the particular case of coaxial circles in a former communication, but is given above for the most general case. Obviously, until this problem was solved, we could not work with gross matter and crystal forms, because in such the axes of the atoms take many directions and the orbits are not coaxial circles.

The average force in the general case of two electrons in circular orbits is expressed as the sum of a series of terms of the inverse powers of the distance between the centers of the two orbits, the series beginning with the inverse first power and including all higher powers. When, however, this total force, which in general is not in the direction of the line joining the centers of the orbits, is resolved along this direction, the first power terms disappear, and the series begins with the inverse square of the distance between centers. If this distance is great enough, the first term of the series is the only term which is effective, so that at great distances the force resembles that of gravitation in this respect.

In all investigations prior to this the forces obtained between atoms have so depended upon the directions of the axes of rotation as to be an attraction or repulsion according to these directions, but these inverse square terms referred to always show an attraction. Although quantities involving the directions of the axes appear in these terms, they occur in such a way as to be ineffective and leave the total force always attractive. This is proved in the case of two crystals as well as other forms of matter including gases, liquids and solids.

The measurements by Mackenzie of the gravitational pull between lead masses, a lead mass and a crystal of calcspar, and finally between two crystals of calcspar seem to establish the fact that the gravitational force is independent of the directions of the axes, at least to the first order of approximation.

A disagreement between the calculated attraction and the existing gravitational attraction appears in the magnitude of the force only.

The calculated force is more than 10^{21} times greater than the existing force, and the difference is so large as to leave no doubt that something is fundamentally in error, and the future prospect of obtaining an all embracing electromagnetic theory depends, in part, upon a proper diagnosis of what is wrong. The hypotheses used are so few that the source of the error may be located, at least with some probability. These assumptions are two, that the electrons revolve in circular orbits in the atoms in their steady states, and, second, that the present form of the electromagnetic equations are applicable to these electrons.

The probability is that these equations *in their present form* do not apply to the electrons. This is the position already taken by a large number of physicists. But, the results indicate rather that there may be a fundamental error at some point in electromagnetic theory than that no such theory is applicable. And, if this can be found, we may still hope for a comprehensive theory, which at the same time embraces atomic phenomena and gravitation as well.

It is found that, by introducing a factor $\frac{1}{2}m\beta_1^2$ into the inverse square terms above mentioned, we then get the correct value of the gravitational pull to within about 1 per cent. It is surprising that a simple factor, having a direct physical significance connected with an electron, should be found that corrects the result. If there were no connection between the expression (54) and the true gravitational force, we would expect the conversion factor to be a pure numeric.

In tracing back to their source the terms from which (54) comes directly, it appears that this factor is introduced or required in those terms which come from the time differentiation of the potentials in distinction to the space differentiation, and that it is required not with every term of $\mathbf{E}$ and $\mathbf{H}$, but in a peculiar manner, so that some terms are altered and others are not.

It is with regret that a solution for the mechanical force based upon new fundamental assumptions embracing this idea is not yet accomplished, and it seems best to publish these results as being of sufficient interest aside from any attempt to build upon a slightly different electromagnetic foundation.

Four properties have been examined by means of modified equations,

1. The equilibrium speed of rings of electrons.
2. The form of the force-equation applicable to atoms at close range, as in the formation of molecules, with application to the diamond as one type of molecule.
3. The elastic forces brought into existence by the compression of the diamond so as not to alter the angles of the space lattice formation, the calculation of the bulk-modulus.

4. The attraction between two diamonds, each 10^{-18} cubic cm. at one cm. apart, oriented in all possible ways with respect to each other.

It is shown that, for rings of electrons, the modified equation gives the same result to a very close degree of approximation as did the original form.

In the fourth case the modified equation gives the gravitational force within about 1 per cent., while the original form gives a force more than 10^{21} times greater than the actual force.

In the third case we obtain the proper order for the value of the bulk-modulus. This result alone goes to show that we are dealing with interatomic forces of the right order of magnitude, and renders it probable that the inverse fourth power terms are the first to be effective in the force-series at the distances concerned between atoms in a crystal. These results lead to a determination of the radius of the orbit of the outside ring in a carbon atom, and confirm former results as to its small dimensions $.885 \times 10^{-10}$ cm.

YONKERS, N. Y.

AN AIR-BLAST RECTIFIER OF HIGH TENSION ALTERNATING CURRENTS.

BY E. R. WOLCOTT AND C. J. ERICKSON.

THAT a partial rectification of a high-tension alternating current occurs on the passage of a discharge from a point to a plate, has been known for many years. However, difficulty has been encountered in attempting to utilize this rectification, particularly if any considerable current flows, as there then results a heavy arc which rectifies but little.

An examination of the effect of a blast of air on this discharge showed that complete rectification and smooth operation could be obtained by means of a current of air flowing from the point to the plate.¹ A method of accomplishing this is illustrated diagrammatically in Fig. 1.

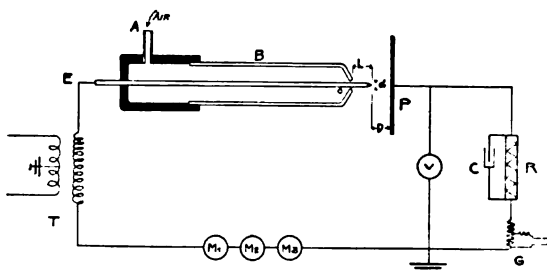


Fig. 1.

A pointed metallic electrode *E* is mounted coaxially within the tube *B*, which is restricted at one end so that a gas entering at *A* is blown through the opening *O* toward the plate *P*. Rectification occurs when the point is some distance within the tube, at the opening of the tube, or extending some distance outside of the tube as shown. In the first two cases, the tube *B* must be made of some insulating material, as of glass; in the last case it may be of metal.²

Mounted for use, this rectifier was connected to one terminal of the secondary of a 2 K.W., 40,000-volt transformer *T*, the other terminal being grounded. The positive current flows from the point to the plate,

¹ E. R. Wolcott and F. Rieber, U. S. Patent No. 1,188,597.

² S. D. Browning and C. J. Erickson, thesis University of California, under direction of Prof. Harmon F. Fischer.

and no discharge results in the opposite direction when the distance between the point and plate, that between the point and air outlet, the air pressure and the voltage are suitably adjusted.

Connected between the other terminal of the transformer and the ground were three milliammeters: M_1 was of the electro-dynamometer type; M_2 was a direct-current instrument of the D'Arsonval permanent field type; M_3 was a hot-wire instrument.

The secondary voltage from the plate P to ground was measured by an electrostatic voltmeter. A General Electric oscillograph was mounted directly in the high-tension circuit between the water resistance R and the ground.

The air condenser C consisted of eleven plates 8 by 3 feet (2.44 by 0.91 meters) and 8 inches (20.3 cm.) apart. Its capacity was 0.00095 microfarads and it was connected in parallel with the water resistance when desired.¹

The diameter of the plate P should be large enough to prevent a discharge from the point to its edges. In the arrangement here described, its diameter was 4 inches (10.16 cm.). The diameter of hole O in the tube B was 0.157 inches (4 millimeters). d , the diameter of the metallic electrode E , was 0.0625 inches (1.57 millimeters). L , the distance between the opening O and the point of the electrode E , was 0.375 inches (9.54 cm.). D , the distance between the plate and the point, was 0.68 inches (1.76 cm.). The frequency of the alternating current was 60 cycles per second.

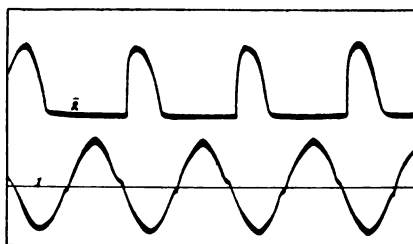


Fig. 2.

With no air flowing out of the opening O of the tube B , curve 1 was obtained at 25 kilovolts. The discharge in this case was an alternating current arc. With air flowing out of the opening O , under a pressure of

¹ The curves shown in Figs. 2 and 3 were obtained with no condenser, those shown in Figs. 4 and 5 were obtained with the condenser connected in parallel with the resistance. One vibrator was connected, in series with a suitable resistance, in the condenser circuit to show the condenser load current. The discharge of the condenser through the parallel resistance is indicated by the drop of the condenser load current curve below the zero line.

2 inches (5.08 cm.) of mercury, curve 2 was obtained. These two curves were not obtained simultaneously so the phase relations as shown are not correct. The direct current reading was 12.3 milliamperes, and the voltage 14.5 kilovolts.

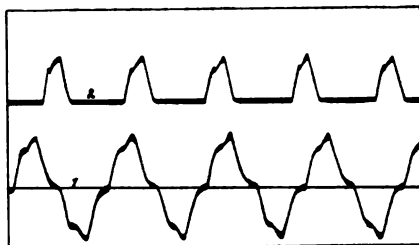


Fig. 3.

This curve differs from Fig. 2 in that a different alternating current generator was used. It will be noticed that the half wave passed by the discharge follows, in general, the wave form of the alternator; but, since a certain voltage is required to start the discharge, it does not include the whole of the half wave. The rise of the rectified wave is more abrupt than that of the original wave.

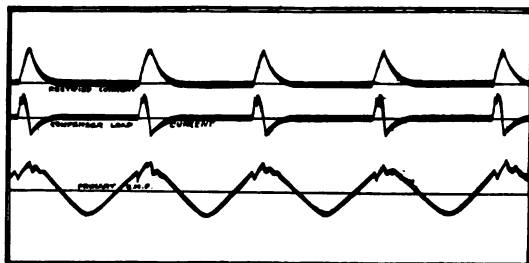


Fig. 4.

These curves were not taken under exactly identical conditions of Figs. 2 and 3 but all three were taken simultaneously and are in phase. The primary electromotive force (E.M.F.) was 241 volts; the secondary voltage was 15,250 volts; the rectified current was 25 milliamperes.

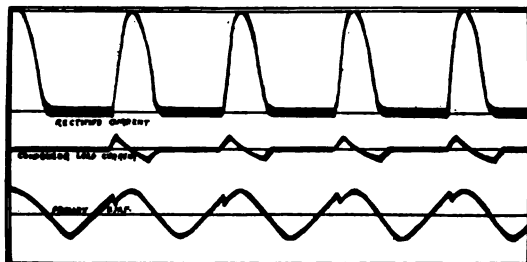


Fig. 5.

These curves were taken under the same conditions as those of oscillogram Fig. 4 except the load was heavier, that is, 104 milliamperes. The sudden drop in primary voltage, at the moment of the application of the load, is plainly evident.

The discharge, when rectification results, is white, rhythmic and snappy, as distinguished from the colored arc which results when no air is flowing. The latter rectifies to a small extent, but in the opposite direction.

Difficulties are encountered trying to operate at too low voltages, say only a few thousand volts. A certain voltage is required to jump the air gap. Likewise there is a drop in voltage across the rectifier in actual operation. Satisfactory rectification has been obtained up to 350 kilovolts, and also at frequencies up to 500 cycles per second.¹

With the point inside of a glass tube *B*, the discharge is more snappy, but the tendency to surge is greater. In this case aluminum electrodes seemed preferable, a larger plate *P* was necessary, and a greater air pressure required.

With the point outside of the tube, satisfactory results were obtained with iron and brass electrodes.

As the gas pressure increases from zero during the discharge, the rectification increases to a maximum, the value of which depends upon the setting of the electrodes and the diameter of the gas outlet.

A further increase of gas pressure beyond this value which gives maximum rectification, produces irregular operation.

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¹ K. V. Laird, Fog Dispersion Tests, Panama-Pacific International Exposition, under the direction of Dr. F. G. Cottrell.

THE THEORY OF ELECTROMAGNETIC MASS OF THE
PARSON MAGNETON AND OTHER NON-SPHERICAL
SYSTEMS.

BY DAVID L. WEBSTER.

INTRODUCTION.

IN the remarkable theory of atomic structure recently proposed by Parson¹ it is assumed that the electron is not spherical, as previously supposed, but a very thin ring, about 1.5×10^{-9} cm. in radius, on which the negative charge revolves at a very high velocity, of the order of that of light. This gives the electron, or magneton, as he calls it, a combination of electrostatic and steady magnetic properties, the steadiness of these magnetic properties being essential to the theory. Their detection by direct experiment with molar magnetic fields was shown by Parson to be very difficult, owing to the fact that the magneton is electrostatically single but magnetically a neutral doublet. Evidence of this sort has, however, been obtained by Grondahl² for free electrons within an iron wire, that seems to agree satisfactorily with calculated effects. It might be supposed that evidence would be obtained also from the fact that the inequality of the magnetic energies of a magneton moving along its axis or perpendicular to it would give a tendency for high speed cathode rays to orient themselves in one plane and produce magnetic and other effects. Or one might expect the same influence in a non-rotating monatomic atom such as one of mercury vapor to produce polarization in the light emitted by it. But as Ehrenfest³ and others have pointed out, such effects would give direct evidence of absolute motion, and therefore are prohibited by the principle of relativity.

Notwithstanding this, the electromagnetic mass and momentum, m , and M , and the magnetic and electrostatic energies T , and W , are very different for motions with the same velocity in different directions in systems not having spherical symmetry, and one may readily prove that $T + W$ is not constant although $T - W$ is so. Therefore in such cases we must have some other forms of mass, momentum and energy, due to some other cause than the electromagnetic field. Much as we

¹ "A Magnetron Theory of the Structure of the Atom," *Smithsonian Miscellaneous Collections*, Nov., 1915.

² *Amer. Phys. Soc.*, Dec., 1916, *Phys. Rev.* [2], 9, 1917.

³ *Ann. d. Phys.*, 23, 204-5, 1907.

may hesitate to speculate on what is inside as small a thing as the magneton, there seems to be no other cause for this extra mass than the changes occurring on acceleration in the internal forces that hold the magneton together. This compels us to test the possibilities of various assumptions about them in search of a possible explanation of this "internal mass." One such explanation is that which I outlined in a discussion before the American Physical Society in December, 1916, and more fully in February, 1917, and which is described in an abstract in the *PHYSICAL REVIEW* of 1917. The purpose of the present paper is to give an actual proof of the assertions made there.

The analysis whose results are outlined in this paragraph is designed primarily for the magneton theory; but it is equally necessary for and applicable to any system of static charges and steady currents (that is, if ρ and ρv are independent of t). Let each pair of charge or current elements be imagined connected by a string or rod that will balance their forces on each other when the system is at rest. When the system is accelerated, the non-radiated part of the electromagnetic field from element A upon element B will appear as though A was not where it actually is, but where it would be if it had not been accelerated since the radiation left it which is now arriving at B . If now the internal force on B due to A is governed by this fictitious position of A rather than its real position, but is of the same strength as though the fictitious distance to A were equal to its real distance, then these internal forces will give a mass effect varying with the direction of acceleration in exactly the required way.

This is not the only system of internal forces that will accomplish the result, and for that reason the internal mass may differ from the mass given by this system by any constant amount. A result of this is that the relation between the mass of the classical electron (if it exists) and its radius is not determined so unambiguously as it is supposed to be, and would change by 50 per cent. if a set of forces of this type were substituted for the hydrostatic tension postulated by Lorentz. If the forces in the magneton are of the type suggested here, it appears that the internal forces balancing magnetic actions will give a negative mass always exactly balancing the positive magnetic mass, and that the mass of the magneton is determined solely by its electrostatic properties and corresponding internal forces. It should, however, be remembered that the internal forces are not definitely known, and therefore that the total mass of any electrical system is really uncertain, though the necessity for some internal mass of any non-spherical system is a direct consequence of the principle of relativity.

In any case, there must be an internal mass, m_i , and momentum, M_i , given presumably by the forces required to hold the system in shape against the electrical forces and these must satisfy the equations:

$$m_i = m - m_e, \quad \text{and} \quad M_i = M - M_e. \quad (1)$$

Thus to find the relation between the mass of the magneton and its charge, current, and dimensions, we must investigate these internal forces in detail.

THE MOMENTUM AND ENERGY OF A MOVING ELECTROSTATIC SYSTEM.

Before undertaking the treatment of the more complicated case of a magneton, let us obtain some idea of how all this may happen by considering a simpler hypothetical case of two electrons of the classical type, of charge $+e$ and radius R held at a distance a which is constant except for Lorentz-Fitzgerald contraction.

We shall denote the electric and magnetic vectors measured in the standard electrostatic and magnetic systems by $\mathbf{e}$ and $\mathbf{b}$ respectively, these letters being used rather than Lorentz's $\mathbf{d}$ and $\mathbf{h}$ because their averages over "physically infinitesimal" volumes are the $\mathbf{E}$ and $\mathbf{B}$ of ordinary electromagnetics, rather than $\mathbf{D}$ and $\mathbf{H}$. The "time" as measured by the distance, ct , that light has travelled since $t = 0$ will be denoted by l , and the units of momentum and mass will be those required by the use of l , that is, the ordinary ones divided by c and c^2 respectively. The value of v/c for any bit of electricity will be denoted by u , and that of the system as a whole by β . Vectors will be denoted by Clarendon type, and their components when treated as scalars will be in italics.

For the general case of any system we have

$$\left. \begin{aligned} \mathbf{M}_e &= \frac{1}{4\pi} \int_{\infty} \mathbf{e} \times \mathbf{b} d\tau, \\ T_e &= \frac{1}{8\pi} \int_{\infty} \mathbf{b}^2 d\tau, \quad W_e = \frac{1}{8\pi} \int_{\infty} \mathbf{e}^2 d\tau, \end{aligned} \right\} \quad (2)$$

where $\mathbf{e} \times \mathbf{b}$ denotes the vector product of $\mathbf{e}$ and $\mathbf{b}$.

The vectors $\mathbf{e}$ and $\mathbf{b}$ may be found by the equations

$$\left. \begin{aligned} \mathbf{e} &= -\nabla\varphi - \dot{\mathbf{a}}, \quad \mathbf{b} = \nabla \times \mathbf{a}, \\ \nabla^2\varphi - \ddot{\varphi} &= -4\pi\rho, \quad \nabla^2\mathbf{a} - \ddot{\mathbf{a}} = -4\pi\rho\mathbf{u}, \quad \nabla \cdot \mathbf{a} + \ddot{\varphi} = 0, \end{aligned} \right\} \quad (3)$$

where ∇ is the vector operator ($k_x D_x + k_y D_y + k_z D_z$), k_x , k_y and k_z being unit vectors. Thus $\nabla\varphi$ is the gradient of the scalar potential φ , and $\nabla \times \mathbf{a}$ and $\nabla \cdot \mathbf{a}$ the curl and divergence, respectively, of the vector potential $\mathbf{a}$, the dot between two vectors denoting the scalar product.

$\dot{\varphi}$ denotes $D_t\varphi$. These equations, in a different notation, and many that we shall derive from them, are given in Lorentz's "Theory of Electrons."

For the special case of the two electrons, we may use Lorentz's method of dealing with a static system in motion. Taking x in the direction of β , we have $\mathbf{u} \equiv \beta$, and $\dot{\varphi} = -\beta D_x\varphi$, $\ddot{\varphi} = \beta^2 D_x^2\varphi$, etc., so that if we let $k = (1 - \beta^2)^{-1/2}$ and $x' = k(x - \beta t)$ we may put (3) in the form

$$\nabla'^2\varphi = -4\pi\rho, \quad \nabla'^2\mathbf{a} = -4\pi\rho\beta, \quad \mathbf{a} = \varphi\beta, \quad (4)$$

or by introducing $\rho' = k^{-1}\rho$ and $\varphi' = k^{-1}\varphi$, $\mathbf{a}' = k^{-1}\mathbf{a}$ we have

$$\nabla'^2\varphi' = 4\pi\rho', \quad \mathbf{a}' = \varphi'\beta \quad (5)$$

with the values of ρ' in the x', y, z system the same as ρ at corresponding points in an x, y, z system when everything is stationary. This, of course, is merely a special case of the relativity transformations.

From these equations, by steps given in full by Lorentz, note 14, we may prove

$$W_s = \frac{1}{8\pi} \int_{\infty} \{k^{-1}(D_{x'}\varphi')^2 + k[(D_y\varphi')^2 + (D_z\varphi')^2]\} d\tau', \quad (6)$$

$$T_s = \frac{1}{8\pi} \beta^2 k \int_{\infty} \{(D_y\varphi')^2 + (D_z\varphi')^2\} d\tau', \quad (7)$$

$$\left. \begin{aligned} M_{xx} &= \frac{1}{4\pi} \beta k \int_{\infty} \{(D_y\varphi')^2 + (D_z\varphi')^2\} d\tau', \\ M_{xy} &= \frac{1}{4\pi} \beta \int_{\infty} D_{x'}\varphi' D_y\varphi' d\tau', \\ M_{xz} &= \frac{1}{4\pi} \beta \int_{\infty} D_{x'}\varphi' D_z\varphi' d\tau'. \end{aligned} \right\} \quad (8)$$

Thus,

$$\left. \begin{aligned} T_s &= \frac{1}{2} \beta \cdot \mathbf{M}_s, \\ T_s - W_s &= -k^{-1} W_{\infty}, \end{aligned} \right\} \quad (9)$$

where W_{∞} is the electrostatic energy of the system at rest, so that $T_s - W_s$ is the same for all directions. (For the general case one may readily prove from the relativity transformations that

$$T_s - W_s = k^{-1}(T_{\infty} - W_{\infty}).$$

To evaluate M_s , we might use equations (8) but shall not do so, as there is a shorter way. For, by equation (2), the result must be the same as if we assume all electric and magnetic energy to be transported as indicated by Poynting's vector $\mathbf{e} \times \mathbf{b}$ and to have momentum equal, in the units of this paper, to its value as energy times the velocity with

which it would have to move to give the required transfer. This statement emphasizes the pragmatic value of this assumption, and is independent of what may or may not be said about actual truth of the assumption. Here we shall limit ourselves to a very slow motion, so that the total magnetic energy, being less than β^2 times the electrostatic, may be neglected.

The assumed flow of energy must be fast enough to accomplish three things: first, it must transport the whole electrostatic energy bodily with the system at a velocity β ; second, since energy is developed in the field at the back of each electron by the internal stresses pulling the electricity along against the retarding electric force, it must be transferred forward around the electron to be given up to the internal stresses again; and third, when the direction of motion is that of a , a similar transfer is necessitated by the force holding them together against their repulsion.

The first part gives $\frac{1}{2}(e^2/R)\beta$ for each electron and $(e^2/a)\beta$ for their mutual energy. For the second part we may notice that as the back of the electron sweeps over any element of volume, the work done by the internal stress equals the product of the volume of the element by the tension stress holding the surface of the electron from expanding. If dS is the element of surface, and x its distance forward of the center the energy given to the electromagnetic field per unit time is

$$\frac{1}{2} \frac{e}{R^2} \cdot e \frac{dS}{4\pi R^2} \cdot \frac{-x}{R} \beta$$

and the distance this must be carried forward by the field is $2x$, making the contribution of this element to M_x equal to $(e^2\beta x^2/4\pi R^5)dS$. Now $(x/R)dS$ is the projection of dS on the y, z plane, and therefore $(2x^2/R)dS$ is the volume in a cylinder parallel to the x axis on this element. Integrating, therefore, we have for the total contribution of this sort to M_x , $\frac{1}{8}(e^2/R)\beta$, making the total mass of one electron the same as one obtained by direct integration.

The last term may likewise be found to be $(e^2/a)\beta$ in the case of motion parallel to the line of centers, and zero for motion perpendicular to it.

Thus the total electromagnetic momentum is $\frac{4}{3}(e^2/R)\beta$ for both electrons separately, and $(2e^2/a)\beta$ or $(1e^2/a)\beta$ for the mutual momentum, according to the direction of motion. The total magnetic energy, T_x , being $\frac{1}{2}\beta \cdot M_x$, undergoes similar changes with direction. Notwithstanding this, the system must be free to turn in any position without external forces or gain or loss of velocity. The next problem is therefore to find the source of momentum and energy that makes up for these changes in M_x and T_x .

To determine this, let us examine the mutual electromagnetic mass from another viewpoint. In the case of acceleration along the line of centers there is a retarding action of the ether due to the delay in the changes of the electrostatic force by the time a required for propagation from one electron to the other. This makes a difference between the forces acting on them, equal to

$$2 \frac{d}{da} \left(\frac{e^2}{a^2} \right) \cdot \frac{1}{2} \dot{u} a^2 = - \frac{2e^2}{a} \dot{u},$$

which, acting for the time $\beta/\dot{u}$ required to gain the velocity β , will produce an impulse

$$- \frac{2e^2}{a} \beta,$$

which must be overcome by the external forces. The mutual electromagnetic mass is therefore the same in this case, that we found by the other analysis.

For acceleration across the line of centers we have a similar effect due in this case to a change of direction and giving a forward component of force on each one equal to

$$\frac{e^2}{a^2} \cdot \frac{\frac{1}{2} \dot{u} a^2}{a} = \frac{1}{2} \frac{e^2}{a} \dot{u}.$$

Also the electric vector radiated from each one, which was zero in the other case, gives a component on the other electron equal to $-(e^2/a)\dot{u}$. Thus, by a calculation exactly like that of the other case, we have a mutual electromagnetic mass of $1e^2/a$, rather than $2e^2/a$. The most plausible way to satisfy equations (1) is to account for the constancy of the total mass by some effect of the forces that hold the electrons together.

In rapid steady motion along the line of centers the distance a is contracted to ak^{-1} , while the electrostatic repulsion remains the same as it is at rest; and therefore the other force, of attraction, may plausibly be assumed independent of this distance. Hence in longitudinal acceleration it may be assumed the same for both in spite of the difference of the electrostatic forces, and we may say

$$m_{ii} = 0, \quad m = m_{ie} = \frac{4}{3} \frac{e^2}{R} + 2 \frac{e^2}{a}. \quad (10)$$

Therefore, since

$$m_{ie} = \frac{4}{3} \frac{e^2}{R} + \frac{e^2}{a} \quad (11)$$

we must have

$$m_{ii} = \frac{e^2}{a}. \quad (12)$$

This is accounted for if we assume that the internal attraction is propagated with the velocity c in such a way that for each one at a time l it is always directed to the position where the other one would be if it had kept, since the time $l - a$, the velocity it had then. This position differs from the real one by the distance $\frac{1}{2}\dot{u}a^2$, and since the magnitude of the force is e^2/a^2 , this effect would give exactly the required drag, $-(e^2/a)\dot{u}$, on the system. That means that this force is propagated with the velocity of radiation but without the perpendicular components characteristic of radiation.

While this assumption is of no interest for its own sake, since the case is a purely hypothetical one, it is of interest in showing the type of assumptions that may be made about the internal forces of the magneton or any other system to account for the uniformity of its mass in all directions which is demanded by equation (1) in the name of the principle of relativity and the experiments on cathode and β rays. Moreover it points toward the conclusion that every set of forces which carry energy with them must also give rise to an amount of mass in any direction equivalent to the energy carried by this set of forces when moving in that direction. Although this mass is not explained on an electromagnetic basis, it is correlated with other forces very much as electromagnetic mass is correlated with electromagnetic forces, as a result of the propagation of the forces by the ether with the velocity of light, though the assumed laws of propagation are distinctly different in the two cases.

An interesting consequence of the necessity for this "internal" mass, as we may call it, is a certain indefiniteness in the total mass of a classical electron. For if the internal stress is really a simple hydrostatic tension, as Poincaré and Lorentz have supposed, then there is no internal mass at all. But if the stress is a system of bonds between opposite elements of the charge, like the bond between the two electrons that we have assumed in the hypothetical case, then the internal energy is three times that of the hydrostatic tension and there is along with it an internal mass of $\frac{1}{2}e^2/R$, making the whole mass equal to e^2/R , instead of $\frac{2}{3}$ that amount.

This hypothesis in the case of a single classical electron would be a complication that is unjustified except perhaps by considerations of stability of shape of the electron.¹ Nevertheless the existence of the mass of this type is certainly necessary in cases such as the magneton,

¹ See Lorentz, l.c., Chapter 5, pp. 214-5. Lorentz notes here the probability that the internal stress in the electron may be more than a simple hydraulic tension, which would give a stable volume but not a stable shape.

in which the internal stresses cannot be purely hydrostatic.¹ In such cases therefore we must always recognize the dependence of the total mass on the nature of the internal stresses, and the resulting uncertainty in any inferences from the mass of the system as to the dimensions.

A GENERAL THEOREM ON "INTERNAL" MASSES.

Let us consider now the more general case of any system in which the charge densities and current densities at all points are constant, except for the changes required by relativity. This includes the case of the magneton or any atom that is not undergoing any change. In such cases we have, so long as the system is at rest, no displacement currents or time changes of ρ and ρu .

There are many possible sets of internal forces that will keep such a system in equilibrium when it undergoes the changes demanded by relativity on acquiring a high velocity, and it may be that there are many of them that would give a constant mass with some reasonable laws governing their behavior when the system is accelerated. The purpose of this paper, however, is not to investigate how many there are, or what laws each one would require, but rather in addition to pointing out the constancy of the mass of a system to find some plausible explanation of the non-electromagnetic part.

Therefore let us prove the following theorem: Let the internal forces of the system whose densities of charge and current do not vary with time be two sets of pairs of forces between each pair of volume elements, one of which sets exactly balances the electrostatic forces between these elements when the system is permanently at rest while the other set corresponds in the same way to the magnetic ones; let these forces be independent of the "apparent distances" between the elements; but vary with apparent direction exactly like the electromagnetic forces to which they correspond; when the system is accelerated, let each force of such a pair act at each instant as though the other element had kept its velocity constant since the instant when radiation would have had to start from it to reach the element in question at the present time; then the resultant of all these forces will combine with the electromagnetic field to produce a mass which is the same in all directions when the system is accelerated from rest.

As a corollary to this we may prove that under these conditions the mass due to the magnetic field is always exactly balanced by a negative mass due to the corresponding internal forces.

¹ The impossibility of assuming in the magneton such a distribution of charge and current as to give no internal forces seems to be proved by the excessive thinness of the ring found.

First, to prove that the effects of the electric and magnetic fields are independent we may find the electromagnetic momentum of such a system for an infinitesimal velocity β , by using the relativity transformations with all terms involving β^2 neglected. Thus, letting primes denote quantities as measured by an observer moving with the system, and therefore equal to the values the corresponding real quantities would have if the system were at rest, we have

$$\begin{cases} \mathbf{e}' = \mathbf{e} + \beta \times \mathbf{b}, & \mathbf{b}' = \mathbf{b} - \beta \times \mathbf{e}, \\ \mathbf{u}' = \frac{\mathbf{u} - \beta}{1 - \beta \cdot \mathbf{u}}, & \rho' = \rho(1 - \beta \cdot \mathbf{u}). \end{cases}$$

Since to the moving observer the system appears steady we have

$$\mathbf{M}' = \frac{1}{4\pi} \int_{\infty} \mathbf{e}' \times \mathbf{b}' d\tau' = 0, \quad (13)$$

for any system as symmetrical as the magneton.

But in any case,

$$\begin{aligned} \mathbf{M}' &= \frac{1}{4\pi} \int_{\infty} [\mathbf{e} \times \mathbf{b} + (\beta \times \mathbf{b}) \times \mathbf{b} - \mathbf{e} \times (\beta \times \mathbf{e})] d\tau' \\ &= \mathbf{M}_e - \frac{\beta}{4\pi} \int_{\infty} [\mathbf{b}_i^2 + \mathbf{e}_i^2] d\tau' + \frac{1}{4\pi} \int_{\infty} [k_{il}\beta b_i b_l + k_{il}\beta e_i e_l] d\tau', \end{aligned} \quad (14)$$

where the subscripts i and l denote the directions perpendicular and parallel to β , respectively.

But since $\mathbf{M}_e - \mathbf{M}'$ represents the electromagnetic momentum due to the velocity β , this equation shows that the steady electric and magnetic fields have no mutual mass.¹ This might be guessed from the fact that they have no mutual energy but does not follow from it without proof.

The electric mass alone is therefore the same as if no currents existed, and the magnetic mass the same as if there were no electrostatic charges.²

¹ Such a theorem has been proved previously by Comstock, *Phil. Mag.* (6), 15, pp. 1-21, Jan., 1908, but the proof given here, based on relativity, seems worthy of attention since it is considerably shorter and more direct and its range of application is different from that of Comstock's theorem.

² Equation 14 enables us to evaluate approximately the ratios of the masses of the electrostatic and magnetic fields of the magneton for each of the principal directions. As we shall prove below, the ratio of the radius of the cross section of the ring to that of the ring itself is extremely minute, and most of the energy and momentum of the field are concentrated very closely around the ring. This enables us to use the formulas for a straight wire as a first approximation. In this way one may readily prove that the momentum of the static charge is approximately the velocity of the magneton times its electrostatic energy in the case of motion along the axis, or times 3/2 the electrostatic energy for motion perpendicular to the axis. Similarly the steady current has a momentum approximately equal to the velocity of the system times the magnetic energy when moving along the axis, or 3/2 of this value when moving perpendicularly to it.

To prove the theorem we now have to find first some system of internal forces that will balance all those of electrostatic origin and at the same time obey the other conditions. Then we may do the same for the magnetic forces.

Such a system for the electrostatic forces must give a force of attraction of magnitude

$$\frac{\rho\rho'd\tau d\tau'}{r^2}$$

between any two elements $d\tau$ and $d\tau'$ whose distance is r when the system is permanently at rest; and the direction of each of them must be toward the position the other element would occupy if its velocity had not changed for a time r before the instant in question. This will put these elements in the condition of the two electrons considered above, and thus make the total direct and indirect electric mass of every pair of elements constant.

To find the magnetic mass, we may treat the system as if every element were electrostatically neutral, but had the same current density that it really has. $\mathbf{b}$ may then be found by equation (4), where

$$\mathbf{a} = \int_{\infty} \frac{[\mathbf{q}]d\tau}{r}, \quad (15)$$

the brackets indicating that the value of $\mathbf{q}$, or $\rho\mathbf{u}$ (the current density vector), for the element $d\tau$ is a "retarded" one, taken for a time an amount r before the time in question.

When the system is permanently at rest, $\mathbf{q}$ does not change with time, and $[\mathbf{q}] = \mathbf{q}$, so that the retarded values present no complications. But if the system is in motion with a low velocity β we must take account of the fact that the distance to the retarded position of each element is not that of the present position, and that the effective volume also is different. By exactly the reasoning given by Lorentz for the case of an electron in motion, we may show that this gives

$$\mathbf{a} = \int_{\infty} \frac{\mathbf{q}d\tau}{[r(1 - \beta_r)]}, \quad (16)$$

where β_r is the component of β in the direction from $d\tau$ to the point where $\mathbf{a}$ is to be found.

To find $\mathbf{b}$, or $\nabla \times \mathbf{a}$, for cases where β_r is small we may remember that if β is constant, $[r(1 - \beta_r)] = r + (\text{terms involving } \beta^2)$, so that $\mathbf{b}$ is the same as when the system is at rest except for such terms. In other cases we must have $\mathbf{b}$ equal to this part plus terms involving $\dot{\beta}$; and since the static part produces no resultant force on the whole system, the part

involving $\dot{\beta}$ must be held responsible for the magnetic mass. We shall neglect all terms involving β^2 and $\beta \cdot \dot{\beta}$.

To find db , the part of b due to the element $d\tau$, we must compute

$$\nabla \times (da) = \nabla \times \frac{q d\tau}{r(1 - \beta_r)}, \quad (17)$$

where q has the proper value for the element $d\tau$ and the symbol ∇ refers to differentiation with respect to the position of the other end of r . Now $[r] = r + [r\beta_r] + \frac{1}{2}[r^2\dot{\beta}_r] = r + r\beta_r - \frac{1}{2}r^2\dot{\beta}_r$, and $[1 - \beta_r] = 1 - \beta_r + r\dot{\beta}_r$, so that the denominator of da is $r(1 + \frac{1}{2}r\dot{\beta}_r)$.

To find $\nabla \times (da)$ let us use polar coördinates with the present position of the element as origin and direction of q as the pole, and with φ measured from the plane that includes the directions of $\dot{\beta}$ and q . With these coördinates

$$\begin{aligned} \nabla \times (da) = k_r \left\{ \frac{1}{r \sin \theta} D_\theta(\sin \theta da_\phi) - \frac{1}{r \sin \theta} D_\phi(da_\theta) \right\} \\ + k_\theta \left\{ \frac{1}{r \sin \theta} D_\phi(da_r) - \frac{1}{r} D_r(r da_\phi) \right\} \\ + k_\phi \left\{ \frac{1}{r} D_r(r da_\theta) - \frac{1}{r} D_\theta(da_r) \right\}, \end{aligned} \quad (18)$$

where

$$da = \frac{q d\tau}{r(1 + \frac{1}{2}r\dot{\beta}_r)} = \frac{q d\tau}{r} - \frac{1}{2}q\dot{\beta}_r d\tau. \quad (19)$$

The part $q d\tau/r$ gives the value b would have if $\dot{\beta} = 0$, that is, if the system were unaccelerated. This part is known to give no resultant force on the system, so that the part to be considered is

$$da' = -\frac{1}{2}q\dot{\beta}_r d\tau. \quad (20)$$

Since the direction of da' is always that of q ,

$$da'_\phi \equiv 0, \quad da'_r = -\frac{1}{2}q\dot{\beta}_r \cos \theta d\tau, \quad da'_\theta = +\frac{1}{2}q\dot{\beta}_r \sin \theta d\tau.$$

Let us now subdivide da' into two parts: da'' , due to $\dot{\beta}''$ the component of $\dot{\beta}$ parallel to q ; and da''' , due to $\dot{\beta}'''$, the rest of $\dot{\beta}$. Thus we have

$$da''_\phi = 0, \quad da''_r = -\frac{1}{2}q\dot{\beta}'' \cos^2 \theta d\tau, \quad da''_\theta = \frac{1}{2}q\dot{\beta}'' \sin \theta \cos \theta d\tau,$$

so that all derivatives of da'' containing da_ϕ or D_ϕ vanish leaving

$$\nabla \times (da'') = k_\phi \left\{ \frac{da''_\theta}{r} + \frac{1}{2r} q\dot{\beta}'' d\tau \frac{d \cos^2 \theta}{d\theta} \right\}$$

or

$$db'' = -\frac{1}{2}k_\phi \frac{q\dot{\beta}'' d\tau}{r} \sin \theta \cos \theta. \quad (21)$$

Since when the system is unaccelerated

$$db = k_{\phi} \frac{qd\tau}{r^2} \sin \theta$$

the effect of the introduction of db'' is like that of moving the element $d\tau$ in the direction opposite to that of q by an amount $\frac{1}{2}\dot{\beta}''r^2$ only inasmuch as this motion decreases θ by $\frac{1}{2}\dot{\beta}''r \sin \theta$ thus changing db by an amount

$$k_{\phi} q d\tau \left\{ -\frac{\cos \theta}{r^2} \cdot \frac{1}{2}\dot{\beta}''r \sin \theta \right\} = db''.$$

The complete effect of this imagined motion of $d\tau$ would include also a change of r and produce 3 times as large a change in the magnetic field. Thus the force on one element due to another accelerated in its own direction is affected by the acceleration of the system only inasmuch as the effective direction—not distance—from each one to the other has its changes due to acceleration delayed by the time r .

For the r component of $\dot{\beta}'''$ we have

$$\dot{\beta}''' \cos \varphi \sin \theta,$$

so that

$$da_{\phi}''' = 0, \quad da_r''' = -\frac{1}{2}\dot{\beta}'''q \cos \theta \sin \theta \cos \varphi d\tau,$$

$$da_{\theta}''' = +\frac{1}{2}q\dot{\beta}''' \sin^2 \theta \cos \varphi d\tau.$$

Here φ derivatives do not vanish, and we have

$$\begin{aligned} db''' &= \frac{1}{2}k_r \frac{q\dot{\beta}'''d\tau}{r} \sin \theta \sin \varphi + \frac{1}{2}k_{\theta} \frac{q\dot{\beta}'''d\tau}{r} \cos \theta \sin \varphi \\ &\quad + \frac{1}{2}k_{\phi} \frac{q\dot{\beta}'''d\tau}{r} \cos^2 \theta \cos \varphi \quad (22) \\ &= \frac{1}{2} \cdot \frac{q\dot{\beta}'''d\tau}{r} \{ \sin \varphi (k_r \sin \theta + k_{\theta} \cos \theta) + \cos \varphi (k_{\phi} \cos^2 \theta) \}. \end{aligned}$$

Now let us compute the change in the vector db produced by the changes of direction involved in a displacement of the element $d\tau$ in the direction opposite to $\dot{\beta}'''$, that is, of the observing point in the direction of $\dot{\beta}'''$, by an amount $\frac{1}{2}\dot{\beta}'''r^2$. These changes are of two sorts, first, those of $\sin \theta$, and second those of the direction k_{ϕ} .

For the first, we have an increase of θ by an amount $\frac{1}{2}\dot{\beta}'''r \cos \theta \cos \varphi$, which changes db by exactly db_{ϕ}''' .

For the second, we have a change of the direction of k_{ϕ} by an angle $\frac{1}{2}\dot{\beta}'''r(\sin \varphi/\sin \theta)$, so that this vector is changed by the introduction of a component of this magnitude directed away from the axis of the coördinate system, so that its r component has this magnitude times $\sin \theta$,

and its θ component this magnitude times $\cos \theta$. Substituting these components for the k_θ of the expression for db we find increments in db exactly equal to the r and θ components of db''' .

Thus these changes of db due to the acceleration component perpendicular to g can also be identified with those produced by a delay by the time r in the changes of direction but not of distance from the point at which db is evaluated to the element $d\tau$. The same statement therefore applies to all magnetic forces between any pair of volume elements.

Thus it appears that the magnetic forces on any element in such a system are always the same as if each other element was situated at the distance it actually is at but in the direction it would have if it had not been accelerated since the last radiation left it that has already arrived at the element in question. This is just the law postulated for the corresponding internal forces. Hence it appears that the part of the mass due to the magnetic field will not only have all dependence on direction of acceleration compensated by the internal forces, but will actually be balanced entirely by them. This completes the proof of the theorem and corollary stated above.¹

This result is of especial interest as applied to the Parson magneton, since it signifies that it is not unreasonable to assume that the magnetic energy of the magneton can not be deduced in any way from its mass, but that the mass is due entirely to its electrostatic energy and the internal forces associated with it. As one may readily see, if the internal forces of either kind are assumed independent of the distance between the elements under all circumstances, the energy of each force, reckoned from zero distance is equal in absolute value to the mutual electrostatic or magnetic energy, but with a plus sign in the former case and a minus in the latter. Hence in this special case we may apply the conclusion so often drawn, though not proved, from relativity, that the mass of each part of the system is equal, in the units used here, to its energy.

¹ A curious paradox arises from the fact that in the case of motion of a magneton perpendicular to its axis the electromagnetic momentum (from equation 14) per unit length of the ring is twice as great in the part of the ring lying parallel to the line of motion as in the part perpendicular to it, and yet during acceleration the former part is subject to no retarding force from the magnetic field. At first sight this would lead one to suspect an error on one side or the other. But the proof of (14) is very direct, and the other assertion rests simply on the fact that the force exerted by any magnetic field on an element of current is perpendicular to the current.

For the solution of this problem it appears that the retarding force on the part of the ring accelerated perpendicular to itself is much greater for acceleration perpendicular to the axis than along it. While the two parts of the ring could exist independently so far as the laws of electrostatics are concerned, the fact that a steady current is impossible without a closed circuit makes it necessary to consider the whole ring in any complete analysis of the magnetic mass.

GYROSCOPIC EFFECTS IN THE MAGNETON.

The question of the existence or non-existence of gyroscopic effects when the magneton is rotated about an axis lying in its plane is given considerable importance by the experiments of Barnett¹ on the magnetization of a bar of iron by rotation, and of Einstein and de Haas² on the converse of this effect. Although both effects are very small and require extremely delicate measurements, these experiments are quantitative as well as qualitative and indicate that the gyroscopic effects of the magnetic particle are of the order of magnitude of those of an electron revolving in an orbit. As Parson³ has pointed out there are serious difficulties in the way of the assumption that magnetism is due to an orbital motion of a electron or ring of electrons of the classical type. For aside from the well-known difficulties connected with radiation, there is the fact that non-coplanar orbits in an atom are unstable, and yet absolutely necessary if most elements, especially light ones, are not to be strongly paramagnetic. Hence arises the necessity for an examination of the gyroscopic properties of the magneton.

Such a calculation is reported in a paper which I presented at the meeting of the American Physical Society in February, 1917, and of which an abstract will be found in the *PHYSICAL REVIEW* of 1917. The net result was that the gyroscopic properties of the magneton are exactly those of a classical electron in an orbit having the same magnetic moment, provided that the assumptions of the present paper are correct.

THE EXPLICIT FORMULA FOR THE MASS.

The radius of the cross section of the ring, or "small radius," R' , as we shall call it, is extremely minute compared to the radius, R , of the ring itself. This simplifies greatly the calculation of the absolute value of the electrostatic energy, which would otherwise be quite difficult. One reason is that it makes the charge distribute itself practically uniformly around the cross section. Another is that the potential at any point on the center of the cross section will be practically the same as if all the charge were on the center line of the ring except for a length on each side of this point that is so short that it may be considered straight. While the truth of these assumptions is not self-evident, it may be proved in the calculation. For calculation, let the assumed straight piece have a length $2xR$, where x is a very small fraction and let $a = R'/R$. The line density of electricity on the ring is $e/2\pi R$. The potential at a point on the center line of the ring is

¹ Science, 30, p. 413, 1909; *PHYS. REV.*, 6, 239-70, Oct., '15.

² Deutsch. Phys. Gesell., Verh., 17.8., pp. 152-70, Apr. 30, '15.

³ L.c.

$$V = \frac{e}{2\pi R} \left[2 \int_0^x \frac{R d\theta}{\sqrt{R^2 \theta^2 + R'^2}} + 2 \int_x^\pi \frac{R d\theta}{2R \sin \frac{\theta}{2}} \right], \quad (23)$$

where the first integral gives the effect of the straight cylinder (treated as having a surface charge) and the other gives that of the rest of the ring. The justification for our assumptions must be the cancellation of all x 's from the result. By direct integration this becomes

$$\begin{aligned} V &= \frac{e}{\pi R} \left[\log \frac{x + \sqrt{x^2 + a^2}}{a} + \log \frac{\tan \frac{\pi}{4}}{\tan \frac{x}{4}} \right] \\ &= \frac{e}{\pi R} \log \left[8a^{-1} \frac{1 + \frac{1}{4} \frac{a^2}{x^2} + \dots}{1 + \frac{1}{8} x^2 + \dots} \right]. \end{aligned}$$

The x terms may now be dropped with no appreciable error, since the fraction a is so inconceivably small that a/x and x can both be negligible at the same time. This leaves

$$V = \frac{e}{\pi R} \log \frac{8R}{R'}.$$

Since the potential is constant within and on the surface of the ring, the electrostatic energy is

$$\frac{1}{2} e V = \frac{e^2}{2\pi R} \log \frac{8R}{R'}. \quad (24)$$

(It may be worth noticing that the magnetic energy may be found by Rayleigh's self-inductance formula¹ as

$$\frac{e^2}{2\pi R} \frac{v^2}{c^2} \log \frac{8R}{R'}$$

or v^2/c^2 times the electrostatic energy, as we should expect.)

Thus the mass of the magneton in ordinary units is

$$m = \frac{eV}{c^2} = \frac{e^2}{\pi c^2 R} \log \frac{8R}{R'}. \quad (25)$$

For the small radius R' , this gives the formula

$$R' = 8Re^{-\frac{\pi R m c^2}{e^2}}. \quad (26)$$

Since $e^2/mc^2 = 2.82 \times 10^{-13}$ cm., this makes R' almost incredibly small if R is as large as the value 1.5×10^{-9} cm., assigned by Parson. This,

¹ Roy. Soc. Proc., 32, p. 104, 1881; A 86, p. 562, 1912.

however, is very uncertain, so no definite value of R' can be assigned at this time.

SUMMARY.

1. It is shown that the relativity principle requires certain assumptions about the internal forces in the magneton, or any other non-spherical system, producing a sort of mass which, added to the electromagnetic mass, will make the total mass constant.

2. Making these assumptions in the most plausible way, the mass of the magneton is $2/c^2$ times its electrostatic energy, with no reference to its magnetic properties.

3. This result leads to a formula for the mass from which the small radius could be determined if the large radius were accurately known.

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REMARKS ON A PAPER BY MR. E. R. STOEKLE ENTITLED
"THERMIONIC CURRENTS FROM MOLYBDENUM."

BY O. W. RICHARDSON.

IN this very interesting paper published in the *PHYSICAL REVIEW* for November, 1916, Mr. Stoekle refers to some experiments I made with tungsten filaments, an account of which was published in the *Philosophical Magazine* (Vol. 26, p. 345, 1913). After pointing out that I made certain experiments which showed that the electron emission could not originate from any process involving the consumption of the hot filament, he says: "His conclusions, however, as to the absence of any appreciable change in the electron emission even upon allowing 'considerable changes in the amount and nature of the gases present in the tube' are not in accord with the observations of I. Langmuir upon the thermionic emission from tungsten nor with the results of the experiments upon molybdenum to be set forth in this paper."

This statement involves a misunderstanding of the language of my paper. None of my conclusions, so far as I am aware, contravene in any way the observations either of Dr. Langmuir or of Mr. Stoekle.

There are two passages in my paper which appear to be particularly relevant to the point at issue. On p. 347 I point out that in a vacuum in which the pressure, apart from mercury vapor, was of the order 10^{-7} mm. the electron current had the same value whether mercury vapor at a low pressure was present or not, and that a difference of 0.4 per cent. in the current, due to this cause, would have been detected. After showing that the emission is not to be attributed to any of the following alternative causes, viz.: (1) evolution of gas by the filaments, (2) chemical action or some other cause depending on impacts between the gas molecules and the filaments, and (3) some process involving consumption of the tungsten, I say on p. 350:

"(4) The only remaining process of a similar nature to those already considered which has not been discussed is the bare possibility that the emission is due to the interaction of the tungsten with some unknown condensable vapor which does not affect the McLeod gauge. This possibility is cut out by the fact that the thermionic emission is not affected when the liquid air and charcoal is cut off and the vapors allowed

to accumulate in the tube, and by the fact that very considerable changes in the amount and nature of the gases present (as by the admission of mercury vapor) have no effect on the emission."

In interpreting this passage it is to be remembered that in writing it I was arguing against a supposititious theory in which I did not believe and for the correctness of which there was no evidence, and that, apart from the allusion to mercury vapor, the vapors and gases referred to are purely hypothetical and imaginary. So far as the effect of mercury vapor on the emission from tungsten is concerned I would direct attention to the following quotation from a recent paper by Dr. Langmuir¹: "Hydrogen, mercury vapor and the inert gases, when pure, have no effect on the electron emission."

¹ Trans. Amer. Electrochem. Soc., p. 352 (April, 1916).

BINAURAL BEATS.

BY G. W. STEWART.

THE first recorded experiments with binaural beats seem to have been made by Dove¹ in 1839. He found that beats were obtained with two forks in unison by separating the forks with one near each ear as well as by presenting both of them to one ear. Obviously these and indeed all binaural experiments have a considerable interest to the physicist, the physiologist, the anatomist, and the psychologist. This breadth of interest has resulted in a scattering of the literature and in serious omissions of recognition of earlier work. Paul Rostosky² has prepared an apparently thorough review of the experiments concerning the results obtained by and conditioned upon bilateral excitation of the organs of hearing. The review includes all results obtained up to and including the year 1896. The excellence of Rostosky's résumé enables the writer to abbreviate greatly references to the work therein described. The policy has been adopted in this paper of accepting results that are amply confirmed by relatively recent workers, without attempting to explain the earlier disagreements which were probably due to individual differences and to the limitations imposed by the particular forms of apparatus.

The chief phenomena observed when two tones differing in frequency are led one to each ear are as follows:³

(a) Beats occur whether the tones are presented to one ear or one to each ear. This was discovered by Dove,⁴ observed subsequently by various experimenters and discovered independently by S. P. Thompson.⁵ This phenomenon is observed by all, even by the partially deaf.

(b) With bilateral presentation of the tones, the minimum intensity observed in each beat does not (even with carefully adjusted equal intensities of the two sources) approximate zero. This is mentioned by S. P. Thompson⁶ but doubtless was observed very much earlier by others.

¹ H. W. Dove, *Repertorium d. Physik*, Bd. 3, p. 404, 1839.

² Paul Rostosky, *Beiträge zur Psychologie und Philosophie*, I, 1905, pp. 173-273.

³ Unless otherwise stated, the difference of frequencies is assumed to be small.

⁴ *Loc. cit.*

⁵ S. P. Thompson, *Phil. Mag.*, 5, IV., 1877, p. 274.

⁶ *Loc. cit.*

(c) When the tones at the ears are alike in intensity and phase, there is a localization in the median plane, the position in this plane varying with the observer and presumably with the apparatus arrangement. Purkyně¹ was the first to record this localization, which has been shown through numerous experiments of other observers, to be a usual accompaniment of binaural perception.

(d) With opposition in phase and equality in intensity, the sound is localized according to S. P. Thompson² at the back of the head. To several observers, with apparatus of a form different than that of Thompson's the localization is in the median plane. Doubtless the position in the median plane is influenced by the method of observation.

(e) The sound is localized on the side of the fork leading in phase when the phase difference is approximately 90° . Thus with phase differences the sound wanders from the median plane at 0° to one side at 90° , to the median plane at 180° , to the other side at 270° and again to the median plane at 360° . Perhaps S. P. Thompson³ first observed the wandering from ear to ear but the periodic displacement of the localization was first most clearly described by Paul Rostosky.⁴ Later workers, who were not aware of Rostosky's results, observed this lateral displacement. Lord Rayleigh⁵ experimenting with forks, recognized the lateral effect and its relation to phase difference, and later⁶ confirmed these observations with experiments conducted by the use of telephones. L. T. More and H. S. Fry⁷ made preliminary experiments which were followed by those of L. T. More.⁸ His results showed unmistakably the lateral displacement. More used an apparatus similar to that earlier adopted by C. S. Myers and H. A. Wilson.⁹ A single source, branched pipes to the ears and an arrangement whereby the lengths of the pipes could be altered in opposite senses, were used. The experiments of Myers and Wilson also gave full confirmation of the lateral effects. Bowlker's¹⁰ experiments do not seem to furnish reliable evidence. All the workers just mentioned were not aware of the splendid experiments of Rostosky published in 1902.¹¹

(f) There is a limit to the frequency of the tone with which the

¹ Purkyně, Prager Vierteljahrsschrift, Bd. 67, p. 91, 1860.

² S. P. Thompson, Phil. Mag., 5, IV., 1877, p. 274, and Phil. Mag., 5, VI., 1878, p. 383.

³ S. P. Thompson, Phil. Mag., 5, XII., 1881, p. 351.

⁴ P. Rostosky, Philosophische Studien, 19, 1902, p. 557.

⁵ Lord Rayleigh, Phil. Mag., XIII., 1907, p. 214.

⁶ Lord Rayleigh, Phil. Mag., XIII., 1907, pp. 316-319.

⁷ L. T. More and H. S. Fry, Phil. Mag., XIII., 1907, p. 452.

⁸ L. T. More, Phil. Mag., XVIII., 1909, p. 308.

⁹ C. S. Myers and H. A. Wilson, Proc. Roy. Soc., LXXX., 1908, p. 260.

¹⁰ T. J. Bowlker, Phil. Mag., (6) XV., 1908, p. 318.

¹¹ *Loc. cit.*

effect just described (e) can be obtained. L. T. More¹ states that a fork of 512 d.v. is near to his limit of accuracy in the judgment by phase differences, that with 1,024 d.v. his judgment became untrustworthy and that with 3,000 d.v. he had no sense whatever of direction. Lord Rayleigh² gives the limiting frequency of his right and left sensations as 768 d.v. C. S. Myers and H. A. Wilson³ state in general terms that with very high frequencies the lateral effects cannot be obtained.

(g) In addition to the maximum at 0° phase difference, there are also two additional maxima occurring one before and the other after opposition in phase, but distinctly perceptible only when the beats are less frequent than one in two to five seconds. These additional maxima were first studied by Paul Rostosky⁴ in 1902, but in the succeeding literature no reference is made to his important contribution. Thus the subsequent discovery of these secondary maxima by the writer was an independent one. The results of the investigation of the latter are presented in this report.

(h) There are several other phenomena of importance, such as the effects obtained by placing forks on the skull, by using forks of large frequency differences, the result of practice of one ear upon the acuteness of the other, and the binaural audibility of two tones each one of which alone may be subliminal. For the present, we will be contented merely with this mention of them.

APPARATUS.

Mirrors were attached on the prongs of tuning forks so that the plane of each mirror was perpendicular to the vibration of its prong. The forks were then mounted so that their vibrations were in perpendicular planes. The light from an arc lamp, by the use of lenses and an aperture, was reflected from two of the mirrors, one on each fork, and formed a small spot of light or an image of the aperture, upon a screen. With vibration of the forks, Lissajou's figures were traced upon the screen, and by these the phase relation of the forks could be ascertained. Glass tubes were mounted, one close to a prong of each fork and were connected by means of two pieces of rubber tubing each a meter in length, to ear tubes such as are used with stethoscopes. In certain experiments the manner of introducing the sound to the ear was changed, but of this mention will later be made. The forks were operated electrically by using a short helical spring contact, 0.3 cm. in radius, of platinum

¹ *Loc. cit.*

² Lord Rayleigh, *Phil. Mag.*, XIII., 1907, p. 214.

³ *Loc. cit.*

⁴ *Loc. cit.*

wire, No. 29 B. & S. gage. The contacts were very quiet and gave a remarkable constancy of amplitude. Indeed the writer has observed the amplitude of a fork so actuated to remain constant, within one per cent., for an hour. No difficulty was experienced because of the presence of both forks in the same room. After many months' work with the apparatus, experimenting upon numerous individuals and confirming completely the experiments of Rostosky and others wherein even greater precautions were taken, the writer can confidently state that the results are in no wise dependent upon any "stray" sounds. Indeed, all the important results were confirmed by leading the tubes into an adjacent room where the forks were not at all audible to the unstopped ears. In practically all the experiments in the same room with the forks, the latter were scarcely, if at all, audible save when the tubes were placed in the ears.

THE RESULTS.

The following results were obtained with the assistance of Mr. F. C. Bruene.

1. The beats with the tones of two forks, one applied to each ear, were heard distinctly. Indeed, of twenty-three inexperienced observers listening for three periods of five minutes each, seventeen noticed the beats distinctly enough to report them. All experienced observers can hear them and record them with very satisfactory accuracy.

2. There existed a distinct wandering of the localization; in front at 0° difference of phase, on the side of the fork with the higher frequency leading in phase from 0° to 180° , and on the side of the slower fork with the higher frequency leading in phase from 180° to 360° , the changes of position being continuous, from front to rear (or within the head) and from rear to front. This wandering when first recognized might be described as occurring in a circular path. A more accurate description occurs in what follows.

3. This rotation of the localization just described was quickly and distinctly observed by some hearers; on the other hand all observers did not recognize it, even with an hour or more of practice. Eleven of the twenty-three uninstructed and inexperienced observers reported their observation of this "rotation" after fifteen minutes' trial (three trials of five minutes each). Six of these eleven noticed the effect within the first five minutes, and three in the second.

4. All of the eleven just mentioned, with the possible exception of one or two, noticed that the localization was much more distinct in the half cycle in which the lead of the faster fork varies from -90° to $+90^\circ$. With experienced observers, the effect is very noticeable.

5. Extended experiments with four experienced observers showed that the most accurate judgment as to phase difference, made by the localization, occurred when that difference is 0° .

6. With the experienced observers the change in the localization at 0° phase difference by the production of a very marked inequality in intensities, is slight. The rotation of the localization was modified by a slight circular shift in the direction of the side of the greater excitation. The effect upon the localization at the sides became very marked, that on the side of the greater intensity becoming more prominent and that on the other almost vanishing.

7. With observers of experience, but who had not yet recognized the secondary maxima,¹ the maxima intensity seemed to occur at 0° phase difference when the beat-period was less than perhaps two seconds. With a greater and gradually increasing period there arose an uncertainty until with a period of perhaps five seconds, the judgment of maximum intensity shifted to a phase difference noticeably less than 180° , or approximately to the first occurring secondary maximum.

8. The rotation of localization with experienced observers occurred as follows: At 0° phase difference the sound was localized as in front and distant, *i. e.*, external to the head. The localization then described in a horizontal plane a path which appeared to be somewhat circular, but in which the apparent distance contracted until the path entered the ear leading in phase. Then at about 180° phase difference the localization passed quickly through the head, into the other ear and from thence around to the front along a path symmetrical to the one just described.

9. When an experienced observer gave his attention to apparent intensity rather than to localization, there appeared three fairly distinct maxima. One occurred at 0° difference of phase, one at $180^\circ - \delta$ and one at $180^\circ + \delta$, δ being less than 45° . The earlier of the additional maxima at $180^\circ - \delta$ phase difference, coincided with the localization in the ear near the higher frequency, and the later one with the localization in the other ear. With a 128 d.v. fork the secondary maxima seemed clearly discernible only if the beat-period exceeded a value of approximately two seconds. The value given is only approximate as the variation in the perception of the secondary maxima is continuous with the changing of the beat-frequency. The significant fact is that the 0° maximum was always present, whereas the secondary ones required a beat-period exceeding at least a second in order to become clearly evident.

10. The localization in the ears as described seemed to depend upon

¹ See (g) and (9).

the perception of the secondary maxima. If they were absent, as at high beat-frequency, the localization seemed confined to the region near the front. In other words the localization in approximately one half the cycle seemed to be due to the secondary maxima or the causes producing them.

DISCUSSION OF RESULTS.

All of the results are not new. (1) was first discovered by Dove as already stated. (2) was in part reported by S. P. Thompson,¹ Lord Rayleigh,² C. S. Myers and H. A. Wilson,³ and L. T. More.⁴ Paul Rostosky was the first to record (2) and (8) and (9) completely. As previously stated in this article, Rostosky's work was unknown to Lord Rayleigh, C. S. Myers and H. A. Wilson, and L. T. More. Thus it chanced that the writer's discovery of the secondary maxima and the rotary character of the localization was an independent one.

None of the phenomena described owed its origin to sounds or noises other than the tones of the forks. Neither did any sound enter the left ear by aerial conduction from the tube entering the right ear. The former statement was tested by conducting the tubes to an adjoining room and by the many alterations in apparatus necessitated by the changing of the forks in experiments described in a succeeding article. The latter statement was verified by the use of tones of very faint audibility and by changing the character of the introduction of sound to the ear (*i. e.*, by the use of binaurals that are not inserted in the ears), without changing the phenomena observed in either case. Reference to Rostosky's experiments will show that he took much more elaborate precautions. Yet our results are in agreement with his at every point. The evidence thus favors the judgment that our precautions were sufficient.

There is an objection to the use of binaurals inserted in the ears, for by partially closing the external meatus they emphasize sounds that are conveyed from the skull to the volume of enclosed air. From the physical point of view, this partial closing of the external meatus would not be likely to change the characteristics of the phenomena and experiment shows this to be the case. Indeed, Rostosky used binaurals that were not inserted into but merely presented in front of the opening of the ear. The writer was also able to detect all the phenomena without the use of inserted binaurals. But there is an additional objection to the inserted binaurals, for there is the chance of conduction from the terminal of the binaural to skull. Experiments in stopping the openings

¹ *Loc. cit.*

² *Loc. cit.*

³ *Loc. cit.*

⁴ *Loc. cit.*

of the binaurals showed that there remained only an exceedingly faint trace of sound which could be fully explained by the failure of the stoppage. The conclusion of the writer is that the results with inserted and with non-inserted binaurals are not qualitatively different.

The results given point definitely to distinct differences between the causes of the localization in front and in the rear, or in the ears. The two halves of the cycle differ in distinctness of localization, in independence of beat frequency, in effects produced by inequalities of intensities, and in the apparent distance from the head, the localization in one half being external and in the other half internal. These differences must be accounted for in any adequate theory.

The fact that some observers by the single source method of C. S. Myers and H. A. Wilson¹ do not obtain such a clearly defined "rotation" but rather a wandering back and forth from the median plane to the sides, is apparently due to the difference in the method of observing. In our experiments, with the continually changing phase difference, the rotation is observed in every case where localization is obtained.

This article gives the chief phenomena characteristic of binaural beats. In succeeding articles will be presented a study of the secondary maxima and a physical theory of all the phenomena involved. It will appear that the experiments do furnish excellent evidence as to the perception of phase by the ear and as to a two-fold nature of the hearing organ.

I wish to acknowledge my indebtedness to Mr. F. C. Bruene for his assistance in the experimental work.

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¹ *Loc. cit.*

THE SECONDARY INTENSITY MAXIMA IN BINAURAL BEATS.

BY G. W. STEWART.

THIS paper is a continuation of a consideration of binaural beats; and contains the results of a study of one of the phenomena involved, namely, the secondary intensity maxima. The physics of hearing has been and is an important study, not because the discovery of new principles in physics are to be expected, but because the progress in the study of audition in the related sciences of psychology, anatomy, and physiology, is dependent upon a clear understanding of the physical aspects involved.

The reader should recall the fact² that when two tones of almost the same pitch are presented one to each ear, there appears an intensity maximum with 0° difference in phase, and secondary maxima at phase differences of $180^\circ - \delta$ and $180^\circ + \delta$. The secondary maxima are perceived clearly only when the beat-period exceeds one or two seconds. They seem to have an origin somewhat different than that of the zero-phase maximum, and therefore a study of them is important.

The results herein presented were obtained with the apparatus already described.³ There were two observers, one to operate the stop-watch at a chosen signal from the other who listened for the appearance of the secondary maxima. The method was to measure the time interval between the two secondary maxima, beginning with the first secondary maximum following zero-phase difference. Thus the interval included opposition in phase, and measured the time for a phase difference change of 2δ .

Results.—The secondary maxima will be considered as occurring at $180^\circ - \delta$ and $180^\circ + \delta$ phase differences, for to an observer watching the Lissajou's figures, it is evident that the phase difference of 180° occurs midway between the secondary maxima. The question arises, does δ change with the frequency of beats? The answer can be found in the following Table I.

¹ G. W. Stewart, *Phys. Rev.*, 2d Series, Vol. IX., No. 6, 1917, pp. 502-508.

² See *loc. cit.*

³ *Loc. cit.*

TABLE I.

Forks of 128 d. v.

Date.	Period of Beats.	δ in Sec.	δ/π .	δ/π .
Aug. 3	27.8		.20	
" 3	12.3	2.60(7)		.21
" 3	12.2	3.50(7)		.29
" 3	13.1	2.75(8)		.21
" 10	35.7	7.9 (4)	.22	
" 10	35.5	7.2 (6)	.20	
" 10	36.0	10.3 (9)	.29	
" 10	13.7	2.95(12)		.22
" 10	14.0	3.6 (11)		.26
" 11	35.2	7.9 (4)	.22	
" 11	13.3	2.68(10)		.20
" 11	13.5	3.1 (6)		.23
Mean.....			.23	.23

The number in parenthesis in the third column indicates the number of actual measurements, the average of which is expressed in the number just preceding. This table shows that with the beat-period increased by the factor three, no definite change takes place in δ . Similar results

TABLE II.

Forks of 42, 63, and 128 d. v.

Fork of 42 d.v.			Fork of 128 d.v.		
Date.	Beat Period.	δ/π	Date.	Beat Period.	δ/π
Aug. 11	20.4	.19 (10)	June 27	23.2	.25 (5)
" 11	20.2	.20 (8)	" 27	10.1	.29 (3)
Aug. 12	22.6	.18 (10)	June 29	23.9	.24 (3)
" 12	23.7	.16 (8)	" 29	31.0	.22 (3)
" 12	22.6	.18 (9)	" 29	16.0	.27 (4)
" 12	24.5	.15 (11)	July 2	35.8	.22 (4)
" 12	21.6	.17 (13)	" 3	27.8	.20
" 12	21.5	.18 (10)	" 3	12.3	.21 (7)
Mean.....		.17	" 3	12.2	.29 (7)
Fork of 63 d.v.			" 3	13.1	.21 (8)
Aug. 3	10.1	.20 (11)	" 10	36.0	.29 (9)
"	10.7	.20 (13)	" 10	35.5	.20 (6)
"	10.8	.18 (11)	" 10	35.7	.22 (9)
Aug. 5	10.2	.18 (4)	" 10	13.7	.22 (12)
"	19.5	.16 (6)	" 10	13.7	.25 (10)
"	19.0	.14 (4)	" 10	14.0	.26 (11)
Weighted mean.....		.18	" 10	13.6	.20 (7)
			" 11	13.3	.20 (10)
			" 11	13.5	.23 (9)
			" 11	35.2	.22 (4)
			Mean.....		.23

were obtained with forks of different frequencies, and can be inspected in succeeding tables where they are presented for another purpose. The data show the inaccuracy of the measurements of 2δ . A maximum would naturally be difficult to locate, for the hearing sense cannot detect small changes in intensity. But, as the succeeding data will show, this is not the only cause for inaccuracy. The constancy of δ with the frequency of beats is an important consideration in the theory of the secondary intensity maxima.

In what manner does δ vary with the frequency of the forks? Pairs of forks having approximately 42, 63, 128, 256, 384 and 469 d.v. were used in the experiments. The results are shown in Tables II. and III.

TABLE III.

Forks of 256, 361 and 469 d. v.

Fork of 256 d.v.			Fork of 361 d.v.		
Date.	Beat Period.	δ/π	Date.	Beat Period.	δ/π
July 13	13.2	.26 (9)	June 26	15.9	.28 (9)
" 13	12.2	.29 (9)	" 26	8.2	.29 (10)
" 14	43.8	.26 (4)	July 20	17.6	.28 (6)
" 14	27.4	.27 (8)	" 20	22.3	.26 (7)
" 14	23.4	.21 (8)	" 20	17.1	.25 (8)
" 14	12.0	.28 (11)	" 27	10.7	.27 (9)
" 14	10.5	.26 (9)	" 27	10.2	.24 (15)
" 14	9.25	.27 (9)	" 27	11.7	.30 (5)
Mean.....		.26	" 27	9.1	.28 (9)
			" 27	12.5	.22 (7)
			Aug. 1	12.7	.30 (6)
Fork of 469 d.v.			Mean.....		
Aug. 5	4.2	.32 (1)			.27
"	4.0	.37 (2)			
"	6.6	.27 (4)			
"	5.6	.35 (2)			
"	5.0	.38 (2)			
"	5.0	.26 (3)			
"	7.0	.27 (3)			
Mean.....		.32			

Here in the third and sixth columns are given the values of δ as fractions of π . The number in parenthesis following each value gives the number of measurements of which that value is the average. The data indicate conclusively the inaccuracy of the measurement. They show that with the same forks and the same beat-frequency, but with observations made on different days, there is a considerable variation. It was found that observations made at different times of day with the observer in different physical states gave different results. Consequently, the observations

on each day were scattered as widely as possible. All of the observations recorded were made by the writer, assisted by Dr. Harold Stiles of the Iowa State College.

Fig. 1 is presented in order to show the character of the variation of δ with the frequency of the forks. While the results are not accurate, the fact that the relation is not linear is indubitable. δ increases less rapidly than in proportion to the frequency.

The reader should be reminded that in all of the observations herein

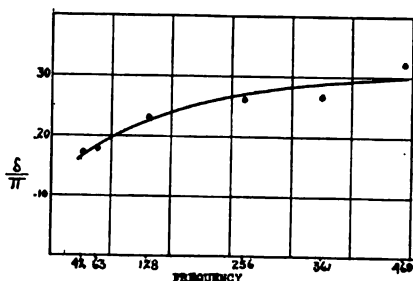


Fig. 1.

reported the two tones were presented by means of stethoscope-binaurals which were inserted in the external meatus of the ear. Objections to this method are stated and answered in the previous article.¹ It is the opinion of the writer that while the quantitative results here presented might be different with open binaurals, yet the conclusions to be drawn

from our experiments are not thereby affected.

Conclusion.—A moment's consideration of the presence of the secondary maxima leads one to assume that the effect is due to conduction from one ear to the other through the head. The first result, viz., the fact that with a given fork frequency δ is independent of the beat-frequency, is distinctly in harmony with this assumption. For, suppose an indefinite number of conduction paths in the skull from the aerial vibration on one side to the ear of the other side. The resultant of these conducted waves will differ in phase from the original aerial vibration by an amount which depends upon the conducting paths and the frequency of the vibration. For any given fork frequency this introduction of phase is therefore constant. (It will be shown in the theory to be published in this journal that this phase is, in fact, δ .) Hence δ should be independent of beat-frequency. Thus the expectation that the explanation of the secondary maxima is to be found in physical theory receives encouragement.

The second result, which shows a nonlinear variation of δ with fork-frequency, might seem in disagreement with physical theory, for with any path between the ears, the retardation in phase by propagation thereon would be proportional to the frequency. This is true for a single path; it is true for a duplex path, but, as can be readily shown, only in

¹G. W. Stewart, *loc. cit.*

case the amplitudes in these paths are equal. In as much as the amplitudes in all the conducting paths could not be equal, these paths cannot be equivalent to a single path of constant length. Hence we see that the two results herein described are not inconsistent with a theory which is based upon skull conduction from one side to the opposite organ of hearing.

I wish to acknowledge my indebtedness to Dr. Harold Stiles for his assistance in the observational work.

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THE THEORY OF BINAURAL BEATS.

BY G. W. STEWART.

AS described in preceding articles on binaural beats,¹ there are several phenomena that invite an explanation in terms of physics. The three outstanding aspects are (1) the existence of the "binaural" beats, herein referred to as "primary" beats which correspond to these heard uniaurally with two forks almost in unison, (2) the distinct perception of secondary intensity-maxima when the binaural beat-period is greater than two to five seconds, and (3) the wandering localization of the sound during each beat. We will consider these three separately and propose a theory which seems to promise a satisfactory explanation.

PRIMARY BEATS.

Dove² in 1839 discovered that when two forks, almost in unison, were presented first to one ear and then, by carrying one about the head, one to each ear, the beats perceived in the first position disappeared and subsequently reappeared as the moving fork approached its final position. I have chosen to call these binaural beats the "primary" beats, for they are always present with observers who are not seriously deaf in one ear. This is not true of the "secondary" beats which occur with clearness only if the primary beat-period exceeds two to five seconds. Separate treatment of the primary beats is justifiable, and in order to make the theory conform strictly to the fact, let us suppose we are dealing with a primary beat-period of approximately one second, or a period where the primary beats alone appear. As already described,³ we have then a maximum with 0° and a minimum with 180° phase differences.

Let us assume that the forks F_1 and F_2 are presented one to each ear, that their amplitudes are equal, that F_1 has the greater frequency, and that there are conducting paths from each fork to the farther ear.

Let ϵ_1 be the retardation in phase in transmission from each fork to the nearer ear. (By the "ear" is meant the physical instrument producing the sensation.)

¹ G. W. Stewart, *PHYS. REV.*, 2d Series, Vol. IX., No. 6, 1917, p. 502 and p. 509.

² H. W. Dove, *Repertorium d. Physik*, Bd. 3, p. 404, 1839.

³ G. W. Stewart, *PHYS. REV.*, 2d Series, Vol. IX., No. 6, 1917, p. 502.

Let ϵ_2 be the phase retardation in transmission to the farther ear.

Let θ be the phase of the slower fork. $\theta = 2\pi ft$, where f is the frequency.

Let $\theta + \epsilon$ be the phase of the faster fork.

Let α be the amplitude at the ear produced by the nearer fork.

Let β be the amplitude at the ear produced by the farther fork.

The the resulting displacement at the ear nearer to the faster fork, at the instant represented by the value θ , is $\alpha \cos (\theta + \epsilon - \epsilon_1) + \beta \cos (\theta - \epsilon_2)$. At the farther ear the displacement is $\alpha \cos (\theta - \epsilon_1) + \beta \cos (\theta + \epsilon - \epsilon_2)$. By expanding we find the former displacement to be $\sin \theta [-\alpha \sin (\epsilon - \epsilon_1) + \beta \sin \epsilon_2] + \cos \theta [\alpha \cos (\epsilon - \epsilon_1) + \beta \cos \epsilon_2]$ and the maximum displacement, or amplitude, to be

$$\sqrt{[-\alpha \sin (\epsilon - \epsilon_1) + \beta \sin \epsilon_2]^2 + [\alpha \cos (\epsilon - \epsilon_1) + \beta \cos \epsilon_2]^2}.$$

The energy of the vibration is proportional to the square of the amplitude. Hence the energy at the ear nearer to the faster fork is,

$$E_1 = \alpha^2 + \beta^2 + 2\alpha\beta \cos (\epsilon - \epsilon_1 + \epsilon_2).$$

E_2 , or the energy at the ear farther from the faster fork, can be written down by interchanging β and α and ϵ_2 and ϵ_1 , or,

$$E_2 = \alpha^2 + \beta^2 + 2\alpha\beta \cos (\epsilon - \epsilon_2 + \epsilon_1).$$

Let $\epsilon_2 - \epsilon_1$ be represented by λ .

Then

$$E_1 = \alpha^2 + \beta^2 + 2\alpha\beta \cos (\epsilon + \lambda) \quad (1)$$

$$E_2 = \alpha^2 + \beta^2 + 2\alpha\beta \cos (\epsilon - \lambda) \quad (2)$$

If the apparent intensity is determined by the sum of E_1 and E_2 , as we will assume, then,

$$E_1 + E_2 = 2(\alpha^2 + \beta^2) + 4\alpha\beta \cos \epsilon \cos \lambda \quad (3)$$

and the apparent intensity will be a maximum at $\epsilon = 0^\circ$ and a minimum at $\epsilon = 180^\circ$. Moreover, since in the very nature of the case β is less than α , the minimum can never be a silence. Thus the theory coincides qualitatively with experiment. Quantitative comparison seems impossible.

Discussion.—It will be observed that the above theory does not designate the points of interference in each organ of hearing, but that it does stipulate the occurrence of the interference in a wholly physical instrument. In other words, peripheral interference is accepted as the cause of the primary maxima, or the "beats." To this extent the theory is not new, for Dove,¹ the discoverer of the primary maxima,

¹ *Loc. cit.*

recognized peripheral interference as a possible explanation. This suggestion has often been repeated. Paul Rostosky,¹ in his theory of the localization and the secondary maxima accepts this non-synchronous variation of the resulting intensity in each ear. But neither in his excellent review² nor in his own contribution just mentioned does Rostosky give a detailed theory explaining the primary maxima and minima.

There may be an objection to the summation of intensities herein employed, but in justification the writer submits four considerations. First, we have the fact that two like tones, each alone faint enough to be just at the limit of hearing, become distinctly audible when simultaneously each is presented to an ear.³ Second, summation seems reasonable when one considers the phenomena of the strengthening of intensity in the numerous binaural experiments recorded in Rostosky's review, just mentioned. Third, the experiments of the writer⁴ upon the accuracy of localization of various frequencies, the head not remaining stationary, gives strong evidence in favor of the intensity-sum as an important factor. In these experiments the frequencies most accurately localized are those in which the theoretical variations of the intensity-sum are the greatest. Finally, the perfect blending of the two tones which is actually observed points distinctly to the summation. According to the theory just proposed, the effect is independent of beat-frequency, that is, the primary maxima and minima ought always to appear, which is in accord with fact. The above simple theory would seem to explain the primary maxima and minima satisfactorily. But the phenomena is much more complex, and these additional complexities will be discussed in a following section.

Peterson⁵ discusses the "nature and probable origin" of primary beats and concludes that they are cortical rather than peripheral in origin. His reasons are based upon the evidence of Cross & Goodwin⁶ to the effect that difference tones cannot be obtained binaurally, the evidence, from experiments on two people each deaf in one ear, that bone conduction must be very small, and upon certain experiments in the counting of the primary beats. The evidence offered by Peterson and the argument in favor of his conclusion are worthy of careful consideration. But in as much as he does not set up any physical theory, a discussion of his paper is probably not appropriate to this article. But the writer should

¹ P. Rostosky, *Philosophie Studien*, 1902, 19, p. 557.

² P. Rostosky, *Beiträge Zur Psychologie und Philosophie*, I., 1905, pp. 173-273.

³ Tarchanow, *St. Peterburger Med. Wochenschos.*, 1878, No. 43, (n.R.).

⁴ G. W. Stewart, *PHYS. REV.*, N.S., Vol. 2, No. 1, July, 1913.

⁵ Joseph Peterson, *Psychological Review*, Vol. XXIII, No. 5, Sept., 1916.

⁶ C. R. Cross & H. M. Goodwin, *Proc. of Academy of Arts & Science*, 1891, 27, 1.

call attention to the fact that the theory of this paper not only assumes a peripheral interference which Peterson denies, but in addition a summation of intensities or a blending which obviously cannot be peripheral.

THE SECONDARY MAXIMA.

The only records of the appearance of the secondary maxima are those of Paul Rostovsky¹ and the writer.² Rostovsky seeks to explain these maxima which appear first in one ear and then in the other, by a consideration of the localization. He concludes that the ratio of the two intensities, represented by equations (1) and (2), determines the degree of the right or left localizations. If one assumes α and β to be of the same order, say 1 to $\sqrt{2}$ or 1 to 2 (the values used by Rostovsky), and then plots the ratio of E_2 , (2), to E_1 , (1), he will obtain a curve similar to Fig. 1. The curve is interpreted by Rostovsky as the degree of right localization, the lower half of the curve, inverted, being the degree of left localization. Thus the maximum right localization appears distinctly or extremely right and gives the impression of a maximum intensity. Hence the two secondary maxima occurring in the neighborhood of a difference of phase of 180° are explained qualitatively; indeed, there also seem to be no objections thereto on the basis of quantitative considerations. Unfortunately, however, Rostovsky makes a fatal error in his theory. His equations similar to (1) and (2) are³ as follows:

$$J_l = a^2 + a_1^2 + 2aa_1 \cos(x + d/2)$$

$$J_r = a^2 + a_1^2 + 2aa_1 \cos(x - d/2).$$

Where the subscripts l and r refer to the left and right sides respectively, and a , a_1 , x and $d/2$ correspond to our α , β , ϵ and λ , respectively. d is a positive quantity, hence J_l reaches a maximum, with changing phase x before J_r , the former occurring at $x = -d/2$ and the latter at $x = d/2$. The faster fork is thus on the left. Moreover, between $x = 0^\circ$ and $x = 180^\circ$, J_l is less than J_r . Experimentally, the secondary maximum appears first (after $x = 0^\circ$) on the side of the faster fork, and hence on the side of the less theoretical intensity. If the maximum in Rostovsky's curve, Fig. 1, represents localization, then (accepting his equations just given) it must represent "left" localization. But this curve is the ratio J_r/J_l and therefore must be interpreted as "right" localization. Hence we find that experiment and his theory do not agree. His error is due to a confusion of sides; in fact, one is disposed to guess (incorrectly) that the side of greater intensity (when $0^\circ < x < 180^\circ$) is that of the faster fork.

¹ Paul Rostovsky, *Philosophie Studien*, 1902, 19, p. 557.

² G. W. Stewart, *Phys. Rev.*, 2d Series, Vol. IX., No. 6, 1917, 509-513.

³ See *loc. cit.*, p. 580.

The writer has considered whether or not an additional assumption of a change in phase during the conduction across the head would lead to a revision of Rostosky's theory that would be in harmony with observation, and he finds a revision upon such an assumption impossible. We are thus led to a complete rejection of the theory of Rostosky.

The writer's theory of the secondary maxima is based upon the outstanding importance of the following experimental facts which have been described in the two preceding articles.¹

1. The secondary maxima do not occur unless the period of the beats is greater than, say, two to five seconds, whereas the presence of the primary maxima is independent of the beat-period.

2. At the secondary maxima the sound seems localized in the ears (first in the one nearer the faster fork), and between the maxima within the head. With the primary maxima the localization is distinctly external to the head.

3. The sound seems to be much rougher at the secondary than at the primary maxima.

4. The movement of the localization is much more rapid between the two secondary maxima than elsewhere in the rotation.

5. If the secondary maxima appear at phase differences of $180^\circ - \delta$ and $180^\circ + \delta$ then, for a given fork pitch, δ is independent of the beat-frequency, but δ does vary with the frequency of the forks used, and this variation is not a linear one.

6. Unequal exciting intensities at the ears have a tendency to eliminate the secondary maximum on the side of the weaker source.

7. When the beat-frequency is much more rapid than one per second, not only do the secondary maxima disappear but localization in the region of 180° phase difference also vanishes.

These facts give a definite indication of the difference in origin of the primary and secondary maxima. But before we admit a complexity to account for this difference in origin, let us consider a physically simple equivalent of the skull and ear organs. Let us assume two physical instruments capable of recording vibrations and connected by numerous sound conducting paths. Is it possible for such an arrangement with two beating tones presented simultaneously one to each ear, to make a record that will show the three maxima and three minima actually experienced in each beat-cycle? Let us first assume that each instrument records intensity or a value proportional to the square of the amplitude at the instrument and that the resulting combined record is proportional to the arithmetical sum of these intensities. Upon this assumption we

¹ G. W. Stewart, loc. cit.

have already found, equation (3), that we may expect only a maximum at $\epsilon = 0^\circ$ and a minimum at $\epsilon = 180^\circ$.

If one assumes that the ear is an instrument that records not intensity but the time integral of the absolute value of the displacement, or $\int_0^{2\pi} |\text{displacement}| d\theta$,¹ he finds that the record in the nearer ear is proportional to

$$|\sqrt{\alpha^2 + \beta^2 + 2\alpha\beta \cos(\epsilon - \epsilon_1 + \epsilon_2)}|$$

and in the farther ear to

$$|\sqrt{\alpha^2 + \beta^2 + 2\alpha\beta \cos(\epsilon + \epsilon_1 - \epsilon_2)}|.$$

Reference to the equations just preceding (1) will show that these values are, in fact, the absolute values of the amplitudes. The total record, or the blended sensation, would be proportional to the sum of these two absolute values. It can be shown that if $\cos(\epsilon_2 - \epsilon_1)$ is greater than $2\alpha\beta/(\alpha^2 + \beta^2)$ this sum will be a maximum at 0° and a minimum at 180° . Obviously, $\cos(\epsilon_2 - \epsilon_1)$ is nearly unity, whereas the term $2\alpha\beta/(\alpha^2 + \beta^2)$ is, in all likelihood, much less than unity. We conclude that the foregoing assumption as to the dependence of the sensation upon the absolute value of the amplitude at the physical instrument leads to the same maximum and minimum as that based upon the dependence of the sensation upon the intensity of vibration. This leads one to doubt whether any assumption can be made as to the record of the instrument, the ear, which will result in a total sensation that has three maxima and three minima in one beat-cycle. At any rate, lacking such an assumption, the conclusion is that we must consider the ear not a single instrument, even for one tone. If, for our present purpose, one can consider each ear to consist of two different physical instruments which act somewhat independently of one another, a distinct gain is made thereby. For, assuming the sensations to be non-interfering, it is obvious that the sensations possible with two somewhat independent instruments are very different than those possible with a single instrument. For, suppose that instrument number one vibrates with the resulting vibration $f(t)$ giving the appropriate sensation, and that instrument number two vibrates with the resulting vibration $F(t)$, giving its appropriate sensation. If the sensations are non-interfering, the total sensation cannot, in general, be the same as had the vibrations $f(t)$ and $F(t)$ been combined in a single instrument.

Our theory will assume the equivalent of two physical instruments

¹ The writer is indebted for the suggestion of this assumption to Professor R. P. Baker, of the Department of Mathematics in the State University of Iowa.

in each ear excited in a mechanically different manner. A pair of like instruments, one on each side, is responsible for the primary maxima and minima, and another pair for the secondary maxima and minima. The former pair is excited chiefly through the ear drum-skin. The latter is excited chiefly through the vibrations of the skull. We will assume the mechanical arrangement to be such that the vibration conveyed by the skull from one fork to the distant ear has its phase determined not only by the equivalent length of path, but also by a change in phase of 180° . We will first discuss the consequence of the foregoing assumptions, and then its agreement with experiments.

The pair of A instruments, assumed excited chiefly through the membrana tympani, will give the primary maxima and minima provided we can also assume a conduction from one ear to the other. This conduction can occur by many paths and has been demonstrated actually to exist,¹ but the methods used could not exhibit the excellence of the conduction. We will assume this conduction as a fact without attempting to describe it in detail. Thus our previous theory, equation (3), can be regarded as applicable to the A instruments, and the primary maxima are completely explained.

The intensities in the B instruments can best be understood by reverting to the equations from which (1) and (2) were obtained. We will use $\alpha_1, \beta_1, E_1', E_2', \epsilon_1'$ and ϵ_1' instead of $\alpha, \beta, E_1, E_2, \epsilon_2$ and ϵ_1 respectively, inasmuch as the values for the B instruments are different. Remembering the assumption that ϵ_2 really becomes $\epsilon_2' + 180^\circ$ we obtain the displacement at the nearer ear, $\alpha_1 \cos(\theta + \epsilon - \epsilon_1') + \beta_1 \cos(\theta - \epsilon_2' - 180^\circ)$ and at the farther ear, $\alpha_1 \cos(\theta - \epsilon_1') + \beta_1 \cos(\theta + \epsilon - \epsilon_2' - 180^\circ)$. The intensities at the nearer and farther ears become, substituting λ' for $\epsilon_2' - \epsilon_1'$,

$$E_1' = \alpha_1^2 + \beta_1^2 - 2\alpha_1\beta_1 \cos(\epsilon + \lambda') \quad (4)$$

$$E_2' = \alpha_1^2 + \beta_1^2 - 2\alpha_1\beta_1 \cos(\epsilon - \lambda'). \quad (5)$$

Taking the sum of intensities as before, we have,

$$E_1'^2 + E_2'^2 = 2(\alpha_1^2 + \beta_1^2) - 4\alpha_1\beta_1 \cos \epsilon \cos \lambda' \quad (6)$$

This sum has a maximum at 180° and a minimum at 0° . If the total sensation varies as the sum of equations (3) and (6), or the total intensity sum of both pairs of instruments A and B, then, no matter what values are assigned to our constants, we could obtain only one maximum and one minimum for one beat-cycle. This can be shown by the usual method of determining maxima and minima. But the assumption that

¹ K. L. Schaefer, *Pflüg. Arch.*, Bd. 61, p. 544, 1895, and also C. S. Myers and H. A. Wilson, *Proc. Roy. Soc.*, 1908, A 80, p. 260.

instruments A and B are differently excited, leads us to assume also a difference in sensation from these excitations. If α_1 and β_1 have nearly the same value the intensity sum of the B instruments will be a maximum at 180° and a minimum, almost zero, at 0° phase differences. We would expect, then, the resulting observed effects at 0° to be chiefly the intensity-sum produced by the A instrument and at 180° to be the combination of the minimum of the A-intensity-sum and the somewhat different maximum of the B-intensity-sum. This is precisely the impression that an observer first obtains. He does not find two secondary maxima but only one in the neighborhood of 180° phase difference. Further, this maximum at 180° as already stated, appears to come from a source located in or on the skull, whereas the maximum at 0° is externally localized. This effect is in entire agreement with the assumption that the B instruments are excited by a skull vibration and the A instruments by an aerial vibration in the external meatus. The fact that the 180° maximum does not appear unless the beat-period is longer than two to five seconds indicates that the excitation of the B instruments requires a longer time, or that the inertia of the system is large, and this is in harmony with the assumption just mentioned.

But how account for two secondary maxima? In discussing the addition of intensities with the A instruments, we tacitly assumed a blending of the sound so that the maximum in either instrument was not noticed. This blending is, in fact, a part of our usual experience. But we have no reason for believing that the usual degree of blending must occur with the intensities at the B instruments. Indeed, the cause of blending (with the A instruments) is not understood. We may, therefore, without making any additional assumptions investigate what the effect would be if the observer is able to recognize the maximum intensity in each B instrument. Equations (4) and (5) show that the maximum intensity in the B instrument located on the side nearer the faster fork occurs before the maximum in the other B instrument by a phase of $2\lambda'$. Therefore, the maximum at the nearer B instrument would occur at $180^\circ - \lambda'$ and at the farther B instrument at $180^\circ + \lambda'$ phase differences. Further, the localization, because the excitation is from the skull, would appear to be first in the nearer and then in the farther ear. Again the theory is in distinct harmony with experiment. Thus, by assuming a difference between the A and B instruments in the manner of excitation and a difference in the degree of blending of the sensations, we can account for the appearance of the secondary maxima.

The difference in localization in the two parts of the beat-cycle, near 0° and near 180° phase difference, will be discussed in detail on a succeed-

ing page, but the comment may here be made that the cause is to be found in the difference in the manner of excitation of the two pairs of instruments.

The difference in the roughness of the sound in the two parts of the beat-cycle may also be explained by differences in excitation, for the excitation via the membrana tympani does, as we know, enable us to detect purity of tone, whereas the excitation directly by the skull would undoubtedly include natural vibrations of the skull itself and thus produce a somewhat rougher sound.

The rapid movement of the localization will also be presently discussed and satisfactorily explained on the basis of our theory.

The fact that δ (or λ' in our theory) is independent of the beat-frequency meets ready explanation for λ' represents the difference in retardation due to difference in the paths to the nearer and farther B ear, and certainly this value depends only upon the frequency of vibration and the conducting paths and not upon the rate at which the phase differences of the sources are changing. Or, in other words, λ' is entirely independent of ϵ and $d\epsilon/dt$.

But how does theory agree with the variation in $2\lambda'$ (our earlier 2δ) with frequency, as shown in Fig. 1 of a previous article?¹ We have assumed a retardation in phase of ϵ_1' in the propagation from the source

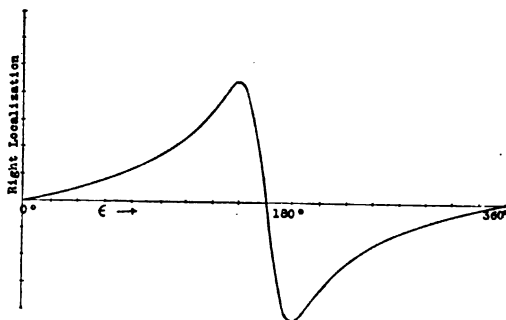


Fig. 1.

to the nearer instrument, and a retardation of ϵ_2' in the path from the source to the farther instrument. The excess retardation in conduction to the farther ear is thus $\epsilon_2' - \epsilon_1'$ or λ' . We can now introduce a helpful simplification without sensible error. Assume ϵ_1' occurs in a single path. Then it will be proportional to the frequency. But ϵ_2' will be a resultant of conduction over an indefinite number of paths, with varying amplitudes. Can ϵ_2' be regarded as occurring in an equivalent single path

¹ G. W. Stewart, *Phys. Rev.*, 2d Series, Vol. IX., No. 6, 1917, pp. 507-513.

and hence be considered as proportional to frequency? As we will see, the answer is a negative one. For let us consider two paths only, along which the vibrations have the amplitudes a_1 and a_2 . The difference in time required to transverse these paths is constant and hence the difference in phase at the distant junction of the paths is proportional to the frequency. Let this phase difference be γ , let r be the resultant amplitude, and let ϵ' be the phase difference between the resultant vibration and the one proceeding along the longer path, say, a_2 .

Then

$$r^2 = a_1^2 + a_2^2 + 2a_1a_2 \cos \gamma \quad (7)$$

and

$$r \sin \epsilon' = a_1 \sin \gamma \quad (8)$$

From these equations we get, in case $a_1 = a_2$,

$$2 \cos \frac{1}{2}\gamma \sin \epsilon' = \sin \gamma$$

or

$$\epsilon' = \gamma/2.$$

Thus with equal amplitudes, or with $a_2 = 0$, ϵ' is proportional to γ and therefore to the frequency. The assumption of equality of amplitudes would therefore give us a change in the phase, ϵ' , that is proportional to the frequency. But the assumption of equality of amplitude is not correct, for different paths would, on account of the differences in length and the presence of damping, have different amplitudes. We are then compelled to reject the suggestion that the various paths can be equivalent to a single path. Our expectation would therefore be that λ' or $\epsilon'_2 - \epsilon'_1$ would change either more rapidly or less rapidly than in proportion to the frequency. Our experiments, as already described, show the latter to be the case.

As already stated, if the intensities of the two sources are far from equal, the general effect of a maximum in the neighborhood of 180° phase difference is retained, but the secondary maximum on the side of the less intensity is very noticeably lessened. This is apparently caused by the fact that there is enough blending of tone to cover up the individual maximum which is, as experiment shows, only recognized at all with practice.

We have thus discussed all the phenomena (excepting those presently to be considered under "localization") and have found that all are distinctly in harmony with the theory herein presented.

THE LOCALIZATION.

Lord Rayleigh¹ is inclined to favor perception of phase difference as one of the explanations of the phenomena of localization. At zero-phase

¹ Lord Rayleigh, Proc. Roy. Soc., 1909, A 83, pp. 61-64.

difference the sound is localized in front from which point it seems to wander about the head in the direction of the side of the tone of higher frequency, then into the ear. It then seems to pass suddenly (at 180° phase difference) through the head, reappearing in the other ear, and then back to the median plane in front by a path similar to that of the first half of the beat-cycle. C. S. Myers and H. A. Wilson¹ from experimental evidence showing the variation of the displacement of the localization with changing phase, accept the view that the lateral displacement is proportional to the difference of the intensities at the ears. If we subtract (2) from (1) we get $-4\alpha\beta \sin \epsilon \sin \lambda$. If now we assume a phase change of 180° to be introduced in transmission we get, by substituting $\lambda + 180^\circ$ for λ , $+4\alpha\beta \sin \epsilon \sin \lambda$. The lateral displacement is then proportional to $\sin \epsilon$ and is zero when $\epsilon = 0^\circ$ or 180° and is a maximum for $\epsilon = 90^\circ$. This is the Myers-Wilson theory of localization. It does offer an explanation of the wandering of the sound toward and away from the median plane, but it is objectionable because by using this physical theory one cannot explain the primary maxima, the secondary maxima, the entrance of latter only when the beat-period is sufficiently long, the external localization in the region, $\epsilon = 0^\circ$, the internal localization in the region $\epsilon = 180^\circ$, the very small effect of excessive inequalities in intensities upon the localization in the region $\epsilon = 0^\circ$ and the marked effect in the region $\epsilon = 180^\circ$, the difference in rapidity of movement in the localization at $\epsilon = 0^\circ$ and $\epsilon = 180^\circ$, and the simultaneous disappearance of localization and secondary beats in the region $\epsilon = 180^\circ$, when the primary beats are much more rapid than one per second.² In fact, the localization in the half cycle $\epsilon = 270^\circ$ to $2\pi + 90^\circ$ differs so much from the localization in the other half that one can hardly accept a theory that does not also make a distinction. In justice to Messrs. Myers and Wilson the statement should be made that their form of apparatus did not lead them to perceive most of the features that have been mentioned.

In the view of the writer the explanation of the localization is not so important as that of the primary and secondary maxima, for while the localization does not occur with all observers even with considerable practice, the presence of the maxima seems common to all. From the physicist's viewpoint, our theory would be that in the phase region, $270^\circ < \epsilon < 2\pi + 90^\circ$, difference of phase at the A instruments³ is a

¹ C. S. Myers and H. A. Wilson, Proc. Roy. Soc., 1908, A 80, p. 260.

² Results given by G. W. Stewart, Phys. Rev., 2d Series, Vol. IX., No. 6, 1917, pp. 502-508.

³ For our present purpose it maybe sufficient to note that the phase of the fork nearer to an ear will furnish the greater portion of the amplitude and thus that the variation in phase difference at the A instruments is largely determined by the phase difference of the forks themselves.

determining factor, and that in the region $90^\circ < \epsilon < 270^\circ$ the non-blending excitation of the B instruments, first with greater intensity on the side of the higher pitch, and then with equal intensity and then with greater intensity on the other side, explains the localization. The explanation of the region $270^\circ < \epsilon < 2\pi + 90^\circ$ by phase difference will not meet the approval of many psychologists, on the ground that sensation depends upon the mode of response of the end organ and not upon the character of the stimulus. But one must not treat too seriously the application of a generalization that has never been definitely established. Moreover, inasmuch as all observers do not obtain the variation of localization herein described, we may assume it to be a "second-order effect" or one whose cause may never have been detected in experiments with sensory stimuli. Peterson¹ considers the localization in this region as caused by phase differences and concludes that the perception of phase difference is cortical in origin. The explanation given above of the localization in the region $90^\circ < \epsilon < 270^\circ$ does not meet with the objection just cited or any other of which the author is aware. Moreover, it should be anticipated that the excitation of the A instruments by the usual route, membrana tympani, etc., would give an external localization and the excitation of the B instruments, directly by the skull, an internal localization, just as is always produced by any vibration given the skull only. The effects produced by unequal intensities at the ears also accord with the theory. For if the localization near $\epsilon = 0^\circ$ is due chiefly to a perception of phase difference by the A instruments the effect of unequal intensity should be slight. Again, if the localization near $\epsilon = 180^\circ$ is caused by the lack of complete blending of the intensities at the B instruments, it would be seriously modified by inequality in the intensities at the ears. Both of these conclusions are verified by experiment. The rapid movement of the localization in the region of 180° is in accord with the theory, which gives the variation in phase difference with localization in the ears as $2\lambda'$ or $2(\epsilon_2' - \epsilon_1')$, for, from physical considerations, we can see that $\epsilon_2' - \epsilon_1'$ must be a small angle. Suppose the equivalent difference of path in the skull bones were 20 cm. If this were in air, the phase introduced thereby for a frequency of 256 d.v. would be about 60° . But the value λ' or δ for this frequency is approximately 45° .² Inasmuch as the velocity in the skull bones must be several times³ that in the air, we find that the theory would suggest an angle even smaller than that found by experiment. Both theory and experiment agree qualitatively that the dif-

¹ J. Peterson, *loc. cit.*

² G. W. Stewart, *Phys. Rev.*

³ H. Frey, *Ztschr. f. Psych. u. Phys. d. Sinnesorgane*, Bd. 28, p. 9, 1902, gives *ten times*.

ference between the phase relations at the two secondary maxima is markedly less than 180° and hence that the passage from the first secondary maximum (following 0° phase difference) to the other would seem to be more rapid than the movement in the remainder of the beat-cycle. The quantitative agreement is however, not satisfactory.

The theory is further confirmed by the fact that with a rapid beat-frequency (more rapid than one per second) not only do the secondary maxima disappear, but the localization in the 180° -phase-difference region disappears also.

ANATOMICAL CONSIDERATIONS.

The query arises, does the structure of the ear permit of the assumptions as made? It would seem that no serious objection arises if we assume the instrument A to be the organs of Corti in the ductus cochlearis, and the instrument B to be the similar organs in the saccule or the utricle. These organs in the cochlea and the saccule and utricle, have a common origin. Herrick¹ states that some physiologists have thought that cochlear and vestibular nerve systems are not wholly distinct and that the sense organs in the saccule may also function as a sound perceptor. This uncertainty concerning the function of the organs in the utricle and saccule on the part of the physiologists does not serve as an objection to our theory. Indeed, the fact that the complex phenomena herein described can be accounted for by the assumption of the existence of the B instruments ought to assist in deciding this uncertainty. The fact that there is a great difference in the central connections of the cochlear and vestibular nerves is in harmony with our contention that the instruments B give sensations which do not completely blend and which do not produce external localization.

But are the vestibular and cochlear organs mechanically different? Obviously they are, for the latter are placed upon a membrane, and are apparently excited by the motions of the fluid in the scala media, of the basilar membrane and of the tectorial membrane. The vestibular organs are located at the points where the membranous labyrinth is fastened to the skull itself. Moreover, the movement of the vestibular fluid must be very small for the vestibular canals are closed. Thus there would seem to be a mechanical difference which corresponds admirably with our assumption that instrument A is excited via the vibration of the membrana tympani and that instrument B is excited by the skull. The only change in our assumption induced by the structure of the ear would

¹ C. J. Herrick, *Introduction to Neurology*, pp. 201-202.

be the possible insertion of the word "chiefly" after each word "excited" occurring in the preceding sentence.¹

RESTATEMENT OF THEORY AND AGREEMENT WITH EXPERIMENT.

By the assumption of two physical instruments A and B on each side, having a difference in mechanical arrangement, a difference in the method of excitation, a difference in the blending of the sensations of the A pair and B pair of instruments and an independence of sensations produced by the two pairs and the additional assumptions that instruments A, the usual instruments of hearing, can perceive phase differences, and that in the skull conduction to the B instruments there is an additional phase change of 180° , we have a theory that explains fairly satisfactorily the presence of the primary maxima, the secondary maxima, the appreciable length of beat-period requisite for the appearance of the latter, the localization in the entire beat-cycle, the difference in the nature of the localization and in the quality of the sound in the two halves of the beat-cycle, the rapid movement of localization between the secondary maxima, the variation of the separation of the secondary maxima with the fork-frequency and its independence of beat-frequency, the difference in the effects of unequal intensities upon the localization in the two halves of the beat-cycle, and finally the simultaneous disappearance of the secondary maxima and the localization in the region of 180° -phase-difference.

Our theory does not lead us to identify the B instruments, but merely to suggest the possibility of these instruments being located in the utricle and saccule. Anatomical and physiological considerations give evidence in favor of the suggestions.

The theory is, of course, incomplete, but it has herein proved to be very successful and hence promising. There are no conspicuous phenomena connected with binaural beats that are not in accord with the theory, but there are minor points that should receive further consideration. For example, S. P. Thompson² has found binaural beats occurring when the tones used have frequencies of almost two to one.

While it is known³ that the secondary maxima and the localization phenomena occur both when the sounds are presented by tubes which

¹ A paper of importance has just come to the writer's notice. D. Richards (*Zeitsch. f. Biol.*, 1916, 66, 579-609) reports upon experiments with a guinea pig in which there were removed (1) both cochlea, (2) both cochlea and one vestibule and (3) both labyrinths. His conclusion is that sound stimuli may be regarded as adequate stimuli for the vestibular apparatus, but that no conclusion can be drawn as to any sensation produced in this way. This contribution is entirely favorable to the theory presented in this paper.

² S. R. Thompson, *Phil. Mag.*, (5), IV, 1877, p. 274.

³ G. W. Stewart, *Phys. Rev.*, 2d Series, Vol. IX. No. 6, 1917, pp. 502-508.

approach but do not close the opening of the external meatus, yet there may be a difference in the quantitative measurement of 2δ with the ear open and closed. The experiments of the writer have been made almost entirely with binaurals that close the external meatus, and accurately speaking his theory has reference to that experimental condition. Nevertheless he believes it to be equally applicable to the open-ear-binaurals.

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ON THE EFFECT OF DISTRIBUTED CAPACITY IN SINGLE LAYER SOLENOIDS.

By J. C. HUBBARD.

THE present paper describes a precise method of measuring the periods of electric oscillations, from the results of which is determined the effective, or distributed capacity of coils of a single layer over a wide range of coil dimensions. The method is checked by a determination, using the measured periods and the calculated inductances and external capacities, of the quantity v , the ratio of the electromagnetic to the electrostatic unit of quantity.

The problem of a coil oscillating in its own free period has proved one of the most difficult in mathematical physics. In addition to its inductance the coil possesses an effective capacity resulting from the fact that different parts of the coil are at different periodically varying potentials. There has been no theory developed except for coils of a single layer, and the theory which has been so developed agrees only roughly and within restricted limits with the experimental data. Drude¹ developed expressions for the wave-length and capacity of a freely oscillating coil and measured the wave-lengths of coils over a wide range of coil dimensions. A more rigorous treatment has been given by W. Lenz² who developed expressions for the wave-lengths of extremely long and of extremely short coils and covered the region between by interpolation.

Another method of finding the free periods consists in measuring the period of a circuit composed of an inductance and an external capacity in series. The square of the period should be a linear function of the external capacity. Thus, $T^2 = 4\pi^2 L(K + K_0)$, where K is the capacity of the external variable condenser, and K_0 is the remaining capacity of the system, composed in part of the distributed capacity of the coil. K_0 may thus be obtained by extrapolation. Such measurements were made by Professor Webster³ who verified the linear relation. Experi-

¹ P. Drude, *Ann. d. Phys.*, (4), 9, pp. 293-339, 1902.

² W. Lenz, *Ann. d. Phys.*, (4), 43, pp. 749-797, 1914. See also, K. W. Wagner, *Das Eindringen einer elektromagnetischen Welle in eine Spule mit Windungskapazität*, *Elektrotechnik u. Maschinenbau*, Wien, Nos. 8 and 9, 1915; *Die Theorie des Kettenleiters nebst Anwendungen*, *Archiv für Elektrotechnik*, Nos. 10 and 11, 1915.

³ A. G. Webster, *An Experimental Determination of the Period of Electrical Oscillations*, *PHYS. REV.*, VI., pp. 297-314, 1898.

ments along this line were projected by the writer in 1911, making use of a new method for measuring periods, with the special object of determining with accuracy the effective capacity of the coils used. Results were presented before the American Physical Society in October, 1912, and in April, 1915."¹

In the meantime Professor G. W. O. Howe² in a paper on the calibration of wave-meters had shown the linear relation to hold down to the free period of the coil and that the effect of the self-capacity of the coil is to add a constant capacity to that of the system.

W. Rietz,³ making use of a Helmholtz pendulum traced the wave form of an oscillating circuit with variable capacity and came to the same conclusion. His results, for the one single-layer coil studied, give only the order of magnitude of the distributed capacity. Frederick A. Kolster⁴ has shown how the distributed capacity of a coil may be found from the apparent inductances at various frequencies.

The present paper is an attempt to supplement the work of Drude, each coil being part of an oscillating circuit of the sort common in wireless telegraphy and instruments, one end of the coil being earthed, and not, as in Drude's experiments, oscillating freely as in the Tesla transformer.

MEASUREMENT OF PERIOD.

The method here used for measuring periods has already been briefly described by the writer.⁵ It makes use of a modification of the apparatus used earlier by Professor Webster and owes its precision largely to the excellent design of his drop chronograph. The modification introduced by the writer consists of an arrangement by means of which an electrometer is made to give deflections of a sign and magnitude depending upon the potential at any desired instant after the interruption of the current in an inductive circuit. If the circuit be oscillating the device enables the oscillations of potential to be traced and furnishes a very precise means of measuring the period of oscillations.

Lever 1 (Fig. 1) closes the circuit through the inductance L , the amount of current being regulated by the non-inductive resistance R . The condenser K is connected across the inductance terminals. A weight is released by an electromagnet and falls freely through a definite vertical distance striking lever 1 and opening the circuit. Oscillations ensue, the period of which depends upon the system K , L and connections, the

¹ J. C. Hubbard, *PHYS. REV.*, N.S., I., pp. 247-249, 1913, VI., p. 58, 1915.

² G. W. O. Howe, *Proc. Phys. Soc. Lond.*, XXIV., pp. 251-259, 1912.

³ W. Rietz, *Ann. d. Phys.*, (4), 41, pp. 543-569, 1913.

⁴ F. A. Kolster, *Proc. Inst. Radio Eng.*, I., No. 2, pp. 19-34, 1913.

⁵ *Loc. cit.*

variable resistance and battery being cut out and earth-connected at *E*. At a definite moment later the projectile strikes lever 2 which, in turn strikes lever 3, lifting it from the contact *P*. Lever 2 is permanently connected with one plate of the air condenser and may be regarded as a potential terminal. The contact *P* is connected with one pair of electrometer quadrants which are disconnected from earth at the moment of release of projectile. As may be seen, connection is established and immediately broken between the electrometer at the moment that lever 2 strikes 3. The phase of oscillation at this moment determines the sign and magnitude of the charge received by the electrometer. The duration of the time of contact between condenser and electrometer must be considered as finite, but the results show that it is either extraordinarily constant or very small compared to the periods used.

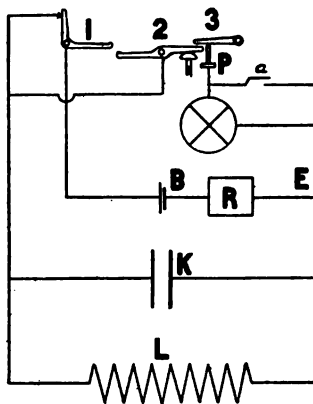


Fig. 1.

The system comprising levers 2 and 3 and the contact screw *P*, in addition to a catch (not shown) to prevent lever 3 from returning to *P* during the electrometer deflection, is mounted on a heavy plate of ebonite which is screwed to a carriage moved vertically by a micrometer screw reading to 0.001 mm. In practice it is customary to determine frequently the highest position for the carriage at which deflections begin to be observed in the electrometer. This is the point at which contact is made with the electrometer at the same instant as the breaking of the circuit and is taken as the micrometer zero. At this point the projectile has a velocity depending upon its height of fall above lever 1. Subsequent readings of the micrometer, referred to this point as origin, thus measure the further excursion of the projectile, from which its average velocity during any interval that oscillations are taking place, and hence the time required, may be found. In making the determination of a period it is customary to locate positions for two successive nodes, then to look for a node at twice this distance, and so on, doubling each time, until a suitable number of nodes, sometimes several hundred, have been covered. The initial and final nodal positions of the screw are determined to 0.0001 cm., and a range of not less than 0.5 cm. is covered, giving an accuracy in the length measurement better than one part in two thousand. The period of oscillation is then $T = s/nV$, where *s* is the distance on the micrometer screw corresponding to $2n$

half waves, and V is the average velocity of projectile during the oscillations which are being measured. It appeared that there were four possible sources of error in the determination of V , each tending to make the actual value less than that found; viz., residual magnetism in the electromagnetic release of the projectile, air resistance, moment of inertia of the levers, and tension of the rubber bands holding the levers in place. By using various values of magnetization in the electromagnet, various velocities up to 325 cm./sec., with consequent variation of air resistance, by trying levers of different dimensions and under different degrees of tension, it is found that these errors are not of a magnitude, singly or collectively, to be detected in the results.

The falling weight, or projectile, is that of Professor Webster's drop chronograph, a description of which has been published.¹

THE ELECTRICAL SYSTEM.

The solenoid, the distributed capacity of which is to be measured, is mounted, axis horizontal, at a distance of several feet from the condenser. The leads from the solenoid to the condenser form a rectangular circuit of known dimensions (Fig. 2), the plane of which passes through the axis

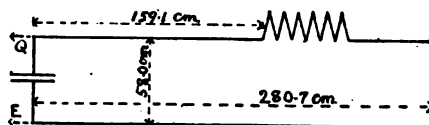


Fig. 2.

of the coil. The upper side of the rectangle is a continuation of the generating line of the coil surface on its lower side, the wire of the coil at the ends making a right angle turn about a peg set into the core. The wire Q , leads to contact with levers 1 and 2.

The air condenser consists of two plates of ground and polished steel held apart by sets of three small glass cylinders of length determined by an optical micrometer. Six sets of cylinders furnish the capacities (Kirchhoff's formula) given in Table III.

Tables I. and II. give the dimensions and calculated values of inductance of the accurately constructed coils. Rosa has given tables from which Nagaoka's formula for the inductance of a current sheet may be corrected to represent the actual inductance of a coil. The following data are necessary:² r = mean radius of coil, b = length from center to center of $n + 1$ turns = $(h - d')n/(n - 1)$, where h = axial length of

¹ *Loc. cit.*

² E. B. Rosa, Bull. Bu. Standards, 8, No. 1, p. 119, 1912-13.

the coil from outside to outside of insulated wire of diameter d' . d = diameter of bare wire, D = pitch of winding = $(h - d')/(n - 1)$. The inductance of the rectangular circuit is calculated from Rosa's formula,¹ in which a = length, b = breadth of rectangle, and ρ = radius of bare

TABLE I
Data for Coils.

Series.	r cm.	d cm.	d' cm.	D/d	Series.	r cm.	d cm.	d' cm.	D/d
E	9.740	.0650	.0840	1.50	K	6.297	.0650	.0840	1.40
F	21.163	.1291	.1529	1.63	L	31.57	.0636	.0844	4.98
G	9.84	.0625	.0834	3.13	M	9.64	.0627	.0840	5.06
H	31.63	.1810	.2095	1.75	N	9.745	.0627	.0840	1.56
I	31.55	.0632	.0830	5.02	O	21.165	.0650	.0860	3.26
J	7.96	.0659	.0855	1.35					

TABLE II.
Dimensions and Inductances of Coils.

Series.	n .	h .	$L' \times 10^{-6}$ Cm.	Series.	n .	h .	$L' \times 10^{-6}$ Cm.
E ₁	886	86.549	30.904	H ₃	70	22.06	3.839
E ₂	750	73.263	25.732	H ₄	50	15.75	2.273
E ₃	625	61.050	20.993	H ₅	35	10.95	1.283
E ₄	500	48.837	16.279	I ₁	162	51.19	12.926
E ₅	375	36.624	11.609	I ₂	100	31.50	6.551
E ₆	250	24.412	7.0403	I ₃	70	21.97	3.850
F ₁	215	45.37	12.637	I ₄	50	15.58	2.288
F ₂	180	37.97	9.993	I ₅	35	10.84	1.292
F ₃	145	30.58	7.442	J	216	19.2	4.41
F ₄	110	23.18	5.035	K	619	56.20	9.694
F ₅	85	17.90	3.448	L	162	51.17	12.946
F ₆	60	12.61	2.027	M	264	83.61	2.796
G ₁	436	85.10	7.763	N	868	84.88	30.202
G ₂	300	58.51	5.126	O ₁	215	45.33	12.674
H ₁	161	50.92	12.861	O ₂	180	37.94	10.021
H ₂	100	31.61	6.539	O ₃	145	30.54	7.465

wire. The wire of the coils of series *E* to *O* was double cotton covered. Coils *E* to *I* and *L* to *O*, inclusive, were wound on built up cores of white pine with the grain parallel to the axis of the coil. Coil *K* was wound on a tube of ebonite of 12.5 cm. external, and 10.1 cm. internal diameter, and of length, 60.5 cm. All cores except that of series *J* were given a light thread in the lathe barely sufficient to hold the tightly wound wire in place. Coil *J* was closely wound on a glass cylinder of thin walls (2 mm.).

The total inductance, L , of the system is obtained by adding the induc-

¹ *Loc. cit.*, p. 155.

tance of the coil, L' , given in Table II., to that of the rectangular circuit, L'' . This would, of course, be strictly true only if the terminals of the coils had been led to the axis before coming off, but the change is so slight as not to appear in any figures retained for total inductance.

Preliminary experiments were conducted to test the effect of changing the dimensions of the rectangular lead circuit and the relative position of coil and condenser. There was no sensible difference in the value of K_D or of v obtained with the same coil in the three circuits used. The dimensions of the circuit used in the final experiments are given in Fig. 2, ρ being 0.0251 cm., and the inductance of the circuit, $L'' = 10,260$ cm.

RESULTS.

The period of oscillation of the electrical system is, by reason of the small resistance in the oscillating circuit and the relatively small capacity, represented by $T = (2\pi/v)\sqrt{LC}$, where L is the combined inductance of coil and rectangular circuit, and C is the combined capacity of the system, consisting of the air condenser K , the capacity of leads and connections K_L , and the distributed capacity of the coil, K_D . Thus,

$$T^2 = \frac{4\pi^2 L}{v^2} (K + K_L + K_D).$$

If there is no appreciable variation of L or of K_D with the frequency, T^2 will be a linear function of K , or $T^2 = \alpha K + \beta$, where $\alpha = 4\pi^2 L/v^2$, and $\beta = (4\pi^2 L/v^2)(K_L + K_D)$. The constants α and β may be determined by the method of least squares, giving

$$\beta/\alpha = K_L + K_D = K_0, \quad \text{and} \quad v = 2\pi\sqrt{L/\alpha}.$$

Table III. is chosen to illustrate the manner in which these quantities are determined, and is an example of the best work done with the drop chronograph, though a large majority of the thirty-four similar tables

TABLE III.

Determination of α and β : Series E₁.

K Cm.	$T^2 \times 10^{10}$, Obs.	$T^2 \times 10^{10}$, Calc.	$\Delta T^2 \times 10^{10}$.	$\Delta T^2/T^2$.
211.32	2.7778	2.7779	-.0001	-.00004
332.04	4.1472	4.1464	+ 8	+ 19
427.47	5.2261	5.2282	- 21	- 40
565.05	6.7912	6.7880	+ 32	+ 37
827.56	9.7587	9.7636	- 49	- 50
1,528.71	17.7164	17.7119	+ 45	+ 25

$$\alpha = .011336 \times 10^{-10}$$

$$\beta = .38237 \times 10^{-10}$$

$$K_0 = \beta/\alpha = 33.73 \text{ cm.}$$

$$L = 25.742 \times 10^6 \text{ cm.}$$

$$v = 2\pi\sqrt{L/\alpha} = 2.9943 \times 10^{10} \text{ cm./sec.}$$

at hand show an accuracy as great, and in only two cases, owing to trouble with the drop chronograph, did the average value of $\Delta T^2/T^2$ exceed one tenth per cent. Table III. illustrates very effectively that, to a high degree of precision, there is neither a variation of inductance or of distributed capacity in the coils as here used.

Having determined K_0 as in Table III., it is next necessary to determine K_L , the capacity of leads, in order that the distributed capacity, K_D , of the coil may be found. The capacity of leads has been directly measured by an electrostatic method.¹ This is not a satisfactory procedure, as it is necessary in making such a measurement to disconnect the condenser and coil from the lead wires, tending to raise somewhat the capacity of the leads. A more satisfactory method consists in finding values of K_0 for geometrically similar coils of different linear dimensions. Suppose one coil is of linear dimensions n greater than another. According to the well-known theorem of electrically similar systems the distributed capacity of the larger coil should be n times that of the smaller. Thus $n(K_D)_1 = (K_D)_n$, whence $(\beta/\alpha)_1 = K_L + (K_D)_1$, and $(\beta/\alpha)_n = K_L + n(K_D)_1$, from which an independent determination of K_L and K_D is possible. If we consider coils of the same core material and of the same ratio of pitch to diameter of bare wire, we may put, after Drude, $K_D = r\phi(h/2r)$, whence $K_0 = \beta/\alpha = K_L + r\phi(h/2r)$. From graphs of K_0 as a function of $h/2r$ (see Tables I., II. and IV.) for the series E, F and H, which are strictly comparable, we may determine K_0 for the corresponding values of $h/2r$. For $h/2r = 1$, for example, we have,

$$\text{E,} \quad K_0 = 32.15 = K_L + 9.74\phi,$$

$$\text{F,} \quad K_0 = 44.85 = K_L + 21.16\phi,$$

$$\text{H,} \quad K_0 = 54.52 = K_L + 31.63\phi,$$

from which $K_L = 22.25$ cm. (See inserted figure in upper right hand corner of Fig. 4.) The value measured directly by the electrostatic method is 23.4 cm., an excellent agreement, being slightly larger, as is to be expected. We then have, $K_D = K_0 - 22.25$. The results are given in Table IV.

It is evident from the results that the factor of chief importance in determining the distributed capacity is the radius of the coil, in all cases the two quantities being of the same order. This agrees with Drude's theory, insofar as it concerns the radius alone, but the form of his expression for distributed capacity does not hold in the present case, the coils here being earthed at one end.

¹ J. C. Hubbard and H. F. Stimson, *PHYS. REV.*, N.S., I., pp. 245-247, 1913.

The values of K_D/r from the results of this paper are plotted in Fig. 3 with corresponding values of $h/2r$. It will be observed that K_D/r is a minimum and very nearly equal to unity when $h/2r = 1$. Drude's

TABLE IV.

Series.	$a \times 10^{13}$.	$\beta \times 10^{13}$.	K_D Cm.	Series.	$a \times 10^{13}$.	$\beta \times 10^{13}$.	K_D Cm.
E ₁	1.3614	46.964	12.24	H ₃	.16892	9.3184	32.92
E ₂	1.1336	38.237	11.48	H ₄	.10028	5.4961	32.56
E ₃	.92476	30.685	10.93	H ₅	.056886	3.2120	32.92
E ₄	.71627	23.973	11.22	I ₁	.57062	30.756	31.65
E ₅	.51132	16.666	10.34	I ₂	.28938	15.735	32.10
E ₆	.31106	9.998	9.89	I ₃	.170015	9.3104	32.51
F ₁	.55547	25.115	22.96	I ₄	.101221	5.4000	30.81
F ₂	.44066	19.455	22.00	I ₅	.057251	3.0750	31.34
F ₃	.32810	14.683	22.50	J	.19416	5.6855	7.03
F ₄	.22219	10.154	23.45	K	.42832	12.915	7.90
F ₅	.15228	7.0226	23.87	L	.57046	30.803	31.75
F ₆	.089730	4.2956	25.62	M	.12342	4.348	13.48
G ₁	.34434	11.874	12.23	N	1.3373	45.700	11.92
G ₂	.22706	7.6549	11.46	O ₁	.55899	25.445	23.30
H ₁	.56543	30.836	32.29	O ₂	.44284	20.215	23.40
H ₂	.28816	15.737	32.36	O ₃	.32973	15.108	23.57

theory, modified to suit the present case, and *assuming the total inductance to be effective*, gives values of distributed capacity in fair agreement (within ten per cent.) with results for the longest coils only. Further reference to Drude's work will be found below. It is interesting to

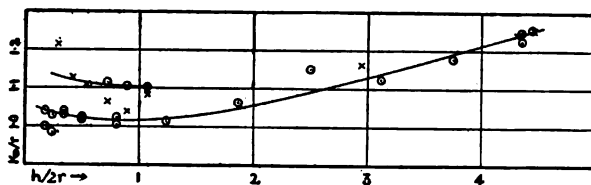


Fig. 3.

note that the increase in distributed capacity on the lower side of the minimum ($h/2r < 1$) is much less rapid than on the upper side. The results would seem to justify Lenz's mode of treatment, since we apparently have two quite independent factors, one predominating greatly in very short coils, the other, in very long coils.

There is no evidence that the variation of ratio of pitch to diameter of wire has a measurable effect on the distributed capacity in the region studied, though some effect is to be expected for coils of a smaller number of turns than those studied here. Such changes as here exist may

quite certainly be attributed to slight changes in the position of the wire leading from the condenser to the lever system.

The nature of the core in the present experiments seems to be of relatively small importance in the magnitude of distributed capacity. For example, the ebonite tube, Series K, gives, for $h/2r = 4.46$, a value of $K_D/r = 1.254$, while for coil E_1 , solid wooden core, $h/2r = 4.44$ and $K_D/r = 1.257$, a difference well within such accidental errors as might occur. On the other hand, Series J, close wound glass cylinder, $h/2r = 1.205$, gives $K_D/r = .884$, a result twelve per cent. smaller than the corresponding value for wooden cores, but, on filling the cylinder with kerosene the value is raised by only one per cent.; in the cases of both the empty and filled cylinder the values lie much closer to those for corresponding coils on wooden cores than do Drude's results for freely oscillating coils on glass cylinders.

In the present experiments, accordingly, the dielectric nature of the core is less important, owing to the fact, as before noted, that one end of the coil is connected to earth, producing a different configuration of the field of electric force from that in Drude's experiments. Fewer lines, relatively, would traverse the core.

To bridge the gap between these experiments and those of Drude the periods of two of the larger coils, with a threefold variation of radius, were measured with $K = 0$, *i. e.*, with the air condenser disconnected. Since $T^2 = \alpha K + \beta$, we have $T_0 = \sqrt{\beta}$. The results follow.

Series L, $\alpha = .0057046 \times 10^{-10}$, $\sqrt{\beta} = .5554 \times 10^{-5}$, T_0 observed $= .57223 \times 10^{-5}$.

Series N, $\alpha = .013373 \times 10^{-10}$, $\sqrt{\beta} = .67602 \times 10^{-5}$, T_0 observed $= .70765 \times 10^{-5}$.

Let Δ be the small positive change introduced in the lead wire capacity by disconnecting the condenser. Then, $T^2 = \alpha\Delta + \beta$, and $\Delta = (T_0^2 - \beta)/\alpha$, giving $\Delta = 3.41$ cm. for Series L, and 3.28 cm. for Series N. Taking as the mean, $\Delta = 3.34$ cm. and calculating $T_0^2 = 3.34\alpha + \beta$, we have for calculated periods of L and N respectively, $.5718$ and $.7082 \times 10^{-5}$ sec., the corresponding observed periods being $.5722$ and $.7076 \times 10^{-5}$ sec.

This is a remarkably good agreement, and shows that the discrepancy between T_0 and $\sqrt{\beta}$ may entirely be explained by the change introduced in lead capacity. It may be concluded that, to a high degree of precision, the square of the period of a coil and condenser circuit is a linear function of the external capacity down to the value of the capacity of the coil.

This conclusion draws attention to a very interesting matter, also discussed in Howe's paper. In all our work we have assumed that the

total inductance of the coil is effective. Drude assumes a distribution of current in a freely oscillating coil such that the total inductance is multiplied by the factor $2/\pi$ to give the effective inductance. The effective capacity of the coil would then be $\pi/2$ times the values we have given. Since the relation $T^2 = \alpha K + \beta$, holds down to $K = 0$, with precision, it would seem that either the total inductance must be effective in a freely oscillating coil, or there must be some exactly compensating change in the effective capacity.

The free periods of all the coils used, earthed at one end, have been calculated by putting $K = -K_L$, giving $T_F = -\alpha K_L + \beta$, and $\frac{1}{2}\lambda_F = \frac{1}{2} 3 \times 10^{10} T_F$. The results follow very closely Drude's expression $\frac{1}{2}\lambda = lf(h/2r, D/d, e)$ if we take for l twice the length of wire in the coil, and for f values in his tables corresponding to values of h equal to twice the axial length of coil for *coils without cores*. The values of f are plotted with corresponding values of $2h/2r = h'/2r$ in Fig. 4. (Series E and H,

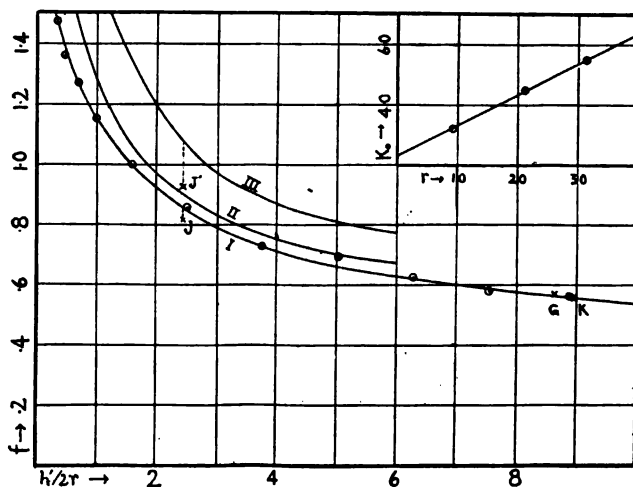


Fig. 4.

only, are plotted for the sake of clearness.) The smooth curve, I., drawn through these points, closely parallels Drude's curve, II., for similar coils without cores, in which $h' = h$. The curve, III., is drawn from Drude's results for corresponding coils on wooden cores, and, as may be seen, lies much farther away from the results of the present paper. The point K, giving the value of f for Series K (ebonite tube), is seen to correspond well with the results of this paper for wooden cores. From Drude's work a value considerably less would be expected. The departure of the value of f for series J (coil wound on glass cylinder), from the curve for wooden cores is seen to be much smaller than the

departure of Drude's corresponding value for a coil on a glass cylinder, J' , from his results for similar coils on wooden cores. All these facts are in further support of the conclusion that, for coils earthed at one end the material of the core has much less effect on the magnitude of the distributed capacity than it has in coils freely oscillating with a potential node at the center.

MEASUREMENT OF v .

As a check on the accuracy of the drop chronograph, a calculation of v , the ratio of the electromagnetic to the electrostatic unit of quantity, has been made from the results. We have,

$$T^2 = (2\pi/v)^2 L(K + K_0) = \alpha K + \beta,$$

from which,

$$v = 2\pi\sqrt{L/\alpha}.$$

Table III. gives an example of v so computed. From the data already given v may be calculated in each case. It will be found that the values are in excellent agreement. Owing to the greater care taken in the construction and measurement of dimensions of the coils of Series E and F we shall use the values of v obtained in those series as examples. We have for the average deviations of r and h , respectively; for E, .003 and .003 cm.; for F, .008 and .005 cm. Table V. gives the values of v , the computed deviation in v , Δv_c , as well as the actual deviation, Δv_0 , from the mean.

TABLE V.

Determination of v .

Series.	$v \times 10^{-10}$.	$\Delta v_c \times 10^{-10}$.	$\Delta v_0 \times 10^{-10}$.	Series.	$v \times 10^{-10}$.	$\Delta v_c \times 10^{-10}$.	$\Delta v_0 \times 10^{-10}$.
E ₁	2.9942	$\pm .0013$	-.0008	F ₁	2.9983	$\pm .0013$	+.0030
E ₂	43	10	- 7	F ₂	39	16	- 13
E ₃	50	11	$\pm$ 0	F ₃	52	13	$\pm$ 0
E ₄	63	13	+ 13	F ₄	43	16	- 9
E ₅	52	11	+ 2	F ₅	45	14	- 7
E ₆	14	—	—	F ₆	39	—	—

The values of v for E₆ and F₆ are rejected because the smallness of periods in these cases made it impossible to carry out measurements with the smaller values of external capacity, giving values of α of a lower order of precision than in the other cases. From the first five results in each series, we have,

$$E, \quad v = 2.9950 \times 10^{10} \text{ cm./sec.},$$

$$F, \quad v = 2.9952 \times 10^{10} \text{ cm./sec.},$$

with an accuracy better than one part in two thousand, a much higher degree of accuracy than has before been obtained by this method. This

is the more interesting because the most authoritative writers on the subject have expressed the opinion that an accuracy greater than one part in one thousand could not be reached by the present method.¹

The best determination of v which has been made by any method is that by Rosa and Dorsey,² who obtained the value, $v = 2.9963 \times 10^{10}$ cm./sec., with an accuracy of one part in ten thousand.

The values of v here obtained are such as to commend the drop chronograph as an excellent instrument for the precise measurement of small intervals of time, for the calibration of wave-meters, and to establish the value of the present method of measuring periods. It would be possible, with small additional labor, to increase the accuracy of the time measurement to one part in ten thousand. This would give a value of v , using properly constructed coils, with a precision probably as great as that of Rosa and Dorsey.

The present method of measuring periods with the drop chronograph is especially useful for getting precise measures of small capacities or of variations of capacity, any capacity which has been introduced appearing as a change in K_0 . In this way the capacities of lead wires, of electrometers, and the mutual effects of conductors have been studied. The apparatus as used in getting the results in this paper is sensitive to a change in K_0 of one part in one thousand, or, of .01 to .04 cm. of capacity. It is obvious that the method is also useful for the precise determination of inductance, and should be especially valuable, because the capacity of the coil is eliminated.

Dielectric constants of insulating liquids have been measured by the writer. For the purpose a small air condenser in a suitable cell was introduced in series with K and the period measured with it empty, or filled with the desired liquid. When conducting liquids are used the damping is so great that precise values of T cannot be obtained by this method. A resonance method has accordingly been developed for the study of dielectric constants of conducting liquids. This method, using a circuit calibrated for wave-length by the drop chronograph, and an audion detector, is capable of yielding good results also for distributed capacity of coils.

Some work has been done on the distributed capacity of coils of more than one layer. This will appear in a later paper.

In conclusion it may be remarked that information is greatly needed concerning the distribution of current and potential in oscillating coils. Until such information is at hand it is probable that the theoretical treatment of the single layer solenoid will not make much headway.

¹ E. B. Rosa and N. E. Dorsey, *Bull. Bu. Standards*, 3, p. 617, 1907.

² E. B. Rosa and N. E. Dorsey, *Bull. Bu. Standards*, 3, pp. 433-604, 1907.

SUMMARY.

The present paper describes a precise method of measuring the electrical periods of circuits, from the results of which are calculated the effective, or distributed capacities of coils earthed at one end. The work covers a wide range of coil dimensions.

1. It is found that the relation between the square of the period and the external capacity is very accurately linear down to the free period of the coil. This relation is new only in the accuracy with which it has been proved.

2. That the distributed capacity of coils earthed at one end is not much affected by the nature of the core.

3. The distributed capacity of such coils is a minimum and practically numerically equal to the radius of coils of length equal to the diameter, increasing, for example, by twenty-five per cent. when the length of coil is four times its diameter. For the practical purposes of wireless telegraphy it would seem sufficiently accurate to consider the effective capacity of the coil as equal to its radius in cm. of capacity, or to $(7/9) \times 10^{-5}$ microfarads.

4. The results, extrapolated to the natural periods of the coils, for coils earthed at one end, and with wooden cores, are in good general agreement with the results of Drude for freely oscillating coils *without* cores and of *twice* the axial length. This gives additional support to conclusion (2).

5. The method is checked by a determination from the results, of v , the ratio of the electromagnetic to the electrostatic unit of quantity. Values are found having an accuracy better than one part in two thousand, establishing the reliability of the present method of measuring electrical periods and commending the drop chronograph as an instrument for the precise measurement of small intervals of time.

6. Some conclusions are drawn concerning the serviceability of the method for the absolute measurement of capacities, inductances, or dielectric constants.

7. It is difficult to reconcile the facts of this paper summarized in (1) and (5) with the general practise of assuming the effective inductance of an oscillating coil to be $2/\pi$ times the total inductance. This question waits on more precise information as to the distribution of current and potential in a freely oscillating coil.

THE INFRA-RED ABSORPTION SPECTRUM OF NAPHTHA- LENE AND OF SOME OF ITS MONO-DERIVATIVES IN SOLUTION.

By A. H. STANG.

INTRODUCTION.

ALTHOUGH much experimental work has already been done on finding the infra-red absorption spectrum of different substances, there remains a very large number of substances whose spectrum is unknown. The results of the infra-red investigations have been very fruitful in solving problems of molecular structure and in testing the energy quantum theory, especially as applied to the theory of specific heats. Thus Bjerrum¹ from a knowledge of the infra-red absorption bands of CO₂ has proposed a solution for the problem of the arrangement of the atoms in the CO₂ molecule, while Nernst and Lindemann² have deduced formulas for the variation of the specific heat with temperature that agree closely with experimental data and are based on the location of infra-red absorption bands of the substances in question.

Still more recently Baly,³ of the University of Liverpool, has proposed a theory which unites the absorption of an organic substance in the infra-red to that in the visible and ultra-violet regions of the spectrum and to the fluorescence and phosphorescence which a body may emit. He has, in particular, predicted from theoretical considerations at what wave-lengths the infra-red absorption maxima of several substances should occur. Among these substances was naphthalene, C₁₀H₈.

The object of this paper is then to investigate the absorption of naphthalene as well as that of several of its mono-derivatives and especially to obtain the positions of maxima absorption. Since these substances are solids at ordinary temperatures, it was determined to obtain their absorption curves when in solutions of carbon disulphide and carbon tetrachloride. These solvents were chosen because they are relatively transparent in the infra-red. From these absorption curves the effect of the solvent and of isomerism will be noted. The agreement of these

¹ Bjerrum, *Verh. der deut. Phys. Gesell.*, 16, p. 737, 1914.

² *Zeit. fur Electro-chemie*, 17, p. 867, 1911.

³ Baly, *Astro-phys. Journal*, 42, p. 4, 1915. *Phil. Mag.*, Vol. 27, p. 632, 1914; vol. 29, p. 223, 1915; vol. 30, p. 510, 1915; vol. 31, p. 417.

absorption maxima with Baly's theory of absorption will also be tested and some conclusions as to the validity of that theory drawn.

APPARATUS.

A fixed arm spectrometer, provided with the Wadsworth¹ constant minimum deviation attachment was used for the work. Fig. 1 gives a diagram of the apparatus. *A* is an adjustable slit. *B* and *E* are 9 cm. diam. 50 cm. focal length spherical mirrors, silvered on the outside. *C* is a rock salt prism, 8 cm. along each face and 8 cm. high, with a refracting angle of $59^{\circ} 58'.75$, obtained from Brashear. The prism was protected from moisture by having a small dish of anhydrous calcium chloride placed upon it and several others on the spectrometer, which was enclosed in a blackened case. When not in use, the prism was covered with a bristol board box painted inside with paraffine which fitted closely to the spectrometer table. The small weight of this box made it preferable to a heavy glass bell-jar. *D* is the Wadsworth plane mirror, while *F* is a plane 10×10 cm. mirror used to change the direction of the convergent light from *E* so that the filar microscope *J* could be used for reading the 35 cm. diam. circle *K*. *C* is a Rubens thermopile, made up of iron constantin junctions with a fixed slit width of 0.03 cm. A Leeds and Northrup sensitive galvanometer was used on closed circuit in series with the thermopile. The source of radiation *L* was a 220-volt Nernst glower. A spherical mirror *M* focused the light on the slit.

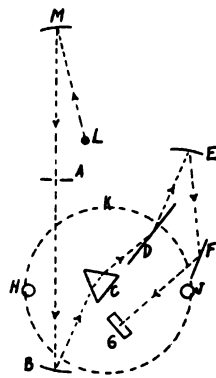


Fig. 1.

The cells used to hold the solutions were made of two plates of polished rock salt about $2 \times 4 \times .4$ cm., between which a clean copper wire bent in U shape was placed. After the plates had been finally polished on an optical grinder with rouge and absolute alcohol, the wire was placed between them and cemented to the plates with SiO_2 . The solvents used did not attack any of the materials in the cell. After filling the cell, a glass plate was cemented over the top to prevent evaporation.

A thin wooden block that ran in vertical ways between stops was provided with two openings somewhat smaller than the cells. The cell containing the solution to be investigated was placed over the upper opening. Over the lower opening a duplicate cell, having the same absorption when empty, was placed filled with the pure solvent. The

¹ Wadsworth, *Phil. Mag.* (5), 38, p. 346, 1894.

elevator was so placed that the cells could be brought in succession exactly to the same position in front of and close to the slit. Between the elevator and mirror *M* interposed a metal screen which could be operated by the observer.

The naphthalene used was recrystallized from alcohol. The beta naphthylamine was obtained from Merck and Co. The alpha and beta naphthol, the nitronaphthalene and the alpha naphthylamine were very kindly furnished by Professor M. Gomberg of the chemistry department. The solvents were obtained from stock in the laboratory. They were listed as c.p. but since the absorption of the solvents does not enter into the final result, absolute purity is not necessary.

PROCEDURE.

A calibration curve for the rock salt prism was made from values given by Paschen.¹ Since the observation room was always kept at 25° C. the values of the refractive index were corrected for temperature by his correction formula.

Before each set of observations, the instrument was calibrated with an optical setting on the .546 μ Hg line and then tested with the Bunsen flame emission maximum which has been accurately located² at 4.40 μ . From the calibration curve, this maximum was always found at a wavelength smaller than 4.4 μ by about 0.4 minute of arc. This discrepancy has been noticed by several investigators³ who attribute it variously to curvature of the image of the slit and to a real difference between a bolometric and an optical setting of the standard line on the slit. Correction was made by assuming that the Bunsen maximum was at 4.400 μ and displacing the calibration curve by the required amount. It may be noted that the .03 cm. width of slit for a 50 cm. focal length mirror amounts to 2.03 minutes of arc.

After the cells had been placed on the elevator and the zero galvanometer reading taken, the metal screen was raised so that light could pass through the cell which contained the solution. When the maximum deflection had been noted, the elevator was raised until the cell containing the solvent came in front of the slit. The deflection now increased and finally came to rest at a value larger than the first. With a linear thermopile such as was used the percentage of energy transmitted was computed by finding the ratio of the deflection through the solution to that through the solvent.

In general, readings were taken one minute apart on the spectrometer

¹ Paschen, *Ann. der Physik*, 26, p. 120.

² Paschen, *Wied. Ann.*, 52, p. 209, 1894; 53, p. 337, 1894; 60, p. 714, 1897.

³ Coblenz, *Carnegie Inst. Publ.*, No. 35, p. 20. Paschen, *Wied. Ann.*, 52, p. 209, 1894.

circle but in the neighborhood of an absorption band they were taken closer together so as to accurately locate the minimum of transmission. Table I. is a sample taken at random from observations through one absorption band. It shows the variation of the intensity of the light caused by the absorption of the solute and of the solvent and by the Nernst glower.

TABLE I.
Infra-red Absorption Data. α Nitronaphthalene in CS₂.

1. Angle from $\lambda = .546\mu$.	2. Wave-length.	3. Solution Deflection, Cm.	4. Solvent Deflection, Cm.	5. Per Cent. Transmitted.
37.00	7.070 μ	1.49	2.01	74.0
38.00	7.160	1.79	3.19	56.1
39.00	7.250	1.42	4.12	33.4
40.00	7.334	1.04	4.58	22.7
40.32	7.360	.93	4.83	19.2
40.62	7.384	.87	4.97	17.5
41.00	7.415	.79	4.93	16.0
41.33	7.440	.84	5.03	16.7
41.61	7.463	.89	5.04	17.7
42.00	7.493	1.00	5.00	20.0
43.00	7.571	1.76	4.95	35.6

The minimum of transmission was always determined from a plot made on a large scale of several values near the minimum. The point where the two lines (Fig. 2) meet was taken as the position of maximum

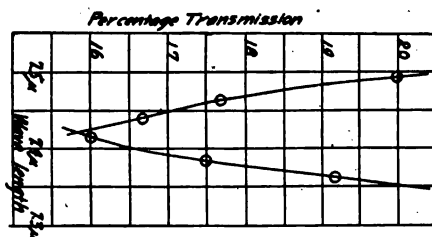


Fig. 2.
 α Nitronaphthalene Absorption Band in CS₂ at 7.42 μ .

absorption. No correction has been made for finite slit width and impurity of the spectrum. Several bands were plotted to an enlarged scale and the correction method proposed by Runge¹ applied. The corrected absorption curve was indeed deeper but there was no shift of the position of the maximum comparable to the experimental error.

In the tables giving the positions of maximum absorption are given

¹ Runge, Zeit. für Math. und Phys., 42, p. 205, 1897.

the estimated limits of error in location after account has been taken of errors that may arise in percentage transmission, plot, calibration curve, adjustment test, and change in temperature.

CHEMICAL STRUCTURE OF NAPHTHALENE AND OF ITS DERIVATIVES.

Naphthalene, $C_{10}H_8$ ¹ consists of two benzene nuclei which have in common two carbon atoms. The accepted formula is given in Fig. 3. The replacement of an H atom in naphthalene gives rise to two isometric mono-derivatives. They are called either α or β derivatives according as the substituent is adjacent to the complex of Fig. 4, common to both groups, or separated from it by a CH group.

α naphthol, $C_{10}H_7(OH) + H_2O$ is represented by the formula of Fig. 5

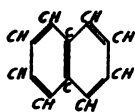


Fig. 3.



Fig. 4.

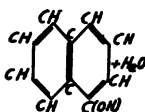


Fig. 5.

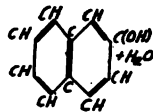


Fig. 6.

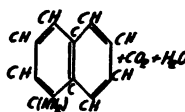


Fig. 7.

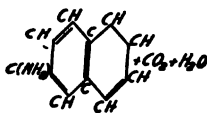


Fig. 8.

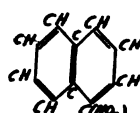


Fig. 9.

Fig. 3. Naphthalene.

Fig. 5. α Naphthol.

Fig. 6. β Naphthol.

Fig. 7. α Naphthylamine.

Fig. 8. β Naphthylamine.

Fig. 9. α Nitronaphthalene.

while β naphthol is shown in Fig. 6. The α and β naphthylamine $C_{10}H_7(NH_2) + CO_2 + H_2O$ correspond to Figs. 7 and 8 respectively and α nitronaphthalene $C_{10}H_7(NO_2)$ has the formula of Fig. 9.

DISCUSSION OF THE RESULTS.

In Fig. 10 are given the transmission curves for naphthalene in CS_2 and in CCl_4 respectively. Table II. gives the location of the absorption maxima. The CS_2 solution gives more bands but except for the band at 3.30μ the location of the common bands agrees within experimental error. Coblenz² has investigated the absorption of naphthalene in CCl_4 in this region and found a band at 3.25μ . The CS_2 band was however located at 3.30μ in several solutions, the observations having been taken more than two months apart. This shift from 3.25μ to 3.30μ must be due to the effect of the solvent itself.

¹ Richter-Smith, Organic Chemistry, vol. 2, p. 390, 1900.

² Coblenz, *loc. cit.*

Since naphthalene consists of two benzene nuclei, it will be of interest to note that Coblenz has found absorption maxima of benzene at 2.7μ , 3.25μ , 4.4μ , 4.9μ , 5.5μ , 6.2μ , 6.75μ , 7.25μ , 7.8μ , 8.67μ , and 9.78μ .

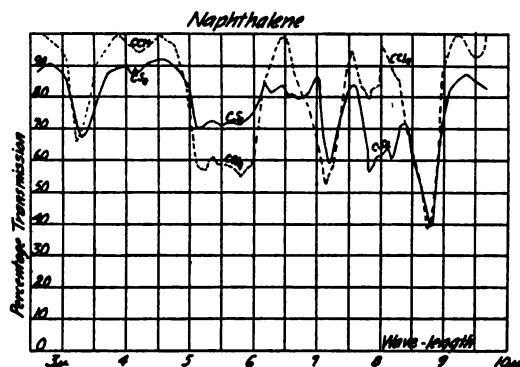


Fig. 10.

TABLE II.
Naphthalene Absorption Maxima.

No.	In CS_2 .	In CCl_4 .	Wave No.
1	$3.30\mu \pm .03\mu$	$3.25\mu \pm .03\mu$	308
2	4.2 .10	4.22 .04	237
3	5.15 .10		194
4		5.27 .03	190
5	5.5 .10		182
6	5.8 .10	5.80 .02	172
7	6.31 .02		159
8	6.75 .10		148
9	7.20 .03	7.18 .03	139
10	7.84 .02	7.84 .03	127
11	8.18 .02		122
12	8.79 .02	8.78 .02	114
13	9.81 .03		102

It is to be noticed that many naphthalene bands occur at the same wave-lengths. These must be due to the vibration of the benzene ring, still present in the naphthalene and are not characteristic of the naphthalene itself.

Baly¹ in his theory has arrived at the conclusion that the wave numbers of the absorption band in the visible or ultra-violet region of the spectrum should be equal to the product of the wave numbers of several infra-red bands by whole numbers. Such a test would serve as a criterion of the worth of his theory. For example, Russel and Lapraik² found three

¹ Baly, *loc. cit.*

² Chem. Soc. of Lon. Trans., 39, p. 171, 1881.

naphthalene absorption bands, the maximum absorption occurring from $.714 \mu$ to $.707 \mu$, from $.636 \mu$ to $.634 \mu$, and from $.619 \mu$ to $.611 \mu$ while the mean wave numbers for these bands are 1407, 1571, and 1626 respectively. The wave numbers of the infra-red bands giving closest agree-

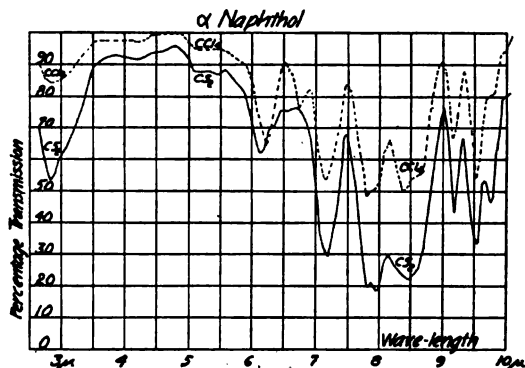


Fig. 11.

TABLE III.

Infra-red Absorption Maxima. Alpha Naphthol.

1. No.	2. In CS ₂ .	3.	4. In CCl ₄ .	5.	6. Wave No.	7.
1	2.85 $\pm$.03 μ		2.87 $\pm$.04 μ		286	
2	6.15	.02			162	D
3			6.24	.03	160	
4			6.76	.03	148	ND
5	7.19	.03	7.19	.03	139	ND
6	7.84	.03	7.82	.03	127	ND
7	7.97	.03			125	
8	8.44	.04	8.40	.03	119	D
9	9.17	.02	9.17	.02	109	
10	9.54	.03	9.54	.02	105	
11	9.73	.03			103	D

Purvis at 3068 (2, 5, 6, 9, 11); at 3324 (5, 6, 8).

ment are for 1407, $139 \times 10 = 1390$, $127 \times 11 = 1397$, $238 \times 6 = 1428$; for 1571 there are no close values; for 1626, $102 \times 16 = 1632$, and $148 \times 11 = 1628$.

The wave numbers of the other bands found by various investigators in the visible region are given below. The numbers in parenthesis after each wave number refer to the infra-red bands listed in Table II. which are very nearly integral factors of this visible wave number.

By Hartley¹ at 3504 (4, 7); at 3670 (4, 7, 11, 13); at 3801 (3, 6, 7, 11); and at 3922 (2, 11).

¹ Chem. Soc. of Lon. Trans., 47, p. 685, 1885.

By Baly and Tuck¹ at 3125 (4, 8); at 3222 (9); at 3700 (1, 4, 8, 10).

By Guthrie² at 3102 (4, 5, 6, 8); at 3167 (7, 11, 13); at 3207 (9, 12); at 3306 (2, 3, 4, 10, 11, 12); at 3488 (4, 7, 9); at 3574 (10, 13); at 3625 (5, 6, 9); and at 3745 (9, 12).

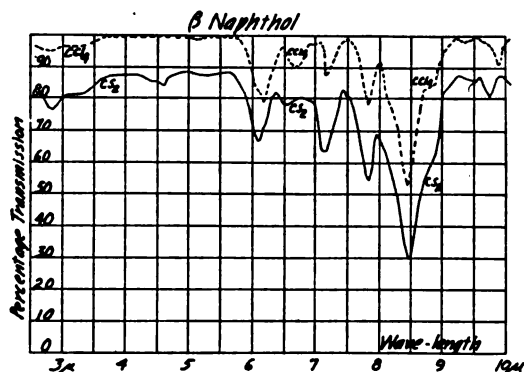


Fig. 12.

TABLE IV.

Infra-red Absorption Maxima. Beta Naphthol.

1. No.	2. In CS ₂ .	3.	4. In CCl ₄ .	5.	6. Wave No.	7.
1			2.77 μ $\pm$.10 μ		361	
2	2.87 μ $\pm$.04 μ				350	
3	6.12 .03		6.18 .03		163	D
4	6.5 .10				154	
5			6.75 .03		148	ND
6	7.18 .03		7.21 .02		139	ND
7	7.84 .02		7.85 .03		128	ND
8	8.47 .02		8.47 .02		118	D
9	9.72 .02				103	D
10			9.90 .06		101	

Purvis at 2996 (8) at 3712 (4, 5, 7, 9).

¹ Purvis, Chem. Soc. of Lon. Trans., 101, p. 1315, 1912.

From these results it is seen that nearly all of the absorption maxima in the visible and ultra-violet have wave numbers which are integral multiples of several infra-red absorption wave numbers.

The constant frequency difference between bands in the visible or in the infra-red which one might expect from Baly's theory have not been found. This detailed discussion of the results has been undertaken to see if the theory does account for the position of the visible absorption

¹ Chem. Soc. of Lon. Trans., 93, p. 1902, 1908.

² Baly, Astrophys. Jour., 42, p. 44.

bands. The proof may be sufficient. It would however be going too far to say that the results are as conclusive as might be desired. From his theory of fluorescence, Baly has predicted that bands would be found at 3.27μ , 3.86μ , 4.72μ , 6.06μ , and at 8.48μ . Only the 3.27μ band has been found, and this band had already been located by Coblenz.

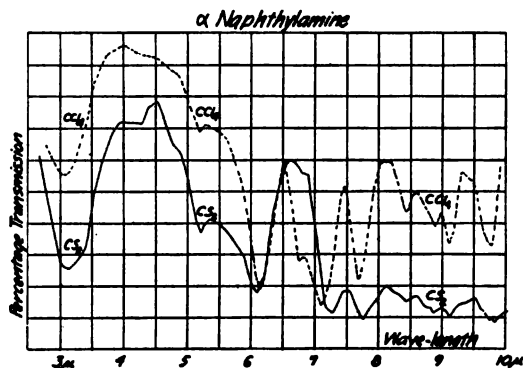


Fig. 13.

TABLE V.

Infra-red Absorption Maxima. Alpha Naphthylamine.

1.	2.	3.	4.	5.	6.	7.
No.	In CS ₂ .		In CCl ₄ .		Wave No.	
1	$3.09 \mu \pm .04 \mu$				323	
2			$3.18 \mu \pm .04 \mu$		314	
3	5.19	.03	5.20	.03	194	N
4	6.10	.03	6.18	.02	163	D
5			6.80	.04	147	
6			7.11	.03	141	
7	7.27	.03			138	ND
8	7.76	.03	7.71	.03	129	D
9	8.44	.03	8.47	.02	118	D
10	8.87	.05	8.92	.03	112	
11	9.11	.02	9.12	.03	110	
12	9.80	.04	9.75	.06	103	ND

Purvis at 3174 (9, 11, 12).

The tables given below for the various isomers list the absorption maxima wave-lengths as found with the two solutions and also show the corresponding wave numbers. In column 7, a band is marked N if it is a naphthalene band, and D if the band is found in both the alpha and the beta derivative. At the bottom of each table is given a list of the wave numbers of the known visible absorption bands and following

each wave number, as before in parenthesis, are the numbers of the infra-red bands which are integral factors of the visible wave numbers. The absorption curves for these compounds are given in Figs. 11 to 15.

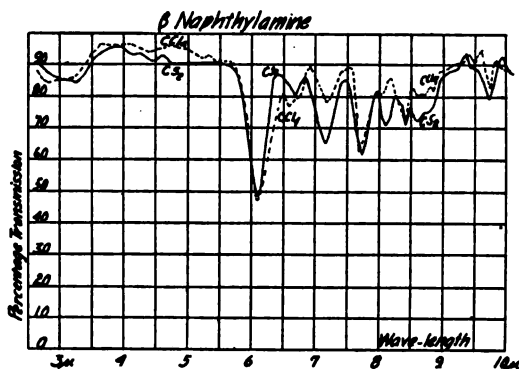


Fig. 14

TABLE VI.
Infra-red Absorption Maxima. Beta Naphthylamine.

1. No.	2. In CS ₂ .	3. In CCl ₄ .	4. Wave No.	5. Wave No.	6. Wave No.	7.
1					350	
2	3.25 μ $\pm$.02 μ	2.86 μ $\pm$.04 μ			308	N
3	4.44 .04	4.44 .04			225	
4	6.13 .03	6.13 .02			163	D
5	6.67 .03	6.64 .04			150	
6	7.18 .03	7.20 .02			139	ND
7	7.73 .02	7.74 .03			129	D
8	8.09 .02				124	
9	8.40 .03	8.41 .02			119	D
10	8.59 .03				116	
11	9.74 .03	9.76 .02			103	ND

Purvis at 2922 (3, 4, 6, 10) and 3470 (5, 6, 9).

CONCLUSIONS.

1. The absorption of naphthalene and of several of its derivatives has been investigated from 2.7 μ to 10 μ in solutions of carbon disulphide and carbon tetrachloride.

2. Bands which are present when one of the solvents is used are nearly always present or indicated by breaks in the curve when the other solvent is used.

3. Although the positions of maximum absorption are generally the

¹ Purvis, Chem. Soc. of Lon. Trans., 101, p. 1315, 1912.

same, in both solutions, the general appearance of the absorption curves may be very different.

4. Naphthalene has several absorption bands which are due to the benzene nucleus.

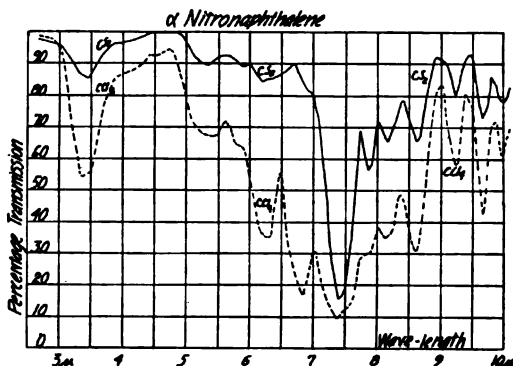


Fig. 15.

TABLE VII.

Infra-red Absorption Maxima. Alpha Nitronaphthalene.

1. No.	2. In CS_2 .	3. In CCl_4 .	4. Wave No.	5.	6.	7.
1	$3.44\mu \pm .02\mu$	$3.42\mu \pm .02\mu$	282			
2	5.29 .04	5.36 .04	188			
3	6.22 .04	6.24 .03	160			
4		6.83 .02	146			N
5	7.42 .02	7.42 .03	135			
6	7.87 .02		127			N
7	8.17 .03	8.20 .04	122			N
8	8.67 .03	8.62 .03	116			
9	9.25 .02	9.23 .03	108			
10	9.67 .02	9.66 .03	103			
11	9.95 .03	9.96 .03	101			

Purvis at 2930 (4, 6, 7, 9, 11).

5. The mono-derivatives of naphthalene have several of the naphthalene absorption bands.

6. There are however many absorption bands present which are characteristic of the derivative itself.

7. Those naphthalene bands occurring in the alpha derivative, are generally present in the beta derivative and there are always several other bands common to both derivatives.

8. The infra-red absorption curve of the alpha derivative is however as a whole different from that of the beta derivative in the same solvent.

9. Nearly all wave numbers of absorption bands for these substances in the visible possess, as integral factors, two or more infra-red absorption maxima wave numbers as Baly's theory predicts.

In conclusion, the writer desires to thank Professor H. M. Randall for proposing the subject and also for many valuable suggestions during the progress of the work.

PHYSICAL LABORATORY,
ANN ARBOR, MICH.,
May 2, 1916.

PROCEEDINGS
OF THE
AMERICAN PHYSICAL SOCIETY.

MINUTES OF THE STANFORD MEETING.

THE eighty-eighth meeting of the American Physical Society was held in Room 370, Physics Department, Stanford University, on Saturday, April 7, 1917, in connection with the meeting of the Pacific Division of the American Association for the Advancement of Science. Morning and afternoon sessions were held. Professor Sanford presided.

After brief addresses by Professor J. C. Merriam, chairman of the Pacific Coast Sub-Committee on Research, and Director Campbell, urging cooperation between the various affiliated societies, the Committee on Research, the State Council of Defence, and similar national organizations, it was unanimously voted to pledge such cooperation on behalf of the physicists of the Pacific Coast. Professor Sanford and the Local Secretary were authorized to organize this cooperation.

The following papers were presented:

Physical Significance of Entropy. W. P. ROOP.

Experimental Data Illustrating Fundamental Theorems of Kinetic Theory. W. P. ROOP.

A Note on the Relationship Between Ionizing Potentials and Atomic Charges. FERNANDO SANFORD.

An Optical Trough for High School and College Demonstration. R. S. MINOR.

The Relation Between E.M.F. of a two-Solution Cell with Calomel Electrodes and the Difference in Specific Inductive Capacity of the two Electrolytes. DAYTON L. ULREY.

The Effect of Occluded Gases on Photoelectric Emission. L. A. WELO.

The Relation Between Current Strength, Voltage, and Luminosity in the Mercury Arc. H. O. RUSSELL.

The Mercury Arc Pump; The Dependence of its Rate of Exhaustion on Current. L. T. JONES and H. O. RUSSELL.

Note on the Use of the Electrometer for Measuring the Conductivity of Gases. J. A. GILBREATH.

A Study of the Stark Effect. J. A. ANDERSON.

Leading Features of the Electric Furnace Spectra of Calcium, Strontium, Barium, and Magnesium. A. S. KING.

Elimination of Pole Effect from Secondary Standards of Wave-Length.
H. D. BABCOCK and C. E. ST. JOHN.

Oscillating Systems Damped by Resistance Proportional to the Square of the Velocity. J. P. VAN ZANDT.

The Surface Tension of Molten Metals. R. H. COON and E. E. HALL.

Description of a New Apparatus for Exciting X-Rays. FRANK RIEBER.

A Mechanical Analogy to Electrostatically Coupled Electric Circuits.
W. J. RAYMOND. (By title.)

Optics of the Microscope. C. W. WOODWORTH.

Note on Electrical Conductivity. F. J. ROGERS.

Rigidity of the Earth from Earthquake Waves. F. J. ROGERS.

A Method of Determining J Based on the Adiabatic Expansion of a Gas.
J. C. SHEDD.

A Hydrodynamical Paradox. GRANDISON GARDNER.

Members were guests of Stanford University for luncheon, and in the evening there was a subscription dinner at Stanford Inn. About thirty-five were in attendance at the sessions and twenty-five at the dinner. At the conclusion of the dinner there was an informal discussion of radiation and atomic theories. At this evening session the following resolution was adopted:

Resolved, that in view of the limited supply of platinum and the great importance of this metal in the manufacture of sulphuric acid, the conversion of ammonia into nitric acid, and many other industrial and scientific purposes closely connected with the national defense, we, the Pacific Coast members of the American Physical Society, urge the State Council of Defense to take immediate steps (1) To conserve our platinum supply; (2) to prevent its waste through the manufacture and sale of platinum jewelry and through other similar unproductive uses; (3) to secure an immediate inventory of our platinum resources; (4) to bring this matter, through suitable channels, to the attention of the National Council of Defence or other branch of the national government, to the end of securing nation-wide conservation of platinum.

The Society also passed a vote of thanks to the authorities of Stanford University for the courtesies and facilities afforded by them.

E. P. LEWIS,
Local Secretary for the Pacific Coast.

ON THE ORIGIN OF THE EARTH'S ELECTRIC CHARGE.¹

BY W. F. G. SWANN.

MEASUREMENTS of the variation of the penetrating radiation, with altitude, point to the upper atmosphere as the origin of a part of this radiation. The whole of the penetrating radiation is probably of the γ -ray type, but the part which reaches the earth's surface from the outer atmosphere

¹ Abstract of a paper presented at the New York meeting of the Physical Society, February 16-17, 1917.

is naturally the most penetrating part. Indeed, it is so penetrating that it passes through a thickness of air which would be equivalent, in absorptive action, to a column of mercury 76 cm. high, if absorption coefficients were simply proportional to density and were independent of material. The γ -ray radiation from the outer layers of the atmosphere will consequently be very "hard," and, in accordance with the known results of laboratory experiments, we must conclude that the negative corpuscles which it emits from the air molecules are emitted almost entirely in the direction of the radiation, and further, that they can have a range in air at least equal to that of the swiftest β -rays from radium products, a range, for example, of 8 meters. The emission of corpuscles by these γ -rays will consequently result, at each point of the atmosphere, in a downward current of negative electricity, which we shall call the corpuscular current. This corpuscular current will charge the earth until the return conduction-current balances the corpuscular current at each point of the atmosphere.

Taking, for the purposes of this abstract, a simplified case where the penetrating radiation considered is all directed vertically downwards, if q is the number of corpuscles liberated per c.c. per second by the penetrating radiation, and h the average distance which a corpuscle travels from its point of origin, the corpuscular current density will be

$$i = qeh,$$

where e is the electronic charge.

If q be taken as 2, which is probably about equal to the number of pairs of ions produced per c.c. per second in a closed vessel as a result of the part of the penetrating radiation in question, and if h be taken as 3 meters, we have

$$i = 2 \times 4.8 \times 10^{-10} \times 800 = \text{about } 8 \times 10^{-7} \text{ E.S.U./cm}^2,$$

which is just of the order of magnitude of the air-earth-current-density, so that on this view, the penetrating radiation from the outer layers of the atmosphere provides a sufficient basis for the explanation of the maintenance of the earth's charge.

The corpuscular current-density, and consequently the conduction current-density, will not necessarily be independent of the altitude, for the factors upon which i depends, viz., the intensity and quality of the penetrating radiation, the number of molecules per c.c. available for possible ionization by the radiation, and the range of the corpuscles set free all alter with the altitude.

A few minor difficulties present themselves if the above view be adopted. Thus, for example, near the surface of the earth, a considerable portion of the whole penetrating radiation comes from the soil, and is directed upwards, but this difficulty disappears when it is remembered that the average "hardness" of the radiation from the soil is very much less than that of the radiation which reaches the earth from the outer layers of the atmosphere. Again, it might appear that the corpuscles set free by the penetrating radiation should, on account of their great energy, produce in the atmosphere many more ions per

second than are actually found to be produced. This difficulty, and others of allied nature become greatly reduced in magnitude, however, when considered in the light of our present knowledge of the action of very swift β -rays when passing through a gas.

The existence of any contribution to the penetrating radiation other than that arising from the radioactive materials in the air and soil has been questioned by some authorities, and it has been supposed that the walls of the ionization chamber are ultimately responsible for such residual ionization as remains when the radiations from the earth and the air are shielded off. To the extent that the "wall effect" arises from an α -particle radiation it could be detected and allowed for, and a probable estimate of the β - and γ -rays radiation accompanying it could be made, if the electroscope were so sensitive that the contributions to the apparent natural ionization were observable as distinct kicks in the recording fiber. The author has succeeded in obtaining the necessary degree of sensitivity for this purpose, and for illustration of the process has obtained photographs showing the rate of emission of the α -particles from a small amount of radioactive material contained in an ionization chamber.

DEPARTMENT OF TERRESTRIAL MAGNETISM,
WASHINGTON, D. C.,
February 3, 1917.

THE EFFECT OF X-RAYS ON THE LENGTH OF LIFE OF *TRIBOLIUM CONFUSUM*.¹

BY WHEELER P. DAVEY.

THE proper use of a physical agent in biological work necessitates the physical measurement of the agent employed. A great deal of work is reported in the literature on the effect of X-rays on various forms of animal life. A study of this literature shows that with very few exceptions the measurements of the dosage have been so incomplete as to render the work of little scientific value. It therefore seemed that the effect of X-rays upon living organisms could be quantitatively studied to some good. The organism studied was the "flour weevil," *Tribolium confusum*. These little beetles are especially adapted to such work as is reported here because they are small, harmless, easy to handle and count in large numbers; they propagate readily, cannot crawl out of glass beakers or small porcelain crucibles, and show little tendency to fly.

It was shown that the effects noticed were really due to X-rays and not to some accidental circumstance, such as static field, ionized air, effect of X-rays on food, etc.

A method has been developed which eliminates the error due to idiosyncrasy, thus making it possible to obtain bio-physical laws of a considerable degree of precision. This method consists in plotting probability curves from the data.

¹ Abstract of a paper presented at the New York meeting of the Physical Society, February 17, 1917.

If the number of individuals used in the experiments is sufficiently large and if they have been gathered from the same brooder at the same time, then corresponding points on the probability curves may be quantitatively compared. The slope of the probability curves is a measure of the idiosyncrasy of the organism to the physical agent employed.

It was shown that X-rays are able to cause premature death if the dose is larger than a certain threshold dose, but that there is a latent interval between the time the X-rays are administered and the time death occurs. This latent interval depends accurately, for any given individual, upon the logarithm of the X-ray dose, according to the formula

$$Y = A - B \log X,$$

where Y is the length of the latent interval and X is the X-ray dose measured in terms of the electrical input of the X-ray tube, material used as anti-cathode, etc. Voltage, current and distance were kept constant, dosage being varied by varying the time. Too much emphasis can not be given to the necessity of keeping the voltage constant, as the X-ray output of the tube varies greatly with the voltage.

With a standard tungsten target Coolidge tube, operating on sine-wave voltage without X-ray filters, the threshold dose for *Tribolium confusum* is 500 milliampere-minutes at 25 cm. distance at 50 kilovolts, *i. e.*, 500 MAM/25³ at 50 KV. RMS.

The formula for the length of the latent interval may be easily derived from an extension of the phycho-physic law if the *resistance* rather than the susceptibility of the organism to X-rays is considered.

It is hoped later to present results on the effect of wave-length of X-rays, and on the sterilizing effect of doses below the threshold lethal dose.

RESEARCH LABORATORY,
GENERAL ELECTRIC CO.,
SCHENECTADY, N. Y.

THE PASSAGE OF PHOTO-ELECTRONS THROUGH METALS.¹

BY K. T. COMPTON AND L. W. ROSS.

EXPERIMENTS by Ladenburg and by Rubens and Ladenburg have shown that the electrons liberated from metals by ultra-violet light escape from a surface layer of the metal, of small but measurable depth, and that the electrons are therefore able to pass a certain distance through the metal and still retain sufficient kinetic energy to enable them to escape from its surface.

We have designed a sputtering chamber in which twelve quartz plates may be successively exposed to the cathode deposit and with which we have been able to secure metal films whose thickness is proportional to the time of sputtering.

¹ Abstract of a paper presented at the New York meeting of the Physical Society, February 16-17, 1917.

These films were then mounted on a rotating disk in a vacuum chamber so that they could be interposed successively in the path of a beam of monochromatic ultra-violet light and the photo-electric current then measured from their emergent side.

After passing through the film, the light fell on a second electrode, producing a current from the latter which was proportional to the amount of light transmitted by the interposed film.

The photo-electric current from the film is given by

$$N = KI_0 e^{-\alpha d} \int_0^d e^{\alpha x} F(x) dx,$$

where I_0 is the intensity of the light entering the film, α is the coefficient of light absorption in the film, d is the thickness of the film and $F(x)$ is the probability that an electron liberated at a depth x in the film may escape from the surface. The form of $F(x)$ has been determined for several simple assumptions regarding the nature of electronic absorption. The simplest assumption is that

$$F(x) = e^{-\beta x},$$

where $1/\beta$ is the average distance which an electron can travel through the metal, normal to its surface, without losing its ability to escape.

Our measurements with platinum films indicate that β is very near the value $0.60(10)^7 \text{ cm}^{-1}$, which corresponds to a mean free path of $1.67(10)^{-7} \text{ cm}$. This is of the order of magnitude of the distance between the atoms in the film. We have detected no difference in the values of β as determined for various wave-lengths, which probably indicates that the escaping electrons lose energy as the result of discrete catastrophies (collisions) rather than by a gradual process.

This value of β is larger than that suggested by the work of Rubens and Ladenburg, but it agrees with the order of magnitude predicted from measurements of the Hall effect in platinum.

If our value of β is correct, it probably disposes of the possibility of explaining the thermionic effect as an integrated photo-electric effect.

PALMER PHYSICAL LABORATORY,
PRINCETON, N. J.

THE ELECTROMAGNETIC MASS OF THE PARSON MAGNETON.¹

BY DAVID L. WEBSTER.

IN the Parson magneton theory² of the atom, and other non-spherical models, the lack of symmetry of the electromagnetic field makes it evident that the electromagnetic mass of a single magneton or whole atom is not the same for

¹ Abstract of a paper presented at the New York meeting of the Physical Society, February 16-17, 1917.

² A. L. Parson, "A Magnetron Theory of the Structure of the Atom," *Smithsonian Miscellaneous Collections*, November, 1915.

accelerations in all directions, even when the magneton or atom is at rest. If this were the whole mass of the system, then as Ehrenfest¹ and others have proved, the system when in motion would orient itself in the position of maximum kinetic energy, and thereby give an indication of its absolute motion to any observer moving with it. The principle of relativity therefore demands the existence of another kind of mass that will vary in such a way as to keep the total mass constant. The origin of this mass must be investigated before any definite statements can be made about the relation between the total observed mass and other properties of the magneton.

To take a simpler problem first, if two equally charged spheres are held at a fixed distance a by a string, the mutual electromagnetic mass of the pair is $2e^2/c^2a$ for accelerations along the line of centers, but only e^2/c^2a , or $1/c^2$ times the mutual electrostatic energy, for accelerations across this line. In very rapid motion the tendency of the electromagnetic forces to turn their line of centers toward the direction of motion will be balanced by the effects of the tension in the string, whose direction, according to relativity, is no longer that of the string itself. Therefore it is reasonable to expect this tension to supply the forces opposing acceleration from rest that give the variable "internal mass." It will do this provided that its changes are propagated with the velocity of light according to the following law: The force of the string on each sphere always acts as though the direction of the other were that of the point where it would be if it had not been accelerated since it was last heard from through signals with the velocity c , but as though its distance were still equal to a . Any electrostatic system with this type of internal forces will have a total mass equal to twice $1/c^2$ times its electrostatic energy.

The same idea may be extended to a steady current system, with strings having the peculiar property of adjusting their forces to balance all changes of direction of the line between the elements of current they connect, but with no change of force for a change of distance. The same postulate on propagation of internal forces will then give a variable mass, but in this case the internal mass will be negative and will always exactly balance the mass due to magnetic action. This is not, of course, the only possible system of internal forces, but is suggested tentatively in the absence of any more plausible one. If it is incorrect, it would be possible to make assumptions leading to either positive or negative values of the resultant mass. The zero value therefore does not represent any sort of lower limit, and is as plausible as any other.

In any system of static charges and steady currents, the fields due to the charges and to the currents may be proved to have no mutual electromagnetic momentum. For the case of the magneton, then, we may combine the above suggestions and conclude that its mass when at rest is most probably equal to $2/c^2$ times its electrostatic energy alone.

JEFFERSON PHYSICAL LABORATORY,
CAMBRIDGE, MASS.

¹ Ann. der Phys., 1907.

GYROSCOPIC EFFECTS IN THE PARSON MAGNETON.¹

BY DAVID L. WEBSTER.

FROM the experiments of Barnett² on magnetization of iron by rotation, and those of Einstein and de Haas³ on the converse of this effect, it appears that the ultimate magnetic particle shows gyroscopic effects of the order of magnitude of those of an electron moving in an orbit of sufficient size and speed to have the required magnetic moment. In view of the difficulties of accounting for stability in the orbits required for observed magnetic phenomena except by the Parson magneton hypothesis, it is necessary to investigate the possible gyroscopic properties of the magneton.

That such properties exist in it appears from a brief consideration of the retarded potentials near a magneton in precessional motion, but the easiest way to obtain an estimate of their magnitude is from a calculation of the moment of electromagnetic momentum⁴ around the axis of the ring. Since at all points very near the circumference of the ring the Poynting vector varies approximately as the inverse square of the distance, r , from it, practically all the energy will be concentrated in this region. At such points the electric vector is $2\rho A/r$ and the magnetic vector is $2\rho Av/rc$, or v/c times the electric. Hence the magnitude of the Poynting vector is $v/4\pi$ times the square of the electric vector, and the electro-magnetic momentum per unit volume is twice v times $1/c^2$ times the electrostatic energy per unit volume. That is, the total moment of this momentum is that of a particle of velocity v and mass equal to twice $\frac{1}{c^2}$ times the electrostatic energy of the system. This mass is exactly that of the magneton as found in the previous paper. In an earlier paper on heat radiation⁵ the magneton has been treated as a ring structure on which the electric charge is free to flow. Continuing this line of argument we should not expect any gyroscopic effects from the internal forces of the ring, since without rotation on its principal axis no unsymmetrical accelerations are introduced by rotation on an axis lying in its plane. The resultant gyroscopic effect is therefore exactly the same as for an electron of the classical type in an orbit equal in size to the ring and in speed to that of the electricity on the ring, as required by the experiments of Barnett and Einstein and de Haas.

JEFFERSON PHYSICAL LABORATORY,
CAMBRIDGE, MASS.

¹ Abstract of a paper presented at the New York meeting of the Physical Society, February 16-17, 1917.

² Science 30, p. 413, 1909, Amer. Phys. Soc., November, 1913, et seq.; PHYS. REV. [2], 6, 239-70, October, 1915.

³ Deutsch. Phys. Gesell. Verk., 17.8, pp. 152-170, April 30, 1915.

⁴ The theorem that the electromagnetic momentum concept can be used for calculation of moments as well as resultant forces can be proved by a method similar to the ordinary resultant proof.

⁵ Proc. Amer. Acad., Jan., 1915.

ON MOSELEY'S LAW FOR X-RAY SPECTRA.¹

BY H. S. UHLER.

THE experimental data were taken from tables compiled by Siegbahn from his own investigations and those of his co-workers. The constants, a and b , in Moseley's law, $\sqrt{\nu_N} = a + bN$, were first calculated by the method of least squares and then the percentage deviations of the computed wave-lengths from the experimental ones were evaluated. This was done for the α and β_1 sub-series of the K series and for the α_1 , β_1 , and γ_1 lines of the L series. In all cases it was shown that Moseley's law does not hold throughout the entire ranges of wave-lengths now known. The deviations at the extreme wave-lengths were always appreciable and amounted to as much as 13.4 per cent. for the $L\text{-}\beta_1$ line of arsenic.

The equation $\sqrt{\nu_N} = A + B \cdot N + C/(D - N)$ was shown to reproduce the given data well within the limits of experimental error. The parameters A , B , C , and D are constants for any one sub-series but change from one sub-series to another. By extrapolating with power series in N it was shown that the characteristic radiations from hydrogen probably have wave-lengths less than 300 angstroms. Attention was called to the fact that, so far as X-ray wave-lengths alone are concerned, it is not necessary to assign the series of natural numbers to the chemical elements from hydrogen to uranium, but that there exist a high order of infinity of other sequences of numbers which may be taken to represent the atomic numbers.

YALE UNIVERSITY,
January, 1917.

THE ELECTRICAL DISCHARGE FROM POINTS.¹

BY JOHN ZELENY.

THE results of some experiments indicate that the electric intensity at the surface of a point discharging a current is independent of the material of the point. Measurements were made in air at different pressures with brass, water, glycerine and methyl alcohol.

The dependence of the electric intensity at the surface of a point when a current can just be maintained, upon the radius of curvature of the point and upon the pressure of the surrounding gas has been investigated for water points in air.

When ions of both signs are supplied by some outside source to the gas near a point, they have a much larger effect upon the current when the point is charged positively than when it is charged negatively.

When points show a retardation in the commencement of a current, this current begins to flow at any voltage above a certain minimum value, only a

¹ Abstract of a paper presented at the New York meeting of the Physical Society, February 16-17, 1917.

certain interval of time after this voltage has been applied, which interval of time decreases rapidly as the value of this applied voltage increases above the minimum value. On lowering the voltage the current stops suddenly at nearly this last value of the voltage. The minimum current that can be maintained with such sensitive points diminishes regularly with the pressure of the gas.

When single alpha rays enter the region near a charged sensitive point, the range of voltage over which they can produce the large temporary increase in ionization, which is used in counting these particles, includes the minimum potential for which a current can be maintained from the point.

YALE UNIVERSITY.

ON THE PRODUCTION OF SOFT X-RAYS BY SLOW MOVING ELECTRONS.¹

BY H. M. DADOURIAN.

ELECTRONS from a hot line-cathode were made to impinge against an aluminium target after falling through accelerating electric fields varying from 20 volts to 1,000 volts. The properties of the resulting X-rays were studied by the "photo-electric" effect of the X-rays.

Evidence was obtained of the generation of soft X-rays by electrons impinging against an aluminium target with velocities corresponding to 20 volts.

If a stream of electrons impinge against a solid with velocities corresponding to a voltage V only a very small proportion of the resulting X-rays have the maximum frequency given by the relation

$$Ve = h\nu$$

When soft X-rays fall upon metal plates from 5 to 20 per cent. of the rays are scattered or reflected.

Thin films of celoidine have been obtained which transmitted from 20 to 40 per cent. of the X-rays.

The "photo-electric" current was found to be proportional to the cathode ray current.

SLOANE LABORATORY,
YALE UNIVERSITY.

EXPERIMENTS CONCERNING "MAGNET-PHOTOGRAPHY."¹

BY L. A. BAUER AND W. F. G. SWANN.

IN the number of the Scientific American for November 4, 1916, experiments by F. F. Mace are recorded showing that certain articles possess the power of affecting a photographic plate when they are allowed to remain in contact therewith under the influence of a magnetic field. A confirmation of some of Mace's results was recorded by the authors at the meeting of the American

¹ Abstract of a paper presented at the New York meeting of the Physical Society, February 16-17, 1917.

Physical Society in December, 1916, and the present paper deals with the continuation of the investigation.

A large number of experiments have been performed at the laboratory of the Department of Terrestrial Magnetism, some with permanent and some with electro-magnets. The details of these experiments are too numerous to be recorded in abstract, but it is concluded that while a magnetic field is not essential to the production of photographs of the kind cited, it probably has an influence in intensifying the effect.

As W. J. Russel and others have shown, wood possesses the power of affecting a plate when allowed to remain in contact with it for some time. The action of the wood is primarily due to the resin in it, and is considerably enhanced if the latter, before use, is stimulated by light.

The action of the resin extends beyond the place of its immediate contact, and in the present experiments, when some stimulated resin was placed on a plate in company with a metallic article, it was found that the image of the article appeared without the employment of a magnetic field, and simply because of the shielding action of the metal on the plate. On evacuating the vessel which contained the article, the effect of the resin extended more uniformly over the plate and was less concentrated in the immediate vicinity of the resin than before; as a result, the image of the metallic article was much more uniform in the case of a vacuum than in that of air.

It has been shown by Russel that metals, especially zinc, when freshly sand-papered, possess the power of influencing a photographic plate when in contact therewith; but in the experiments here described, this action is overpowered by the shielding action of the metal as regards the influence of the resin.

When, in the experiment with resin and the metallic article described above, the resin was replaced by pieces of wood, no apparent image of a set of *metallic articles* appeared after a 7-day exposure in a vacuum, but the absence of effect in this case is probably only one of degree, since the wood contains resin. It is interesting to observe, however, that when the same experiment was carried out under the influence of a magnetic field, distinct impressions were obtained in a vacuum after an exposure of 6 days.

The experiments are being continued.

DEPARTMENT OF TERRESTRIAL MAGNETISM,
WASHINGTON, D. C.,
February 3, 1917.

THE CRYSTAL STRUCTURE OF ALUMINIUM AND SILICON.¹

BY ALBERT W. HULL.

THE author's method of applying X-ray crystal analysis to microscopic crystals has been briefly described in this journal in connection with the crystal structure of iron.² It consists in sending a narrow beam of mono-

¹ Abstract of a paper presented at the New York meeting of the Physical Society, February 16-17, 1917.

² PHYS. REV., 9, 84, January, 1917.

chromatic X-rays through finely powdered crystals, and photographing the diffraction images of the slit, produced by reflection in all the possible planes belonging to the crystal structure. To make sure that all orientations are equally represented in the aggregate of tiny crystals, the powder is made as fine as possible and kept in continuous motion during the exposure. The positions and intensities of the lines on the photograph are compared with the theoretical values for different possible arrangements of atoms, until an arrangement is found which represents completely the observed lines.

In the preliminary calculations it is assumed that the scattering electrons are concentrated at the centers of the atoms. This gives wrong intensities for the lines, but causes no error in their positions. The relative intensities of the line should be completely accounted for by the position and condition of the electrons in the atoms, and since the number of lines that can be photographed is very large, this offers a powerful means of testing theories of atomic structure. It is hoped to report the results of such an investigation in the near future.

Aluminium.

Fine filings from chemically pure sheet aluminium, in a thin-walled glass tube of 2 mm. diameter, were exposed for 3 hours to a narrow beam of rays from a Coolidge tube with molybdenum target, running at 40,000 volts, 9 milliamperes. The diffracted rays were photographed on Eastman X-ray film with calcium tungstate intensifying screen, bent in a circle of 11.1 cm. radius. The rays were filtered, before striking the crystals, by a paste of powdered zircon crystals 0.3 mm. thick, which completely absorbs the β lines of the K radiation and leaves, as a practically monochromatic radiation, the α doublet, $\lambda = .710 \text{ \AA.}$ and $\lambda = .714 \text{ \AA.}$ The reflection of this doublet in the different crystal planes gives the lines which appeared on the photograph.

They are tabulated in the accompanying table, together with the corresponding spacing (calculated from the angle of the line and the wave-length) and the theoretical spacing for the assumed lattice. The last column gives the indices of the plane which produced each line.

Aluminium has usually been considered to belong to the isometric system. The X-ray data shows that it must be tetragonal. The unit of structure is a *centered rectangular prism*, with square base $2.85 \text{ \AA.} \times 2.85 \text{ \AA.}$ and height $4.05 \text{ \AA.}$, with an atom at each corner and one in the center of the prism. The structure is thus similar to that of iron, except that the unit is a rectangular parallelopiped instead of a cube. The number of atoms per unit parallelopiped is

$$n = \frac{[\text{density}] \times [2d_{100}]^2 \times [2d_{001}]}{\text{mass of atom}} = \frac{2.70 \times 2.85^2 \times 4.05}{2.69 \times 1.663} = 1.99.$$

This is equal, within the limit of error of the measurements, to 2, which is correct for the assumed structure.

Silicon.

Fine crystalline silicon, pulverized in a steel mortar, was mounted in the same manner as aluminium, and exposed for 4 hours to filtered Mo rays at 32,000

volts, 8 milliamperes. The results are given in the table. The lattice is identical with that of diamond, *i. e.*, two-face centered lattices intermeshed, the one being displaced with reference to the other along a cube diagonal a distance of $\frac{1}{4}$ the length of the diagonal. The side of the cube is 5.43 Å., as compared with 3.56 Å. for diamond, and the distance between nearest adjacent atoms is 2.35 Å. as compared with 1.54 Å. for diamond.

The number of atoms per unit cube of the lattice is

$$n = \frac{\rho [4d_{100}]^3}{M} = \frac{2.34 \times 5.43^3}{28.1 \times 1.663} = 8.00,$$

which is correct for the assumed lattice.

Aluminium.						Silicon.					
Distance of Line from Center, Cm.	Spacing of Plane, Ångströms.		Intensity of Line.		Indices of Plane.	Distance of Line from Center, Cm.	Spacing of Plane, Ångströms.		Intensity of Line.		Indices of Plane.
	Experimental.	Theoretical.	Experimental (Estimated).	Theoretical.			Experimental.	Theoretical.	Experimental (Estimated).	Theoretical.	
3.45	2.33	2.33	100	100	011	2.58	3.13	3.14	100	100	111
3.99	2.02	2.02	60	64	{ 001	4.21	1.93	1.93	80	344	110
					{ 110	4.97	1.64	1.64	75	148	311
5.67	1.426	1.426	50	86	{ 100			1.57	0	0	111 (2)
					{ 112	6.00	1.36	1.356	25	118	100
6.68	1.21	1.21	60	142	{ 211	6.54	1.25	1.25	45	108	331
6.95	1.17	1.17	20	45	{ 013	7.39	1.11	1.11	50	370	211
8.09	1.01	1.01	5	28	{ 011 (2)	7.84	1.05	1.04	40	116	{ 511
					{ 001 (2)						{ 111 (3)
8.86	.93	.93	25	101	{ 110 (2)	8.59	.96	.96	20	152	110 (2)
					{ 301	8.98	.92	.92	30	144	531
					{ 213	9.66	.86	.86	25	262	310
9.11	.90	.90	25	96	{ 111	10.01	.83	.83	10	63	533
					{ 114			.82	0	0	311 (2)
					{ 130	10.62	.79	.79	5	77	111 (4)
10.05	.82	.82	10	85	{ 132						{ 711
					{ 102	11.00	.76	.76	10	110	{ 551
10.66	.78	.78	15	104	{ 011	11.58	.73	.73	20	414	321
					{ 105	11.90	.71	.71	15	148	{ 731
					{ 321						{ 553
11.75	.71	.71	2	34	{ 100 (2)			.66	0	47	100 (2)
					{ 112 (2)			.64	0	45	733
					{ 411	13.33	.64	.64	5	172	411
12.30	.68	.68	4	128	{ 233	13.60	.63	.63	5	98	{ 751
					{ 215						{ 111 (5)

RESEARCH LABORATORY,
GENERAL ELECTRIC COMPANY,
SCHENECTADY, N. Y.

"It will be noted that the structure here given for Aluminium is identical with a face-centered cubic lattice of side 4.05 Å., a fact which had escaped notice, owing to a numerical error in preliminary calculations."

IONIZATION POTENTIALS IN HYDROGEN, NITROGEN AND NITROUS OXIDE.¹

BY FRED M. BISHOP.

THE ionization potential of certain gases has been studied under conditions which tend to minimize the photo-electric effect on the receiving electrode due to radiations in the tube.

Two different forms of apparatus were used, one a modification of the one used by Franck and Hertz in which plate electrodes were substituted for cylindrical ones. The other piece of apparatus contained an aluminium plate and quartz window and photo-electrons were used.

Ionization in hydrogen began at 11 volts and a second and more intense type set in at about 15.8 volts. No ionization could be detected in nitrogen until an accelerating potential of 7.5 volts was applied. The method was extended to nitrous oxide to see if the combination of one atom with another in a compound had any effect on its ionizing potential. Curves for nitrous oxide did not differ appreciably from those in nitrogen.

YALE UNIVERSITY.

NEW METHODS FOR COUNTING THE ALPHA AND THE BETA PARTICLES.¹

BY ALOIS F. KOVARIK.

THE principle of the method is the discharge to a point properly sensitized and is due to Geiger. The method of observation used by Geiger and by myself with L. W. McKeethan in former researches has been the optical method: observing the sudden movement of the fiber in a string electrometer when an alpha or a beta particle caused a flow of electricity to the point. The C. T. R. Wilson's inclined leaf electroscope has also been adapted for the purpose.

I have found recently a very convenient instrument for the same purpose to be Zeleny's electroscope.² In the latest form, the leaf is hinged and the instrument is very sensitive. Its sensitiveness is increased by using a potential difference between the plate and the leaf and case. The leaf is connected directly to the discharging point. The apparatus so arranged is very convenient also for lecture demonstrations.

Another method of observation is based on the utilization of the current to the point in connection with a sensitive telephone. If the discharging point is connected to earth through a sensitive telephone, the discharges are made just audible. The effect was greatly magnified by adapting an audion amplifier. The discharging point is connected to earth through the primary coil of a small induction coil (telephone amplifier). One end of the secondary coil is connected to the grid of the audion bulb while the other end is connected to one plate of a

¹ Abstract of a paper presented at the New York meeting of the Physical Society, February 16-17, 1917.

² PHYS. REV., 32, p. 255, 1910.

condenser, the other plate being earthed. The plate of the audion is connected in series with a sensitive telephone and a battery of about 50 volts, the negative end being earthed. The filament is lighted from a 6-volt battery whose positive pole is earthed. When the number of particles counted is fairly large, this method seems even more accurate than the observation of the fiber of the string electrometer.

Experiments are under way to modify this method so that the particles may automatically register themselves on a chronograph.

It is not difficult to make a point which will respond to alpha particles and give their number accurately but for the beta particles, greater care must be taken to have the point carefully made and to have it quite sensitive. The experiments under way show that their number is also correctly given when proper precautions (see former publications) are taken. Gamma rays also may be studied by these methods and just what these observations signify is being investigated.

YALE UNIVERSITY.

ON THE MEASUREMENT OF " h " BY MEANS OF X-RAYS.¹

BY F. C. BLAKE AND WILLIAM DUANE.

IN April, 1915, Duane and Hunt presented a paper to this Society describing experiments with X-rays in which it was shown that the equation

$$Ve = h\nu$$

(where V represents a constant voltage applied to the tube, e , the elementary charge and h , Planck's action constant) holds true when, and only when ν represents the *maximum* frequency of the X-rays produced.²

In fact we may define " h " to be the kinetic energy of an electron divided by the maximum frequency of the radiation it can produce, when it hits an atom.

The law furnishes us with one of the most accurate methods we have of determining h , e being supposed known.

The authors of this paper have begun a series of experiments designed to determine the value of h under widely different conditions, and also the variation in the amount of X-radiation when different substances are used as targets.

Many improvements have been made in the methods of measurement. A new spectrometer has been designed, with Mr. Franklin L. Hunt's assistance, which enables us to read the positions of both the crystal and the ionization chamber to within 10" of arc, corrected as usual for eccentricity by means of two verniers at the ends of a diameter. As we make measurements with the

¹ Abstract of a paper presented at the New York meeting of the Physical Society, February 16-17, 1917.

² Dr. D. L. Webster and Dr. A. W. Hull have since verified the law under different conditions.

ionization chamber first on one side and then on the other of the zero line, thus avoiding the necessity of determining the zero point at all, and as the smallest angle measured amounts to about 6° , the error from reading angles does not exceed 1/10 per cent. for each individual determination.

The electrical current through the X-ray tube comes from a high tension storage battery of 20,000 cells. An electrostatic voltmeter (kindly loaned to us by Dr. Webster and Dr. Clark) measured the voltage applied to the tube. This we calibrated by means of a measured current flowing through a manganin resistance of 898,000 ohms. To avoid errors due to changes in the constants of the voltmeter we calibrated it over the range used each time we made a determination, thus referring each measurement of a voltage to that of a current and a resistance. We estimate that each voltage measurement is correct to within 1/10 per cent.

The errors in measuring the voltage and the "glancing angle" are small as compared with the error that depends upon the interpretation one gives to the shape of the curve near the point at which the radiation vanishes. (Slides were shown illustrating these curves.) The shape of the curve is largely due to the widths of the slits and source of the rays (focal spot on X-ray target).

Using Millikan's value of $e = 4.774 \times 10^{-10}$ our values of h vary from 6.507×10^{-27} to 6.559×10^{-27} according to the estimates we make of the effects of the widths of the slits and source.

HARVARD UNIVERSITY,
February, 1917.

DETERMINATIONS OF LUMINOUS EFFICIENCY AND THE MECHANICAL EQUIVALENT OF LIGHT.¹

BY W. W. COBLENTZ AND W. B. EMERSON.

IN connection with the value of the mechanical equivalent of light as determined by using the visibility curve of the average eye, the radiation constants of a black body, and the brightness of a black body at various temperatures,² it was of interest to obtain further data upon this important constant. The present measurements were made upon a standardized vacuum tungsten lamp obtained from the Photometric Section of this Bureau. The lamp was operated (stationary) at 1.23 watts per candle, giving 26.5 c.p. or 26.5×10^{-8} lumens per cm^2 . The total radiation was 293.5×10^{-8} watt per cm^2 at a distance of 1 m. from this lamp.

The radiant luminous efficiency was determined by means of a linear thermopile (of bismuth and silver) and a luminosity screen having a transmission curve which coincides very closely with visibility curve of the average eye.³

¹ Abstract of a paper presented at the New York meeting of the Physical Society, February 16-17, 1917.

² PHYS. REV. (2), 9, p. 87, 1917.

³ The constituents of the solution are:

Cupric chloride ($\text{CuCl}_2 \cdot 2\text{H}_2\text{O}$).....	5.7 grams
Cobalt ammonium sulphate [$\text{Co}(\text{NH}_4)_2(\text{SO}_4)_2 \cdot 6\text{H}_2\text{O}$].....	1.2 "
Potassium chromate (K_2CrO_4).....	0.16 gram
Nitric acid (HNO_3), 1.3 c.c. sp. gr. 1.05.....	0.123 gram
Water (H_2O).....	100 mols (c.c.)

This solution is used in a glass cell 1 cm. in thickness backed by a cell of water 3 to 4 cm. in thickness.

The various corrections were of course applied. The weighted mean value of the luminous efficiency is $L_v = .0142$ per cent. for a "40-watt" vacuum tungsten lamp operated at 1.23 watts per candle.

The *mechanical equivalent of light* is obtained from a knowledge of the total radiation, the luminous efficiency, and the candlepower of the lamp used. The luminous equivalent is $(293.5 \times 10^{-6} \times .0142) = 4.16 \times 10^{-6}$ watt per cm^2 . Equating this to the candlepower data (26.5×10^{-4} lumens = 4.16×10^{-6} watt) gives a value of 1 *lumen* = .00157 *watt* of radiant energy of maximum visibility, which is in close agreement (3 per cent.) with the value given in the main part of the paper,¹ which gives further checks upon this value.

Our conclusions are therefore in agreement with Ives and Kingsbury² that the value of the mechanical equivalent of light is close to 1 *lumen* = .0016 *watt* of radiant energy of maximum visibility.

BUREAU OF STANDARDS,
WASHINGTON, D. C.,
January 24, 1917.

THE ELECTRO-MAGNETIC MASS OF THE PARSON MAGNETON.³

By C. DAVISSON.

EXPRESSIONS are obtained for the electro-magnetic mass of the Parson magneton for small velocities parallel and at right angles to its axis. These, in general, are not the same and vary with the velocity of the charge in the ring. It is shown, however, that when the velocity of the charge in the ring is equal to that of light the directional variation of the mass disappears. The masses parallel and at right angles to the axis are the same and given by

$$m = \frac{1}{c^2} \frac{e^2}{8\pi\rho} \bar{K},$$

where c = velocity of light,

e = charge of the magneton (Heaviside units),

ρ = radius of ring,

¹ PHYS. REV. (2), 9, p. 87, 1917.

² PHYS. REV. (2), 8, p. 254, 1916.

³ Abstract of a paper presented at the New York meeting of the Physical Society, February 16-17, 1917.

$$\bar{K} = \frac{2}{\pi} \int_0^{\pi/2} \frac{d\theta}{(1 - \bar{u} \sin^2 \theta)^{1/2}},$$

$$\bar{u} = 1 - \frac{r^2}{4\rho^2},$$

and

r = radius of cross-section of ring.

CARNEGIE INST. OF TECHNOLOGY,
PITTSBURGH.

INTENSITIES OF X-RAYS OF THE L SERIES AS A FUNCTION OF VOLTAGE.¹

BY DAVID L. WEBSTER AND HARRY CLARK.

IT was shown by one of us² that all of the lines of the K -series of the X-ray spectrum of rhodium appear only at voltages greater than a definite critical value. This is the quantum voltage corresponding to a wave-length equal to or slightly shorter than the shortest line of the series. Above this critical voltage all of the lines of the series were found to increase in the same ratio with increase of voltage. In this paper are presented the results of preliminary experiments of a similar nature on the L -series of platinum. The apparatus and methods of work were similar to those referred to above.

Friman's³ graphs of the square root of frequency against atomic number may be divided into two classes according to slope and curvature; thus suggesting that the L -series consists of two groups, L_1 , containing lines α_1 , α_2 , β_2 and β_3 , and L_2 , containing η , β_1 , β_3 , β_4 , γ_1 , γ_2 , γ_3 and γ_4 .

We have studied the intensities of α_1 , β_2 , β_3 , β_4 and γ_1 as functions of voltage and have found that α_1 and β_2 have the same critical voltage, 11.45 k.v. $\pm$.05. This is the quantum voltage for the line β_3 , the shortest line of the (supposed) series. β_2 , β_4 and γ_1 have the same critical voltage, 13.20 k.v. $\pm$.05, the quantum value for a wave-length between γ_1 and γ_2 . Accordingly γ_2 , γ_3 and γ_4 (not yet examined) belong to neither L_1 nor L_2 .

We have found also that the intensities of the lines of both series are given by

$$I = k(V - V_c)^{1/2},$$

where V_c is the critical voltage of the series to which the line belongs and k is constant for any one line. The relation between the values of k for various lines has not been determined. This formula holds also for the K -series lines of rhodium according to the experiments referred to above.

The value of k derived from our experiments is 6.53×10^{-27} erg. seconds.

HARVARD UNIVERSITY.

¹ Abstract of a paper presented at the New York meeting of the Physical Society, February 16-17, 1917.

² PHYS. REV., June, 1916.

³ Phil. Mag., Nov., 1916.

APPLICATION OF THE PHOTO-ELECTRIC CELL AS A PYRHELIOMETER.¹

BY W. W. COBLENTZ.

ABOUT 10 years ago Ångström² described a new method for studying solar radiation, based upon diffusion of the violet and ultra-violet rays by the atmosphere. He covered his pyrheliometer with a blue glass. In this manner he was able to study atmospheric diffusion of solar radiation independently of the water vapor present, and obtained some very interesting data on the variation of the solar constant.

In view of the fact that his instrument was lacking in sensitivity, a potassium photoelectric cell (made by Dr. Kunz) was tested in view of its high sensitivity in the blue-violet. It was found that the galvanometer deflection was not proportional to the stimulus; but for high intensities there were indications that the correction might be found quite uniform. A cell of improved construction gave direct proportionality.

The cell was found too sensitive (battery only 2 volts) to be used with a d'Arsonval galvanometer. Using a higher voltage and milliammeter, the current which passed through the cell, when exposed (through blue glass to reduce the intensity) to the sun was sufficient to damage the cell.

The experiments thus far indicate that by using a less sensitive photo-electric substance the application of this instrument as a pyrheliometer deserves further study.

BUREAU OF STANDARDS,
WASHINGTON, D. C.,
February 3, 1916.

ON THE INSTABILITY OF ELECTRIFIED LIQUID SURFACES.¹

BY JOHN ZELENY.

THE behavior of a liquid whose surface has become unstable owing to electrification has been studied by means of photographs taken with instantaneous illumination, and also from so-called moving pictures taken at the rate of 800 per second.

When the unstable condition is reached on an alcohol drop a fine thread of liquid about 0.004 mm. in diameter is pulled out of the surface at the rate of about 8 meters per second. The thread of liquid breaks up into minute droplets at the rate of about a million or more per second. These droplets spread out in their flight so as to fill a conical volume and the mist thus formed appears colored by transmitted and by reflected light.

The length of the threads obtained from an alcohol surface is usually under a millimeter but threads 15 mm. long have been obtained from a glycerine surface. With a surface of the order of one square millimeter only one thread

¹ Abstract of a paper presented at the New York meeting of the Physical Society, February 16-17, 1917.

² Ångström, *Nova Acta Reg. Soc. Sci. Upsala*, IV, Vol. 1, 1907.

is usually formed at any instant, but with a sufficiently high voltage several threads may be produced at once.

When highly electrified drops are made to issue from the end of a capillary tube the electrical field in the neighborhood undergoes rapid changes in direction and magnitude because of the flying material and the liquid masses assume some curious shapes which undergo modifications with extreme rapidity.

YALE UNIVERSITY.

BIO-PHYSICS.¹

BY WHEELER P. DAVEY.

THE application of chemistry to the study of biological problems has been of far-reaching importance. There is every reason to believe that a similar application of physics will be of like value. An outline of the present knowledge of the physics of the living cell will be given, together with some of the theory of the physical structure of cells. The effects of surface tension, adsorption, diffusion and osmosis upon the life of cells will be given. The relation of other branches of physics to biological problems—especially the study of medicine—will be given.

RESEARCH LABORATORY,
GENERAL ELECTRIC CO.

TUNGSTEN ELECTRODE LOSS IN THE HYDROGEN ARC.¹

BY G. M. J. MACKAY AND C. V. FERGUSON.

DUFFIELD² has made a great many determinations of the loss in weight of the electrodes of the carbon arc under varying conditions, and has concluded that the loss of an atom of carbon from the cathode is accompanied by the simultaneous emission of four electrons. From our experience with arcs between tungsten electrodes in various gases we are led to believe that there is no such amount of tungsten lost relative to the transfer of current, and that the phenomena involved do not necessarily require the emission of anything but electrons from the negative electrode.

A quantitative experiment to illustrate the magnitude of the different factors was performed by weighing the tungsten buttons used as electrodes of an arc in hydrogen at approximately 300 mm. pressure before and after operation.

The apparatus consisted of a spherical glass bulb about 12 cm. in diameter. Tungsten leads were sealed in, tipped with removable tungsten buttons of approximately equal size and about 2 mm. apart. The arc was operated from a 500-volt D.C. machine, and was started by heating the electrodes by an auxiliary high potential transformer. The arc was run continuously for 1,265 minutes, the volt ampere characteristics being

¹ Abstract of a paper presented at the New York meeting of the Physical Society, February 16-17, 1917.

² Proc. Roy. Soc., 92, pp. 122-143, 1915.

Initial.....201 volts, 1.40 amperes,
 Final.....245 volts, 1.40 amperes.

The dimensions of the electrodes were

	Positive.	Negative.
Length.....	3.17 mm.	3.81 mm.
Diameter.....	3.17 "	2.03 "
Initial weight.....	0.4435 g.	0.3785 g.
Final weight.....	0.4085 g.	0.4026 g.
Difference.....	-0.0350 g.	+0.0241 g.

The cathode had thus gained 75 per cent. of the material lost by the anode. This interchange could not be accounted for by convection currents since the arc being horizontal, the tendency would be for the metal vapor to go upwards to the top of the bulb. Apparently, then, the tungsten particles given off by the anode become positively charged and migrate towards the cathode.

The temperature of the electrodes was measured by an optical pyrometer of the Holborn-Kurlbaum type as follows:

	Positive.	Negative.
Back shoulder.....	2681° K.	2469° K.
Middle of side.....	2734	2681
Front shoulder.....	3056	3200
Face.....	3151	3112

Assuming the whole of the weight lost to have come from the face of the anode, the loss in grams per square centimeter per second would be $7.9 \cdot 10^{-6}$ which corresponds to the rate of evaporation in vacuum at 3040° K., less than the temperature found. At 3151° K. the rate would be $28 \cdot 10^{-6}$.

It might be expected that the normal rate of evaporation would be reduced by the gas present as in the nitrogen-filled lamp. There is evidence, however, that hydrogen lowers the rate of evaporation to a lesser extent than nitrogen which at atmospheric pressure reduces the rate to about 1 per cent. of the rate in vacuum. It is particularly difficult, also, to entirely eliminate water vapor from hydrogen, so that some loss of tungsten may be attributed to attack from this source.

It would therefore appear that the following conclusions may be drawn:

1. The cathode though at a very high temperature gains more weight than it loses.
2. The anode loses weight at a rate not appreciably greater than would occur from normal evaporation of the metal due to its temperature alone.
3. The metal leaving the anode becomes positively charged and migrates to the cathode. The number of atoms carried for each unit charge is about 10^{-4} .

RESEARCH LABORATORY,
 GENERAL ELECTRIC COMPANY,
 SCHENECTADY, N. Y.

A NOTE ON THE RELATION BETWEEN IONIZING POTENTIALS AND ATOMIC CHARGES.¹

BY FERNANDO SANFORD.

THE ionizing potentials necessary to produce the single-line spectra of a number of elements have been determined. It was shown by the present writer at the last meeting of the Pacific Physical Society that on the assumption that the electrons which radiate light and X-rays are moving in approximately circular orbits about a positive central charge, the magnitude of this central charge may be computed from the formula $Q = 2.95 \times 10^{-12} / \sqrt{\lambda}$, where λ is the wave-length of the radiation emitted. It is accordingly possible to calculate the central positive charges for the electrons which give the single-line spectra and to compare these charges with the potential necessary to set up this radiation.

In the following table are given the wave-lengths of the single-line spectra

I.	II.	III.	IV.	V.
Element.	λ .	Q .	V Obs.	V Calc.
Mercury.....	2537	5.86	4.89	4.81
Zinc.....	3076	5.32	4.04	4.06
Cadmium.....	3260	5.17	3.81	3.85
Magnesium.....	2852	5.53	4.35	4.35
Calcium.....	4227	4.54	2.94	2.97
Strontium.....	4608	4.35	2.69	2.70
Barium.....	5536	3.96	2.24	2.16

as collected by Langmuir in a paper in Trans. Am. Electro-Chem. Soc., XXXIX., 152, with their central positive charges calculated from the above formula, while in the fourth and fifth columns are given the ionizing potentials in volts as determined experimentally and as calculated from the equation $V = (Q - 2.42) 1.4$ where V is the ionizing potential in volts and Q is the positive charge in electrostatic units multiplied by 10^{10} .

It will be seen from the above table that when the ionizing potentials are plotted against the positive charges of the respective atoms a straight line relation will be found.

This is even more strictly true if the theoretical ionizing potentials of the single line spectra of the alkali metals as given by Langmuir in the same paper are plotted against their respective central charges.

A STUDY OF THE STARK EFFECT.¹

BY J. A. ANDERSON.

FROM the standpoint of astrophysics it is highly important to be able to examine for Stark effect the lines of such elements as Fe, Ni, Cr, Ti, V, etc., elements which are specially prominent and important in the solar spec-

¹ Abstract of a paper presented at the Stanford University meeting of the Physical Society, April 7, 1917.

trum. Such work requires fairly high dispersion, for the spectra are all rich in lines, and consequently the source must be reasonably bright. An apparatus which works quite well has been constructed recently, and with it some results of interest have already been obtained. The essential feature of the apparatus is the cathode, which is a metal cylinder 12 mm. in diameter and about 15 mm. long, surrounded by a silica tube which extends about 2 cm. beyond the cathode. The luminous discharge is thus confined within the silica tube and is observed through a slot extending from the end of this tube to the surface of the cathode. This cathode and a suitable anode are inclosed in a bell-jar which is exhausted until the dark-space has a length of 5 to 8 mm. Direct current of from 10 to 100 milliamperes at 6,600 volts supplied by a set of high-potential generators is used, the maximum field strength being about 16,000 volts per cm. When the current passes, the cathode heats up rapidly and can easily be melted; in general, however, the current is so regulated that the temperature of the cathode is between 600° and 1000° C. In the region λ 3800 to λ 4600, most of the lines of the metals enumerated above can be recorded in an exposure of 15 minutes, using a dispersion of about 4 Ångström units per mm.

The majority of the lines of these metals show no appreciable Stark effect. Exceptions are:

1. The lines $\lambda\lambda$ 4098, 4111, 4129, 5275, 5298, and 5329 of chromium, which break up into components in a complicated way, the maximum separation being about one fourth that given by H δ .

2. A group of about a dozen lines of nickel in the region λ 3940– λ 4027; these do not break up into components, but the wave-length is changed; *i. e.*, each line shifts bodily either to the red or to the violet.

3. Five faint iron lines in the λ 4100 region, which behave like the nickel lines.

An effect similar to that for nickel has been found for 32 lines of the secondary spectrum of hydrogen in the region λ 4370– λ 4565.

LEADING FEATURES OF THE ELECTRIC FURNACE SPECTRA OF CALCIUM, STRONTIUM, BARIUM, AND MAGNESIUM.¹

BY ARTHUR S. KING.

THE furnace spectra of calcium, strontium, barium, and magnesium have been studied by the method already employed for several other elements, the spectra produced at temperatures ranging from 1650° to 2500° being photographed for the interval λ 2600 to λ 7200. Material is thus obtained for a classification of lines according to the temperature at which they appear and the rate of increase in intensity with rise of temperature. Certain lines can be selected which are useful as temperature indicators in the spectra of other sources, lines strong at the lowest furnace temperature being of special interest. The fact that a considerable proportion of the lines in these spectra are known

¹ Abstract of a paper presented at the Stanford University meeting of the Physical Society, April 7, 1917.

to be connected by series relations furnishes an opportunity to note the behavior of series lines at various temperatures. While the members of the same series are affected similarly, large differences in behavior are found between separate series. A triplet series of calcium was found which is favorably situated to test the change, among the members of the same series, of relative intensity with wave-length at different temperatures. As the temperature rose, the violet members of the series were clearly strengthened to a greater degree than those of longer wave-length. This agrees with a previous observation for the principal series of cesium in the furnace, and indicates a shift of the maximum with change of temperature similar to that shown in the spectrum of an incandescent solid. The close resemblance in structure between the spectra of calcium, strontium, and barium permits a comparison of the homologous lines of the three elements. Such lines are found to be affected in the same way in the furnace spectra.

From the point of view of wave-length measurement, the elimination of dissymmetry in line structure and the resolution in the furnace of groups of lines which form diffuse blends in the arc in air are of especial interest in these spectra.

ELIMINATION OF POLE EFFECT FROM SECONDARY STANDARDS OF WAVE-LENGTH.¹

BY HAROLD D. BABCOCK AND CHARLES E. ST. JOHN.

THE arc recommended by the International Committee on Wave-Lengths has been tested for pole effect. This disturbing influence is found to persist even at the center of the arc to the amount of $0.006 \text{ \AA}$. The 6 mm., 6 ampere Pfund arc is also affected to about the same extent at its center, but by making it 12 mm. long and using 5 amperes or less, a central zone is found free from pole effect. That the values thus obtained are the fundamental wave lengths of the lines, is confirmed by observations upon arcs having negative poles of various constitution, operated so as to be free from pole effect. The importance of using fundamental wave-lengths as standards is pointed out.

AN OPTICAL TROUGH FOR HIGH-SCHOOL AND COLLEGE DEMONSTRATIONS.¹

BY RALPH S. MINOR.

THE optical trough is rectangular in form, about 3 inches wide and $3\frac{1}{2}$ inches high and 26 inches long in size. The bottom is of metal and the sides and one end are plate glass. The other end carries a planomeniscus lens of less than 2 mm. thickness. The trough is suspended on a special tripod stand and the height may be regulated by means of an adjustable collar so that the trough may be rotated about a vertical axis after the height has been fixed.

¹ Abstract of a paper presented at the Stanford University meeting of the Physical Society, April 7, 1917.

This optical trough with its various accessories is designed to serve three purposes: First, as a piece of demonstration apparatus for a first course in physics, second, for the demonstration of a variety of phenomena which are beyond the scope of the first course; third, as a piece of laboratory apparatus for a course in college physics. The optical trough supplements the Hartl optical disk and has the advantage of using the entire wave front and not merely a section of it. The visibility of a beam of light passing through the trough is slightly greater than that produced by the Hartl disk with the same illumination.

The accessories include: plane and spherical mirrors, a plano-convex glass lens, double convex water lens, double concave air lens, trial case lenses and cylinders for studying the refraction of the eye, image and object screens for studying refraction at a single spherical surface, and models of both Huyghens and Ramsden eye pieces which may be used either in air or in water.

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NOTE ON THE USE OF THE ELECTROMETER FOR MEASURING THE CONDUCTIVITY OF GASES.¹

BY J. A. GILBREATH.

IN carrying out experiments on the conductivity of gases, a tube containing two parallel plate electrodes separated by a few millimeters is sometimes used. One of the plates is connected to an electrometer and the other to one terminal of a storage battery. H. S. Anderson used such a tube in working with the photoelectric effect and reports finding a current through the tube in a direction opposed to the E.M.F. of the battery.² This effect can be explained on the basis of induction. The two parallel plates constitute a condenser. If the E.M.F. of the battery decreases, the potential of the connected plate falls and this releases a part of the bound charge on the second plate. Thus, if one plate is connected to the positive pole of the battery it would have a positive charge and the second plate would have an equal negative charge. If the potential of the positive plate diminish it is unable to hold so large a bound charge on the second plate. Part of the negative charge from the second plate then flows into the electrometer giving the appearance of a current opposed to the E.M.F. of the battery.

If a battery is furnishing current, its E.M.F. is very apt to decrease. In the article referred to the cells were furnishing current through a water resistance which constituted a potentiometer system by which any desired E.M.F. could be applied to the tube. Under these conditions it is not strange to find a sensitive electrometer showing a negative "current" when a positive potential is applied.

¹ Abstract of a paper presented at the Stanford University meeting of the Physical Society, April 7, 1917.

² *Phys. Rev.*, Vol. XXXV., October, 1912.

In using a tube of this kind with an electrometer whose sensibility was 4,000 divisions per volt the writer found that a potentiometer arrangement for varying the voltage was not satisfactory. The cells continually ran down, showing the negative effect. It was found more satisfactory to vary the voltage by varying the number of cells used. Before connecting the battery to the tube a small current was drawn through a high resistance for two or three minutes then the battery was allowed to recover for two or three minutes, and then connected to the tube. The negative effect then entirely disappeared and the electrometer showed no deflection.

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ERRATUM.

Vol. IX , April, 1917, abstract by I. G. Priest entitled " A Proposed Method for the Photometry of Lights of Different Colors, II.," p. 341, fourth line from bottom, $V^{\wedge}$ should read V_{λ} .

On page 344,

"Table of Values of Ratio $\frac{\text{Candle power at 64.91 volts}}{\text{Candle power at 99.86 volts}}$ "

should be heading for table on page 345.

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